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AN

INTRODUCTION

TO

NATURAL PHILOSOPHY;

DESIGNED AS A

TEXT-BOOK

FOR THE USE OF

STUDENTS IN COLLEGE.

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PREFACE.

THE present revision of Olmsted's Natural Philosophy has been made in accordance with the plan of the former editions, the object being to produce a text-book in physics adapted to the requirements of a college course; it is believed that the work as now presented includes all that is necessary in this department of a liberal education.

While students in technical schools may need special treatises upon various subdivisions of physics, the general course here offered will prove a sufficient preparation for such technical work.

The rapid growth of science has made it necessary to add materially to the size of the volume, although many facts of interest and descriptions of practical applications have been necessarily omitted that the subjects treated of might be presented within reasonable limits. The aim of the editor has been to present the *principles* of physics, and not to compile an exhaustive treatise. For this reason, also, only such new engravings have been added as were necessary to make clear the meaning of the text and to impress the facts upon the memory; the professor in charge of the department is thus left free to pursue such course of experimental illustration as his own experience and the facilities at his command render most desirable.

The Appendix (Applications of the Calculus) having found favor with many, is retained unchanged.

Part I., Mechanics, has been but slightly altered, and therefore the indebtedness to Professor Joseph Ficklin, expressed by the author of the revision of 1870, is properly acknowledged here.

BROOKLYN, N. Y., August, 1882.

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NATURAL PHILOSOPHY.

INTRODUCTION.

Art. 1. Classification of Physical Sciences.—The material world consists of two parts—the *organized*, including the animal and vegetable kingdoms; and the *unorganized*, which comprehends the remainder. Organized matter is treated of in *Physiology*, and in those branches of science usually called *Natural History*. Unorganized matter forms the subject of *Natural Philosophy* and *Chemistry*. *Chemistry* considers the internal constitution of bodies, and the relations of their smallest parts to each other. *Natural Philosophy* deals principally with the external relations of bodies and their action upon one another. If, however, the bodies are so large as to constitute *worlds*, of which the earth itself is one, this science takes the name of *Astronomy*.

The word *Physics* is much used to include both Natural Philosophy and Chemistry; but sometimes it is applied to the branches of Natural Philosophy, except Mechanics. According to the latter use of the word, Natural Philosophy is divided into two general subjects, Mechanics and Physics.

2. Definitions relating to Matter.—

A *body* is a separate portion of matter, whether large or small.

An *atom* is a portion of matter so small as to be indivisible.

A *particle* denotes the smallest portion which can result from division by mechanical means, and consists of many atoms united together.

A *Molecule* is the smallest portion of any substance which can exist in a free state, and is made up of atoms.

Mass is the quantity of matter in a body, and is usually measured by its weight.

Volume signifies the space occupied by a body.

Density expresses the relative mass contained within a given

volume. Thus, if one body has twice as great a mass within a certain volume as another has, it is said to have twice the density.

Pores are the minute portions of space within the volume of a body, which are not filled by the material of that body. All matter is porous, some kinds in a greater and some in a less degree.

Force is the name of any cause which gives, or tends to give, motion to matter, or which changes, or tends to change, motion already existing.

3. Properties of Matter.—

(1.) *Extension.*—Every portion of matter, however small, has length, breadth, and thickness, and thus occupies space. This is its extension.

(2.) *Impenetrability.*—While matter occupies space, it excludes all other matter from it, so that no two atoms can be in exactly the same place at the same time. This property is called impenetrability.

The two foregoing are often called *essential* properties, because we cannot conceive matter to exist without them.

(3.) *Divisibility.*—Matter is *divisible* beyond any known limits. After being divided, as far as possible, into particles by mechanical methods, it may be still further reduced by chemical action to atoms, which are too small to be in any way recognized by the senses.

(4.) *Compressibility.*—Since pores exist in all matter, it may be compressed into a smaller volume. Hence all matter is *compressible*, though in very different degrees.

(5.) *Elasticity.*—After a body has suffered compression, it shows, in some degree at least, a tendency to restore itself to its former volume. This property is called elasticity. A body is said to be *perfectly elastic* when the force by which it recovers its size is equal to that by which it was before compressed. The word elasticity is used generally in a wider sense than is given in the above definition, namely, the tendency which a body has to recover its original *form*, whatever change of form it may have previously received. Thus, if a body is stretched, bent, twisted, or distorted in any other way, it is called *elastic,* if it tends to resume its form as soon as the force which altered it has ceased. *Torsion* is the name of the elastic force which tends to untwist a thread or wire when it has been twisted.

(6.) *Attraction.*—This is the general name used to express the universal tendency of one portion of matter towards another. It receives different names, according to the circumstances in which it acts. The attraction which binds together atoms of different kinds, so as to form a new substance, is called *affinity,* and is

discussed in Chemistry; that which unites particles, whether simple or compound, so as to form a body, is called *cohesion;* the clinging of two kinds of matter to each other, without forming a new substance, is called *adhesion;* and the tendency manifested by masses of matter toward each other, when at sensible distances, is called *gravity.*

(7.) *Inertia.*—This is also a universal property of matter, and signifies its tendency to continue in its present condition as to motion or rest. If at rest, it cannot move itself; if in motion, it cannot stop itself or change its motion, either in respect to direction or velocity.

4. **Branches of Natural Philosophy.**—Natural Philosophy is generally divided into *Mechanics, Hydrostatics, Pneumatics, Sound, Magnetism, Electricity, Heat,* and *Light.*

Mechanics treats of the motion and equilibrium of bodies, caused by the application of force. Since there are three conditions of matter, solid, liquid, and gaseous, it is convenient to divide the general subject of Mechanics into three branches.

1st. The mechanics of solids, also called *Mechanics.*

2d. The mechanics of liquids, called *Hydrostatics.*

3d. The mechanics of gases, called *Pneumatics.*

All the other branches of Natural Philosophy (often called Physics) treat of various phenomena caused by *minute vibrations* in the particles of matter. These vibrations are excited in different ways, and when transmitted to us, affect one or more of our senses. Thus, *sound* consists of such vibrations as affect the sense of hearing; and *light* is another mode of vibration, that affects only the sense of vision.

It was formerly customary to regard magnetism, electricity, heat, and light, as so many kinds of *imponderable* matter, that is, matter having no sensible weight, and thus distinguished from solids, liquids, and gases, which are the different forms of *ponderable* matter. But it is now known that when forces are applied to matter, they not only produce the visible forms of motion, but may be made to develop either sound, magnetism, electricity, heat, or light; and that most of these modes of motion may be transformed into others, and each may be made a measure of the force which is employed to produce it.

PART I.

MECHANICS.

CHAPTER I.

MOTION AND FORCE.

5. Classification of Motions.—Motion is change of place, and is either *uniform* or *variable*. In *uniform* motion equal spaces are passed over in equal times, however small the times may be. In *variable* motion the spaces described in equal times are unequal. Such motion may be either *accelerated* or *retarded*. In *accelerated* motion the spaces described in equal times become continually greater; in *retarded* motion they become continually less. Motion is said to be *uniformly accelerated* if the increments of space in equal times (however small) are equal; and *uniformly retarded* if the decrements are equal.

Velocity is the space described in the *unit* of time. In Mechanics, one second is much used as the unit of time, and one foot as the unit of space; hence, velocity is the number of feet described in one second.

6. Uniform Motion.—When motion is uniform, the number of feet described in one second, multiplied by the number of seconds, obviously gives the whole space. Let $s =$ space, $t =$ time, and $v =$ velocity; then $s = t\,v$; $\therefore t = \frac{s}{v}$, and $v = \frac{s}{t}$. If this space is compared with another, s', described in the time t', with the velocity v', then $s : s' :: t\,v : t'\,v'$; or briefly, in the form of a variation, $s \propto t\,v$. In like manner $t \propto \frac{s}{v}$, and $v \propto \frac{s}{t}$.

If two bodies, moving uniformly, describe *equal* spaces, then $s = s'$; $\therefore t\,v = t'\,v'$; $\therefore t : t' :: v' : v$. That is, in order that two bodies may describe equal spaces, their velocities must vary inversely as the times during which they move.

7. Questions on Uniform Motion.—

1. A ball was rolled on the ice with a velocity of 78 feet per second, and moved uniformly 21 seconds; what *space* did it describe? *Ans.* 1638 feet.

2. A steamboat moved uniformly across a lake 17 miles wide, at the rate of 20 feet per second; what *time* was occupied in crossing? *Ans.* 1*h.* 14*m.* 48*s.*

3. On the supposition that the earth describes an orbit of 600 millions of miles in 365¼ days, with what *velocity* does it move per second? *Ans.* 19 miles, nearly.

4. Three planets describe orbits which are to each other as 15, 19, and 12, in times which are as 7, 3, and 5; what are their *relative velocities?* *Ans.* 225, 665, and 252.

8. Momentum.—The product of the mass of a body and its velocity is called *momentum.* Thus let $m =$ momentum, $q =$ the mass, and $v =$ the velocity, and we have $m = q\,v$, $q = \frac{m}{v}$, and $v = \frac{m}{q}$.

If the momentum of one body equals that of another, then, since $m = m'$, $q\,v = q'\,v'$, $\therefore q : q' :: v' : v$. That is, in order that the momenta of two bodies should be equal, their masses must vary inversely as their velocities.

Since there are two elements entering into the momentum of a body—namely, its *mass,* usually expressed in pounds, and its *velocity,* expressed in feet per second—therefore momentum cannot be measured either in pounds or in feet, being in nature unlike either. The word *foot-pound* is employed for the unit of momentum whenever the unit of mass is a pound and the unit of velocity is a foot per second.

9. Questions on Momentum.—

1. A ship weighing 336,000 lbs. is dashed against the rocks in a storm, with a velocity of 16 miles per hour; with what momentum did she strike? *Ans.* 7,884,800 foot-pounds.

2. A ball weighing 1 oz. is fired into a log weighing 53 lbs., suspended so as to move freely, and imparts a velocity of 2 ft. per second. Assuming that the log and ball have a momentum equal to the previous momentum of the ball alone, required the velocity of the ball. *Ans.* 1,698 ft. per second.

3. Suppose a comet, whose velocity is 1,000,000 miles per hour, has the same momentum as the earth, whose velocity is 19 miles per second; what is the ratio of their masses? *Ans.* 1 : 14.6.

4. Two railway cars have their quantities of matter as 7 to 3, and their momenta as 8 to 5; what are their relative velocities? *Ans.* As 24 to 35, or nearly 5 to 7.

5. The momentum of a cannon-ball was 434 foot-pounds; what must be the velocity of a half-ounce bullet, in order to have the same momentum? *Ans.* 13,888 feet.

10. Classification of Forces.—The principal forces in nature are the following:

1. *Attraction* in its several forms. *Cohesion* and *chemical affinity* are the forces which bind together the particles and atoms of bodies, and *gravity* is that which everywhere near the earth causes bodies to fall toward it, or to press upon it.

2. *Elasticity.*—This is a force which, in many kinds and conditions of matter, tends to repel the particles from each other.

The forces, whether attraction or repulsion, which exist among the atoms or molecules of a body, are called *molecular* forces.

3. *Muscular force.*—All living beings are endowed with this force, by which they put in motion bodies around them, and by acting upon other bodies, are enabled also to move themselves from place to place.

4. *Matter in motion.*—If a body which some force has put in motion impinges on another body, it imparts motion to it, and is therefore itself a force. This is true not only of ordinary visible motions, but of those small and often invisible vibrations, which manifest themselves as sound, heat, &c. Gravity, or any other force, may cause heat, and heat may cause light and electricity. Thus, any form of motion is a force, and it can be employed to produce other forms.

11. Impulsive and Continued Forces and their Effects.—An *impulsive* force is one which has no sensible continuance, as the blow of a hammer. A *continued* force is one which acts during a perceptible length of time. Continued forces are subdivided into *constant* and *variable.* A *constant* force has the same intensity during the whole time of its action; a *variable* force is one whose intensity changes.

Keeping in mind the property of inertia, we associate different kinds of motion with the forces which produce them, as follows:

1. An *impulsive* force causes *uniform* motion.
2. A *continued* force, *accelerated* motion.
3. A *constant* force, *uniformly accelerated* motion.
4. A *variable* force, *unequally accelerated* motion.

If the force is applied in a direction opposite to that in which the body has a previous uniform motion, the connection is the following:

5. An *impulsive* force causes *uniform* motion, or *rest.*
6. A *continued* force, *retarded* motion.
7. A *constant* force, *uniformly retarded* motion.
8. A *variable* force, *unequally retarded* motion.

In cases 1 and 5, it is obvious that, the impulse being given,

the body is left to itself, and cannot change the state of motion or rest impressed on it.

In 2, 3, and 4, it must be considered that the force at each instant adds a new *increment* to the uniform motion which the body would have had if the force had ceased ; and if the force is constant, those increments are equal ; if variable, they are unequal.

In 6, 7, and 8, the same statements may be made in regard to decrements. It is also plain that in these three last cases, if the force continues to act indefinitely, the motion will be retarded until the body comes to a state of momentary rest, and then is accelerated in the direction of the force.

12. Measure of Force.—The intensity of an *impulsive* force is measured by the *momentum* which it will produce or destroy ; that is, $f \propto m$. But $m \propto q\,v$; $\therefore f \propto q\,v$. Hence, if q is constant, $f \propto v$. If, then, an impulse is applied to a given *mass*, the intensity of that impulse is measured by the *velocity* which it imparts or destroys.

Thus, if a force gives to a mass q a velocity v, the same force would give to one-half the mass, or $\frac{q}{2}$ a velocity $2\,v$; for $f = q\,v$, and also $f = \frac{q}{2} \times 2\,v$. So also if a force f gives a velocity v to a mass q, a force $2\,f$ would be required to give to the same mass a velocity $2\,v$.

But in the case of a *constant* force, the momentum depends not only on the intensity of the force, but on the time during which it is applied ; that is, $f\,t \propto m$, and $f \propto \frac{m}{t}$. If the mass of the body is given, then, as in the case of an impulsive force, q being constant, $f\,t \propto v$, and $f \propto \frac{v}{t}$.

A constant force produces uniformly accelerated motion, since the increase of velocity during any unit of time is constant. The unit of time usually chosen is one second and the increase in velocity per second is called the *acceleration* due to the force. If a force f produces an acceleration of 20 ft. per second, and another force f' produces an acceleration of 40 ft. per second, the mass being supposed the same in both cases, the force $f' = 2\,f$; or if a force f acting upon a body at rest, gradually increases its velocity from 0 to 20 ft. at the end of the second, and if f' will increase the velocity of the *same* body from 0 to 40 ft. at the end of the second, then $f' = 2\,f$.

If a force f gives to a mass q an acceleration of 50 ft., and

force f' gives to mass $3\,q$ an acceleration 50 ft., then $f' = 3\,f$. If a force f gives to a mass q an acceleration 30 ft., and f' gives to $2\,q$ an acceleration 60 ft., then $f' = 4\,f$.

To express the measure of a *variable* force, let t be a constant and infinitely small portion of time; then the force varies as the mass multiplied by the increment of velocity imparted in that time divided by the time.

13. The Three Laws of Motion.—All the phenomena of motion in Mechanics and Astronomy are found to be in accordance with three first principles, which Newton announced in his Principia, and which are to be regarded as forming the basis of mechanical science. They may be named and defined as follows:

1. The law of *inertia.*—A body at rest tends to remain at rest; and a body in motion tends to move forever, in a straight line, and uniformly.

2. The law of the *coexistence of motions.*—If several motions are communicated to a body, it will ultimately be in the same position, whether those motions are simultaneous or successive.

3. The law of *action and reaction.*—If any kind of action takes place between two bodies, it produces equal momenta in opposite directions; or, every action is accompanied by an equal and opposite reaction.

The truth of these laws cannot be established, except approximately, by direct experiments, because gravity, friction, and the resistance of air, interfere more or less with every possible experiment. They are to be learned rather by a careful study of the phenomena of motion in general. We see an approximation to the *first* law, in rolling a ball on a horizontal surface; first, on the earth, then on a floor, and again on smooth ice, the motion approaching toward uniformity as obstructions are diminished, and gravity producing no direct effect, because acting at right angles to the line of motion. The discussion of the *second* law is reserved for Chapter III. The *third* law is illustrated by a variety of cases in collision, attraction, and repulsion. Suppose that a body A, being in motion, strikes directly against B, which is at rest; it is found that B acquires a certain momentum, and that A *loses* (that is, *acquires* in an opposite direction) an equal amount. The same is true if B is in motion, and A either overtakes or meets it. In the collision of two railroad trains, it is immaterial as to the effects which they will respectively suffer, whether each is moving towards the other, or whether one is at rest, provided that in the latter case the moving train has a momentum equal to the momenta of the two trains in the former case. When a magnet attracts a piece of iron, each moves towards the other with the

same momentum. A spring between two bodies A and B drives A from B with as much momentum as B from A; and the sudden expansion of burning gunpowder, which propels the balls when a broadside is fired, causes an equal amount of motion of the ship in the opposite direction.

14. Force of Gravity.—Every mass of matter near the earth, when free to move, pursues a straight line towards its centre. The force by which this motion is produced is called *gravity;* either the gravity of the body or the gravity of the earth; for the attraction is mutual and equal, in accordance with the third law of motion. It is easy to understand why a small mass should attract a large one, as much as the large mass attracts the small one. Let A consist of *one* atom of matter, and B, at any distance from it, consist of *ten* atoms. If it be admitted that A attracts *one* atom of B as much as *that one* atom attracts A, then the above conclusion follows. For A attracts *each* of the ten atoms of B as much as *each* of the same ten attracts A; so that A exerts ten units of attraction on B, while B exerts ten units of attraction on A. The same reasoning obviously applies to the earth in relation to the small bodies on its surface.

15. Relation of Gravity and Mass.—At the same distance from the centre of the earth, *gravity varies as the mass.* This is because it operates equally on every atom of a body; hence the greater the number of atoms in a body, the greater in the same ratio is the attraction exerted upon it. That gravity varies as the mass is also proved from the observed fact, that in a vacuum it gives the same velocity, in the same time, to every mass, however great or small, and of whatever species of matter. For a constant force, acting for a given time, is measured by the *momentum* which it produces (Art. 12), and that momentum, if the velocity is the same, varies as the mass; therefore the force also varies as the mass to which it imparts the given velocity.

If a body is not free to move, its tendency towards the earth causes *pressure;* and the measure of this pressure is called the *weight* of the body. Weight is usually employed as a measure of the mass in bodies. The foregoing relations are embodied in the following expressions: $g \propto q$; and $w \propto q$.

16. Relation of Gravity and Distance.—*At different distances above the earth's surface, gravity varies inversely as the square of the distance from the centre.*

The demonstration of this proposition is reserved for astronomy, where it is shown by the movements of the bodies in the solar system that this law applies to them all.

The moon is 60 times as far from the earth's centre as the distance from that centre to the surface: therefore the attraction of the earth upon the particles of the moon is 3600 times less than upon particles at the surface of the earth. At the height of 4000 miles above the earth, gravity is four times less than at the surface. But the heights at which experiments are commonly made upon the weights of bodies bear so small a ratio to the radius of the earth, that this variation is commonly imperceptible. At the height of *half a mile*, the diminution does not amount to more than about $\frac{1}{4000}$th part of the weight at the surface. For, let r = the radius of the earth = 4000 miles, nearly; and let x be the height of the body, w its weight at the earth's surface, and w' its weight at the height x. Then,

$$w : w' :: (r+x)^2 : r^2 :: r^2+2rx+x^2 : r^2.$$

$$w : w-w' :: r^2+2rx+x^2 : 2rx+x^2 \therefore w-w' = \frac{w(2rx+x^2)}{r^2+2rx+x^2} \, (A).$$

But when x is a small fraction of r, x^2 may be neglected, and the formula becomes $w - w' = \dfrac{w \times 2x}{r+2x}$ . . . . (B).

Let x be *half a mile;* then $\dfrac{w \times 1}{4000+1} = \frac{1}{4001}$th part of the whole weight; or, a body would weigh so much less at the height of half a mile than at the surface of the earth. But if the height were as great as 100 miles above the earth, the loss should be calculated by formula (A), since the other would give a result too small by one per cent. or more, according to the height.

What loss of weight would a body sustain by being elevated 500 miles above the earth? *Ans.* $\frac{17}{81}$, or more than $\frac{1}{5}$ of its weight.

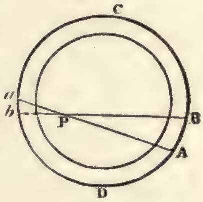
The relation of gravity to distance is expressed by the formula $g \propto \dfrac{1}{d^2}$; and as $g \propto q$ also, it varies as the product of the two; that is, $g \propto \dfrac{q}{d^2}$; or *gravity towards the earth varies as the mass of the body directly, and as the square of the distance from the earth's centre inversely.*

17. Gravity within a Hollow Sphere.—A particle situated *within a spherical shell* of uniform density, is equally attracted in all directions, and *remains at rest.* This is true, because, in every direction from the particle, the mass varies at the same rate as the square of the distance, so that attraction increases for one reason, as much as it diminishes for the other; which is proved as follows:

Let the particle P (Fig. 1) be at any point within the spherical shell $A\ B\ C\ D$. Let two opposite cones of revolution, of very small angle, have their vertices at P, and suppose the figure to be

a section through the centre of the sphere and the axis of the cones. Then $A\,B$ and $a\,b$ will be the major axes of the small ellipses, which are the bases of the cones, and which may be considered as plane figures. By geometry, $A\,P : P\,B :: P\,b : P\,a$; and the angles at P being equal, the triangles are similar; hence the angles B and a are equal. Therefore, the bases of the cones are similar ellipses, being sections of similar cones, equally inclined to the sides. By similar triangles, $\overline{A\,P}^2 : \overline{P\,b}^2 :: \overline{A\,B}^2 : \overline{a\,b}^2$. Let q and q' represent the masses of the thin laminæ which form the bases; then, since similar ellipses are to each other as the squares of their major axes, we have

Fig. 1.



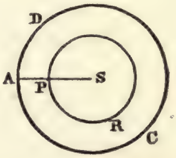
$$q : q' :: \overline{A\,P}^2 : \overline{P\,b}^2, \text{ or } \frac{q}{\overline{A\,P}^2} = \frac{q'}{\overline{P\,b}^2}.$$

But $\frac{q}{\overline{A\,P}^2}$ and $\frac{q'}{\overline{P\,b}^2}$ represent the attractions of the bases respectively on the particle (Art. 16); and since these are equal, the particle is equally attracted by all the opposite parts of the spherical shell.

18. Gravity within a Solid Sphere.—Within a *solid sphere* of uniform density, weight varies directly *as the distance from the centre.*

Let a particle P (Fig. 2) be within the solid sphere $A\,D\,C$; and call its distance from the centre d. Now, by the preceding article the shell exterior to it, $A\,D\,R$, exerts no influence upon it, and it is attracted only by the sphere $P\,R\,S$. Let q represent the quantity of this sphere; then gravity varies as $\frac{q}{d^2}$. But $q \propto d^3$; $\therefore g \propto \frac{d^3}{d^2} \propto d$. Hence, in the earth (if it be supposed spherical and uniformly dense, though it is neither exactly), a body at the depth of 1000 miles weighs *three-fourths* as much as at the surface, and at 2000 miles it weighs half as much, while at the centre it weighs nothing.

Fig. 2.



Comparing this proposition with Art. 16, we learn that just at the surface of the earth a body weighs more than at any other place without or within. Within, the weight diminishes *nearly* as the distance from the centre diminishes; without, it diminishes as the square of the distance from the centre increases.

At the surface of spheres having the same density, weight varies as the radius of the sphere. Let r be the radius of the sphere, and

q its mass; then, since $g \propto \frac{q}{r^2}$, in this case it varies as $\frac{r^3}{r^2} \propto r$. Therefore, if two planets have equal densities, the weight of bodies upon them is as their radii or their diameters. If a ball two feet in diameter has the same density as the earth, a particle of dust at its surface is attracted by it nearly 21 millions of times less than it is by the earth.

19. Questions for Practice.—

1. How much weight would a rock that weighs ten tons (22,400 lbs.) at the level of the sea, lose if elevated to the top of a mountain five miles high? *Ans.* 55.8952 lbs.

2. If the earth were a hollow sphere, and if, through a hole bored through the centre, a man were let down by a rope, would the force required to support him be increased or diminished as he descended through the solid crust, and where would it become equal to nothing?

3. How much would a 44-pound shot weigh at the centre of the earth; how much at a point half way from the centre to the surface; and how much 100 miles below the surface?

4. Suppose a 32-pound cannon-ball, fired with the velocity of 2000 feet per second, to have the same momentum as a battering-ram whose weight is 5760 pounds; find the velocity of the latter. *Ans.* 11.11 ft. per sec.

5. Suppose light to have weight, and one grain of it moving at the rate of 192,000 miles per second, to impinge directly against a mass of ice moving at the rate of 1.45 feet per second, and to stop it; required the weight of the ice.

Ans. 99877.832 lbs., or nearly 44½ tons, reckoning 7000 gr.=1 lb.

6. If a ball of the same density with the earth, $\frac{1}{10}$th of a mile in diameter, were to fall through its own diameter toward the earth, what space would the earth move through to meet the ball, the diameter of the earth being taken at 8000 miles?

Ans. $\frac{1}{80000000000}$ inch, nearly.

7. If a hole were bored through the centre of the earth, what would be the conditions of the motion of a stone dropped into the hole?

In its descent towards the centre, the force of gravity would continually decrease till at the centre it became zero; but though this force *decreases* in *intensity*, it will at each instant *increase* the previously existing velocity, though by decreasing increments, so that the stone will have its greatest velocity at the centre of the earth: it will then, in an inverse order, suffer continually *increasing* decrements of velocity until it finally comes to rest at the other surface of the earth, when it will return under similar conditions.

CHAPTER II.

VARIABLE MOTION.—CONSTANT FORCES.

20. Relation of Time and Acquired Velocity.—When a body moves with uniform motion, $S = t\,v$ (Art. 6). When a body moves with uniformly varied motion the case is somewhat different.

Let us consider the case of a body that moves with uniformly increasing velocity. Suppose the body to start from rest and at the end of the 1st second to have acquired a velocity of 10 ft. per second; that is to say, a velocity which would carry it over 10 ft. per second during the next and each succeeding second, if the force ceased to act at the end of the first second. Now since the velocity is supposed to increase uniformly, we shall have at the end of the 2d second a velocity of 20 ft., at the end of the 3d a velocity of 30 ft., and so on.

Hence the first law of motion under the action of a constant force: *In uniformly accelerated motion the acquired velocities vary as the times.*

21. Space Passed Over.—Since the body started from rest and gained uniformly in velocity till it acquired a velocity of 50 ft. per second at the end of the 5th second, it is evident that its *average* velocity was 25 ft. per second; for at the start its velocity was 0 and at the end was 50; at an interval of one second after starting it had a velocity of 10, and one second before the end of the time considered it had a velocity of 40; two seconds after starting the velocity was 20, and two seconds before the end of the time the velocity was 30. Thus the less velocity at any given interval is balanced by the greater velocity during the corresponding interval of the pair, and we are thus enabled to find the distance passed over, by multiplying the *average* velocity, of 25 ft. per second, by the time, 5 seconds, giving the space 125 feet.

22. Space Described during 1st Second.—We have considered the velocities at intervals of one second, but we could have chosen smaller intervals as well, and no matter how small we make our unit of time, the law holds good. Now, during the first second the body acquired a velocity of 10 ft., and if we suppose the first second to be divided into 10 equal intervals, we may

apply the same analysis to these as to the five full seconds already considered; and we find the average velocity to be $\frac{10 + 0}{2}$, or 5 ft.: hence, since the body moved for one second with a velocity which would average 5 ft. per second, it must have moved over 5 ft.; hence, *a body starting from rest will, under the action of a constant force, move during the first second over a space equal to one-half the velocity acquired at the end of that second.*

23. Space Described during any Second.—The space described during *any* second is one-half the velocity impressed upon the body by the *constant force* during that second, plus the space described by reason of velocity already impressed upon the body by previous action of the force.

24. Relations of Time, Space, and Acquired Velocity.—It is necessary to know all the possible relations between the space, time, and acquired velocities. Let us now examine the relations between time and space. During the first second the body, in the case already given, moves over 5 ft. and acquires a velocity of 10 ft.; during the 2d second it will move over 10 ft. in consequence of the velocity already impressed, and over 5 ft. additional because of the continued action of the force, making a total of 15 ft. At the beginning of the 3d second the velocity is 20 ft., and the body will move over 20 ft. in consequence of this, together with 5 ft. more on account of the continued action of the force; and so on to the end of the time.

Hence, we have—

Times.	Ac. vel. at beginning of interval.	Spaces described during interval.	Total spaces.
1st Sec.	0	5	For 1 sec. 5
2d "	10	15	" 2 " 20
3d "	20	25	" 3 " 45
4th "	30	35	" 4 " 80
5th "	40	45	" 5 " 125

Examining the above results, and calling the space described during the 1st second S, we have the space during

$$\begin{aligned} 2 \text{ secs.} &= S \times 4 = S \times 2^2 \\ 3 \quad " &= S \times 9 = S \times 3^2 \\ 4 \quad " &= S \times 16 = S \times 4^2 \\ 5 \quad " &= S \times 25 = S \times 5^2 \end{aligned}$$

That is to say, *The spaces described under the action of a constant*

force are proportional to the squares of the times during which the force acts.

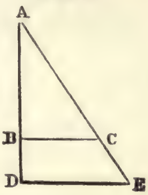
Acquired velocities are proportional to the times, and therefore the spaces must be also proportional to the squares of the acquired velocities.

25. Laws of Uniformly Accelerated Motion.—To recapitulate; when bodies move under the action of a constant force, the following relations exist between space, time, and velocity:

1. *The acquired velocities vary as the times.*
2. *The spaces vary as the squares of the times.*
3. *The spaces vary as the squares of the acquired velocities.*

As an aid to the memory, the following analogy may be employed. Let s be the space described, v the velocity acquired by a body moving from rest for the time t, s' the space described, v' the velocity acquired at any other period t'; then, from what has already been demonstrated, if t and t' be represented by the lines $A\ B$ and $A\ D$ (Fig. 3), and v and v' by the lines $B\ C$ and $D\ E$, drawn at right angles to them, s and s' will be represented by the triangles $A\ B\ C$ and $A\ D\ E$. For $A\ B\ C : A\ D\ E :: A\ B^2 : A\ D^2$; or, as $B\ C^2 : D\ E^2$; or $s : s' :: t^2 : t'^2$, or as $v^2 : v'^2$. The velocity acquired varies as the time; from the similar triangles $A\ B\ C$, $A\ D\ E$, we have $B\ C : D\ E :: A\ B : A\ D$, or, $v : v' :: t : t'$.

FIG. 3.



26. Formulæ.—Let us represent by f the acceleration due to a force, that is to say, the increase of velocity per second due to the action of the force; then the space passed over during the 1st second, if starting from rest, would be $\frac{1}{2} f$, as deduced in Art. 22. Calling the total space s, time in seconds t, and acquired velocity v, we have, from the above laws,

$$f : v :: 1 : t. \quad \therefore t = \frac{v}{f} \quad \text{and } v = f\,t. \quad \therefore f = \frac{v}{t}.$$

$$\tfrac{1}{2} f : s :: 1^2 : t^2. \quad \therefore s = \tfrac{1}{2} f\,t^2 \text{ and } t = \sqrt{\frac{2\,s}{f}}. \quad \therefore f = \frac{2\,s}{t^2}.$$

$$\tfrac{1}{2} f : s :: f^2 : v^2. \quad \therefore s = \frac{v^2}{2\,f} \quad \text{and } v = \sqrt{2\,f\,s}. \quad \therefore f = \frac{v^2}{2\,s}.$$

27. Applications of the Formulæ.—

1. A body moves from rest during 6 seconds with acceleration $f = 20$ ft.; what space does it pass over, and what velocity does it acquire? *Ans.* $s = 360$ ft., $v = 120$ ft. per sec.

2. A train starts from rest and moves with acceleration of 2 ft. per second, and finally acquires a speed of 60 miles per hour, or 88 ft. per second. How long did it take to acquire this speed, and what distance was run in so doing? Here $f = 2$, and $v = 88$.

Ans. 44 sec.; space = 1936 ft.

The mean value of the acceleration due to the action of gravity is usually given as 32.2 ft. per second, and is generally represented by g. If we substitute g for f in the general formulæ, when treating of falling bodies, we will have $t = \frac{v}{g}$, $v = g\,t$, $s = \frac{1}{2}\,g\,t^2$, $t = \sqrt{\frac{2\,s}{g}}$, $s = \frac{v^2}{2\,g}$, and $v = \sqrt{2\,g\,s}$.

3. A body falls from rest and reaches a point 257.6 ft. below. How long time was it falling and what velocity did it acquire?

Ans. $t = 4$ sec; $v = 128.8$ ft. per sec.

4. A body moves from rest during 4 seconds and acquires a velocity of 300 ft. What was the acceleration, and what space was passed over? *Ans.* $f = 75$ ft. per sec.; $s = 600$ ft.

5. A body moves from rest during $3\frac{1}{2}$ seconds and passes over 147 feet. What was the acceleration, and what the final velocity?

Ans. $f = 24$ ft. per sec.; $v = 84$ ft. per sec.

6. A train after running $\frac{1}{2}$ mile has acquired a velocity of 30 miles an hour. What was the acceleration, and how long had it been moving? *Ans.* $f = \frac{1}{4}$ mile per minute; $t = 2$ minutes.

28. Uniform and Uniformly Varied Motion Combined.—Thus far we have assumed the body to start from rest. If the condition be changed and the body be considered as having a uniform motion at the time the action of the constant force begins, we have merely to combine the formula for that motion with that of uniformly accelerated motion already used. Thus, if a body is thrown downward with a force which gives it a velocity of 40 ft. per second, how far will it fall in 4 seconds, and what velocity will it have at the end of that time? Under the action of the downward impulse alone, it would move over 4×40 ft. $=$ 160 ft. Under the action of gravity it would move over $16.1 \times 4^2 = 257.6$; combining these two effects, we have 417.6 ft. as the total distance passed over in the given time. Designating the velocity due to the impulse, usually called the "initial velocity," by v, we have total space $s = v\,t + \frac{1}{2}\,g\,t^2$; and also final velocity $v' = v + g\,t = 40 + 128.8 = 168.8$ ft. per second.

29. Uniformly Retarded Motion.—In like manner we can determine the results when a constant force acts to retard

velocity already imparted, by merely taking the difference of the two effects.

A body receives an impulse of 100 ft. per second, and is retarded by a constant force whose acceleration is 10 ft. per second; how far will the body move in 5 seconds? We now have $s = v\,t - \frac{1}{2} f\,t^2$; $s = 100 \times 5 - 5 \times 25 = 375$.

30. Applications of Formulæ for the Fall of Bodies.—
1. A body falls 6 seconds; what *space* does it pass over, and what *velocity* does it acquire?

Ans. $s = 579.6$ ft., $v = 193.2$ ft. per sec.

2. *How far* must a body fall to acquire a velocity of 50 feet per second, and *how long* will it be in falling?

Ans. $s = 38.8$ ft., $t = 1.55$ sec.

3. A body fell from the top of a tower 150 feet high; *how long* was it in falling, and what *velocity* did it have at the bottom?

Ans. $t = 3.05$ sec., $v = 98.2$ ft.

4. If a ball be thrown upward with a velocity of 100 feet per second, what *height* will it reach? *Ans.* 155.3 ft.

5. Suppose a body to fall during 3 seconds, and then to move uniformly during 2 seconds more, with the velocity acquired: what is the whole *distance* passed over?

The space fallen through is $16.1 \times 9 = 144.9$ feet. The velocity acquired is $32.2 \times 3 = 96.6$ feet. The space described uniformly is $96.6 \times 2 = 193.2$ feet. Therefore the whole space is $144.9 + 193.2 = 338.1$ feet.

6. A ball fired perpendicularly upward was gone 10 seconds, when it returned to the same place; how *high* did it rise, and with what *velocity* was it projected? *Ans.* $s = 402.5$ ft., $v = 161$ ft.

31. Space in any Given Second or Seconds of Fall.— If it be required to find how far a body will move during any specified unit or units of time, proceed thus: suppose it to be required to determine how far the body will move during the 7th second; for the whole 7 seconds,

$$s = \tfrac{1}{2} f\,t^2 = \tfrac{1}{2} f \times 7^2;$$

for six seconds, $$s' = \tfrac{1}{2} f\,t'^2 = \tfrac{1}{2} f \times 6^2;$$

$$s - s' = \tfrac{1}{2} f\,(t^2 - t'^2) = \tfrac{1}{2} f\,(7^2 - 6^2) = \frac{13\,f}{2}.$$

Suppose we are required to determine the space described during the last three seconds:

$$s = \tfrac{1}{2} f\,t^2 = \tfrac{1}{2} f \times 7^2;$$
$$s' = \tfrac{1}{2} f\,t'^2 = \tfrac{1}{2} f \times 4^2;$$
$$s - s' = \tfrac{1}{2} f\,(t^2 - t'^2) = \tfrac{1}{2} f\,(7^2 - 4^2) = \frac{33\,f}{2}.$$

1. How far does a body move in the 14th second of its fall? *Ans.* 434.7 ft.

2. A body had been falling 2 minutes; how far did it move in the last second? *Ans.* 3847.9 ft.

3. What space was described in the last two seconds by a body which had fallen 300 feet? *Ans.* 214.1 ft.

4. A body had been falling $8\frac{1}{2}$ seconds; how far did it descend in the next second? *Ans.* 289.8 ft.

32. Calculation for Projection Upward or Downward.—A body projected downward describes $t\,v$ feet by the force of projection, and $\frac{1}{2}\,g\,t^2$ feet by the force of gravity (Art. 28). A body projected upward describes $t\,v$ by the force of projection; but this is diminished by $\frac{1}{2}\,g\,t^2$, which gravity would cause it to describe in the same time (Art 29). Therefore the formula for space described by a body projected downward is $s = t\,v + \frac{1}{2}\,g\,t^2$; by a body projected upward, the formula is $s = t\,v - \frac{1}{2}\,g\,t^2$.

1. A body is projected downward with a velocity of 30 feet in a second; *how far* will it fall in 4 seconds? *Ans.* 377.6 ft.

2. A body is projected upward with a velocity of 120 feet in a second; *how far* will it rise in 3 seconds? *Ans.* 215.1 ft.

3. Suppose at the same instant that a body begins to fall from rest from the point D (Fig. 4), another body is projected upward from B with a velocity which would carry it to A; it is required to find the point where they would meet.

FIG. 4.

Let C be the point where the bodies would meet; and let $A\,B = a$, $B\,D = b$, $D\,C = x$; then will $A\,D = a - b$, $A\,C = a - b + x$.

Now the time of descending through $D\,C = \left(\frac{2\,x}{g}\right)^{\frac{1}{2}}$; and the time of ascending through $B\,C$ (= time down $A\,B$ — time down $A\,C$) $= \left(\frac{2\,a}{g}\right)^{\frac{1}{2}} - \left(\frac{2\,(a-b+x)}{g}\right)^{\frac{1}{2}}$; but the time down $D\,C$ must be equal to the time up $B\,C$; hence we have

$$\left(\frac{2\,x}{g}\right)^{\frac{1}{2}} = \left(\frac{2\,a}{g}\right)^{\frac{1}{2}} - \left(\frac{2\,(a-b+x)}{g}\right)^{\frac{1}{2}}, \text{ or } x^{\frac{1}{2}} = a^{\frac{1}{2}} - (a-b+x)^{\frac{1}{2}};$$

$$\therefore\ (a-b+x)^{\frac{1}{2}} = a^{\frac{1}{2}} - x^{\frac{1}{2}}, \text{ and } a-b+x = a+x-2\,(ax)^{\frac{1}{2}};$$

$$\therefore\ 2\,(a\,x)^{\frac{1}{2}} = b, \text{ or } 4\,a\,x = b^2, \text{ and } x = \frac{b^2}{4\,a}.$$

4. Suppose a body to have fallen from A to B (Fig. 5), when another body begins to fall from rest at D; *how far* will the latter body fall before it is overtaken by the former?

Let C be the point where one body overtakes the other, and let $AB = a$, $BD = b$, $DC = x$; then $AC = a + b + x$. Now time down $DC = \left(\frac{2x}{g}\right)^{\frac{1}{2}}$, and time down $BC =$ time down $AC -$ time down $AB = \left(\frac{2(a+b+x)}{g}\right)^{\frac{1}{2}} - \left(\frac{2a}{g}\right)^{\frac{1}{2}}$; but at the moment when the lower body is overtaken, time down $DC =$ time down BC, or

$$\left(\frac{2x}{g}\right)^{\frac{1}{2}} = \left(\frac{2(a+b+x)}{g}\right)^{\frac{1}{2}} - \left(\frac{2a}{g}\right)^{\frac{1}{2}};$$

$$\therefore\ x = \frac{b^2}{4a}.$$

FIG. 5.

33. Questions on Falling Bodies.—

1. The momentum of a meteoric stone at the instant of striking the earth was estimated at 18435 foot-pounds, and it had been falling 10 seconds; from what height did it fall, and what was its weight? *Ans.* 1610 ft.; 57.2 lbs.

2. An archer wishing to know the height of a tower, found that an arrow sent to the top of it occupied 8 seconds in going and returning; what was the height of the tower? *Ans.* 257.6 ft.

3. In what time would a man fall from a balloon three miles high, and what velocity would he acquire?
Ans. $t = 31.4$ sec.; $v = 1011.1$ ft.

4. A body having fallen for $3\frac{1}{2}$ seconds, was afterwards observed to move with the velocity which it had acquired for $2\frac{1}{2}$ seconds more; what was the whole space described by the body?
Ans. 478.9 ft., very nearly.

5. Through what space would the aeronaut (in Question 3) fall during the last second? *Ans.* 995 ft.

6. A body has fallen from the top of a tower 340 feet high; what was the space described by it in the last three seconds?
Ans. 299.5 ft.

7. Suppose a body be projected downward with a velocity of 18 feet in a second; how far will it descend in 15 seconds?
Ans. 3892.5 ft.

8. A body is projected upward with a velocity of 65 feet in a second; how far will it rise in two seconds? *Ans.* 65.6 ft.

9. With what velocity must a stone be projected into a well 450 feet deep, that it may arrive at the bottom in four seconds?
Ans. 48.1 ft. in a second.

10. The space described in the fourth second of fall was to the space described in the last second except four, as 1 : 3; what was the whole space described by the body? *Ans.* 3622.5 ft.

11. A staging is at the height of 84 ft. above the earth. A ball thrown upward from the earth, after an absence of 7 seconds, fell on the staging; what was the velocity of projection?

Ans. 124.7 ft. per second.

12. A body is projected upward with a velocity of 483 feet in a second; in what time will it rise to a height of 1610 feet?

Ans. $t = 3.8$ sec., or 26.2 sec.

13. From a point 386.4 feet above the earth a body is projected upward with a velocity of 161 feet in a second; in what time will it reach the surface of the earth, and with what velocity will it strike? *Ans.* $t = 12$ sec., $v = 225.4$ ft.

14. A body is projected upward with a velocity of 64.4 feet in a second; how far above the point of projection will it be at the end of 4 seconds? *Ans.* 0 ft.

15. A body is projected upward with a velocity of 128.8 feet in a second; where will it be at the end of 10 seconds?

Ans. 322 ft. below the point of projection.

34. Determination of the Acceleration due to a Given Force.—Thus far forces have been represented by their accelerations f., but we have no definite idea as to *how much* acceleration a *force* of 9 lbs. would impress upon a *mass* of 13 lbs. By a force of 9 lbs. is meant a constant pressure or tension which would be exactly balanced by a weight of 9 lbs. or by a spring which would sustain such weight.

If a force will produce an acceleration of f ft. per sec. in a mass m, the same force would produce an acceleration of only $\frac{1}{2}f$ ft. per sec. in a mass $2m$, or $2f$ ft. per sec. in a mass $\frac{m}{2}$.

If a force will produce an acceleration f ft. per sec. in a mass m, twice that force would produce an acceleration of $2f$ ft. per sec. in the same mass.

Now a *force* of 9 lbs. acting upon a *mass* of 9 lbs. produces an acceleration of 32.2 ft. per sec. in the case of a falling body. But this same force, if acting upon twice as much matter, or a *mass* of 18 lbs. could impress only one-half as much acceleration or $\frac{32.2}{2}$. From this example, we see that the acceleration f, due to a force, is such part of the acceleration which gravity would impress as the force F in lbs., is of the mass W in lbs.; hence $f = \frac{F}{W} \times 32.2$, and this value of f must be substituted in the formulæ of Art. 26 when applied to problems like the following:

If a body whose weight is 40 lbs. is moved horizontally by a constant force of 10 lbs., how far would it move during 5 seconds,

and what velocity would it acquire, no friction nor other resistances being considered?

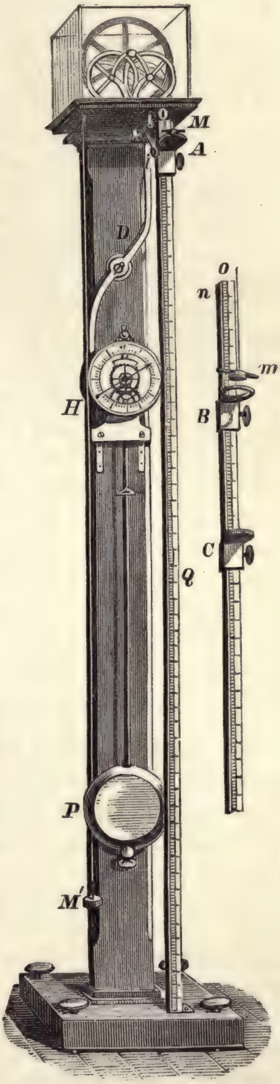
The acceleration of the mass 40 lbs., due to the force 10 lbs., is $f = \frac{10}{40} \times 32.2 = 8.05$ ft. per sec.

Hence,

$s = \frac{1}{2} f t^2 = \frac{8.05}{2} \times 5^2 = 100.6$ ft., and

$v = f t = 8.05 \times 5 = 40.2$ ft. per sec.

Fig. 6.



35. Atwood's Machine.—All the facts of uniformly accelerated or retarded motion may be illustrated with sufficient accuracy by Atwood's machine, since in accordance with what has just been shown, we can render the motion of the parts as slow as we choose, and therefore bring the velocities of the moving masses within the limits of observation. From the base of the instrument, which is furnished with leveling screws, rises a substantial pillar, about seven feet high, supporting a small table upon the top (Fig. 6).

Above the table is a grooved wheel, delicately suspended on friction-wheels, and protected from dust by a glass case. Two equal poises, M and M', are attached to the ends of a fine cord, which passes over the groove of the wheel. As gravity exerts equal forces on M and M', they are in equilibrium. To set them in motion, a small bar m is placed on M, which will immediately begin to descend, and M' to rise. But this motion will be slower than in falling freely, because the force which gravity exerts on the bar must be communicated to the poises, and also to the revolving wheel over which the cord passes. By increasing the poises M, M', and diminishing the bar m, the motion may be made as slow as we please. H is a simple clock attached to the pillar for measuring seconds, and for dropping the poise M at the beginning of a vibration of the pendulum. Q is a scale of inches

extending from the base to the table. The stage A may be clamped to any part of the scale, in order to stop the poise M in its descent, as represented at C. The ring B, which is large enough to allow the poise, but not the bar, to pass through it, is also clamped to the scale wherever the acceleration is to cease.

Let M be raised to the top, and held in place by a support, and then let the pendulum be set vibrating. When the index passes the zero point, the clock causes the support to drop away, and the poise descends. The pendulum shows how many seconds elapse before the bar is arrested by the ring, and how many more before the poise strikes the stage. From the top to the ring the motion is accelerated by the constant fraction of gravity acting on it; from the ring to the stage the poise moves uniformly with the acquired velocity. Moreover, the resistance of the air is so much diminished when the motion is slow, that a good degree of correspondence is found to exist between the experiments and the results of calculation.

If we disregard the mass of the wheel as not sensibly affecting the results, which we may do in practice if the weights are heavy as compared with it, we may illustrate the action of the machine by the following case: Suppose the weights to be $7\frac{1}{2}$ ounces each. They will balance and no motion will result. Now lay a weight of one ounce upon one of them, and, equilibrium being destroyed, motion will ensue. In this we have a *force* of one ounce moving a mass of *sixteen* ounces, and the resulting velocity will be determined by reference to paragraph 34, to be about two feet at the end of the 1st second, a velocity readily noted.

36. Work.—If a force moves a body over a distance b ft. against a resistance of a lbs., *work* is said to be done; hence the measure of the work done involves the resistance overcome and the distance moved. The unit of work which will be used hereafter, is the work done in raising a weight of 1 lb. through a height of one foot, and this unit is called a "*foot-pound.*"

If 10 lbs. be raised through 1 ft., ten foot-pounds is the work done, or if 1 lb. be raised through ten feet, the work is ten foot-pounds also.

The product of the resistance in pounds by the distance in feet through which the body is moved against the resistance, *is the measure* of the work done.

Ex. 1. A body is moved on a horizontal plane against a resistance of 50 lbs., due to friction; how much work is done in moving it over 100 ft.? *Ans.* 5000 foot-pounds.

Ex. 2. A train weighing 100 tons moves 30 miles, resistance being 8 lbs. per ton; how much work is expended in this case?

Ans. 126,720,000 foot-pounds.

37. Living Force.—In order to give motion to a body work must be done upon it; if after having acquired a certain velocity resistance be opposed to its motion, it will in its turn do work in overcoming such resistance. In order to determine the work which a moving body is capable of doing, we have merely to determine the height from which the body must fall to acquire the given velocity, and the product of such height in feet by the weight of the body in pounds will be the work which it can do before coming to rest; for the work it can do is only that which has been stored up in it, which we may call "accumulated work;" that is to say the work done in raising it to the height determined, or what is the same thing, the work done upon it by gravity while falling through this height.

Let u = number of units of work accumulated in a body whose weight is w, and velocity v ft. per second. Put s for the height in feet from which the body must fall to acquire the given velocity; then $u = w\,s$, but by the laws of falling bodies, $s = \frac{v^2}{2\,g}$.

$$\therefore\ u = \frac{w\,v^2}{2\,g}.$$

The expression $\frac{w\,v^2}{g}$ is called the living force of the body; therefore, *one-half the living force is the measure of the work accumulated in a body.*

The force of gravity g acts upon every particle of a body alike; therefore the greater the number of these particles, or the greater its mass, the greater will be its weight, or $w = q\,g$ from which we have $q = \frac{w}{g}$, which, in the expression for living force, gives us $\frac{w\,v^2}{g} = q\,v^2$, while momentum $= q\,v$. To compare these, suppose a ball weighing *one* pound to move with the velocity of 2000 feet, and another ball weighing *two* pounds to move with the velocity of 1000 feet, then the *momentum* $(q\,v)$ of the first equals that of the second. But the *living force* $(q\,v^2)$ of the first is twice as great as that of the second; for $1 \times \overline{2000}^2 : 2 \times \overline{1000}^2 :: 2 : 1$. To give a missile greater *velocity* is more advantageous than to increase its *mass*. A 40-pound ball with 1400 feet velocity, is 7 times more efficient in penetrating the walls of forts and the hulks of ships than a 280-pound ball with 200 feet velocity, though the momentum is the same in each case.

38. Applications of Living Force to Work.—

1. A body weighing 100 lbs. is moving at the rate of 50 ft. per second. What space will it move through against a resistance of 5 lbs. before it comes to rest?

Let $s =$ the space in feet; then as 5 lbs. has been overcome through distance of s feet, the work done is $5\ s$ foot-pounds $= u$; but $u = \frac{w\,v^2}{2\,g}$;

$$\therefore\ 5\,s = \frac{100 \times 50^2}{2 \times 32.2},\ \text{from which we find}$$

$$s = 776 + \text{ ft.}$$

2. Required the time before the body will come to rest, in the last problem.

The resistance, 5 lbs., is a *constant force*, and the body will move with uniformly retarded motion. If it has a velocity of 50 ft. per second at the start and 0 at the end of the time, its average velocity is 25 ft. per second; hence, $\frac{776}{25}$ will give the number of seconds required, or $t = 31.04$ sec.

3. A railway train weighing 100 tons has a velocity of 30 ft. per second, resistances being 8 lbs. per ton; what distance will the train move, on a level, after steam is shut off?

Ans. 3494— ft.

4. Two men are pulling a boat ashore by a rope, one at each end, A being in the boat and B on the shore; how will the time of bringing the boat ashore compare with the time in which A would pull it ashore alone, were the other end of the rope fixed to an immovable post?

5. Suppose the rope to pass from A in one boat to B in another equal boat; how fast will B's boat move? will A's boat have the same velocity as when B was on the shore?

39. Measure of Force.—In Art. 12 force is said to be measured by *momentum*, or $f \propto q\,v$; and in Art. 37 it is said to be measured by the *work* performed, or $f \propto q\,v^2$. But these statements are not to be considered as inconsistent with each other; for in the first case, force has reference to *inertia;* in the second case it has reference to *work*. When a force acts on a body that is free to move without obstruction (which is, however, only a supposable case), the effect is perpetual; the body will move on uniformly forever. If the force had been greater, the velocity would have been greater in the same ratio. But when resistances oppose (as is always true in practice), then the force is expended in overcoming them, and this is the *work* to be performed; and if the force ceases to operate, the motion will at length cease also;

but, as has been shown, the space passed over, and therefore the work performed, will vary as the square of the velocity.

When force is employed to perform work, it is by some writers called *energy*, to distinguish it from force as used in producing momentum.

CHAPTER III.

COMPOSITION AND RESOLUTION OF MOTION.

40. Motion by Two or More Forces.—Motion produced by a single force, either impulsive or continued, has been already considered. But motion is more generally caused by several forces acting in different directions.

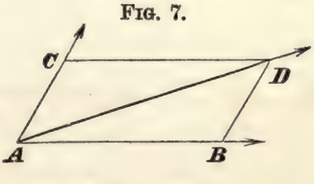
When two or more forces act at once on a body, each force is called a *component*, and the joint effect is called the *resultant*. Forces may be represented by the *straight lines* along which they would move a body in a given time; the lines represent the forces in two particulars, the *directions* in which they act and their *relative magnitudes*. Whenever an arrow-head is placed on a line, it shows in which of the two directions along that line the force acts.

41. The Parallelogram of Forces.—This is the name given to the relation which exists between any two components and their resultant, and is stated as follows:

If two forces acting at once on a body are represented by the adjacent sides of a parallelogram, their resultant is expressed by the diagonal which passes through the intersection of those sides.

Suppose that a body situated at A (Fig. 7) receives an impulse which, acting alone, would carry it over $A\ B$ in a given time, and another which would carry it over $A\ C$ in the same length of time. If both impulses are given at the same instant, the body describes $A\ D$ in the same time as $A\ B$ by the first force, or $A\ C$ by the second, and the motion in $A\ D$ is uniform.

Fig. 7.



This is an instance of the coexistence of motions, stated in the second law of motion (Art. 13). For the body, in passing directly from A to D, is making progress in the direction $A\ C$ as rapidly as though the force $A\ B$ did not exist; and at the same time it advances in the direction $A\ B$ as fast as though that were the

only force. When the body reaches D, it is as far from the line $A\ B$ as if it had passed over $A\ C$; it is also as far from the line $A\ C$ as if it had gone over $A\ B$. Thus it appears that both motions $A\ B$ and $A\ C$ fully coexist in the progress of the body along the diagonal $A\ D$. That the motion is uniform in the diagonal is evident from the law of inertia; for the body is not acted on after it leaves A.

It is evident that a single force might produce the same effect; that force would be represented, both in direction and magnitude, by the line $A\ D$. The force $A\ D$ is said to be equivalent to the two forces $A\ B$ and $A\ C$.

42. Velocities Represented.—The lines $A\ B$ and $A\ C$ are described by the components separately, and the line $A\ D$ by their joint action, *in the same length of time.* Hence the *velocities* in those lines are as the lines themselves. In the parallelogram of forces, therefore, two adjacent sides and the diagonal between them represent—

1st. The relative *directions* of the components and resultant;

2d. Their relative *magnitudes;* and

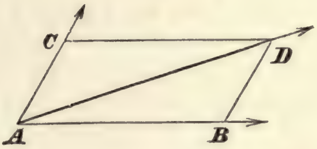
3d. The relative *velocities* with which the lines are described.

43. The Triangle of Forces.—For purposes of calculation, it is more convenient to represent two components and their resultant by the sides of a triangle, than by the sides and diagonal of a parallelogram. In Fig. 7, $C\ D$, which is equal and parallel to $A\ B$, may represent in direction and magnitude the same force which $A\ B$ represents. Therefore, the components are $A\ C$ and $C\ D$, while their resultant is $A\ D$; and the angle C in the triangle is the supplement of $C\ A\ B$, the angle between the components. Care should be taken to construct the triangle so that the sides representing the components may be taken in succession in the directions of the forces, as $A\ C$, $C\ D$; then $A\ D$ correctly represents their resultant. But, although $A\ C$ and $A\ B$ represent the components, the third side, $C\ B$, of the triangle $A\ C\ B$, does not represent their resultant, since $A\ C$ and $A\ B$ cannot be taken successively in the direction of the forces. It is necessary to go back to A in order to trace the line $A\ B$. It should be observed, that though $C\ D$ represents the *magnitude* and *direction* of the component, it is not in the *line* of its action, because both forces act through the same point A.

Three forces produce equilibrium when they may be represented in direction and magnitude by the three sides of a triangle taken in order.

For, when three forces are in equilibrium, one of them must be equal to, and opposite to, the resultant of both the others. But the forces $A\ C$ and $A\ B$ (Fig. 8) produce the resultant $A\ D$; therefore the equal and opposite force $D\ A$, since it is in equilibrium with $A\ D$, is also in equilibrium with $A\ C$ and $A\ B$, or $A\ C$ and $C\ D$. Hence the three forces $A\ C$, $C\ D$, and $D\ A$, taken in order around the figure, produce equilibrium.

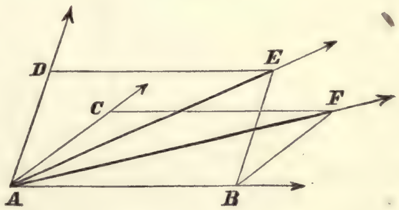
FIG. 8.



44. The Forces Represented Trigonometrically.—Since the sides of a triangle are proportional to the sines of the opposite angles, these sines may also represent two components and their resultant. Thus, the sine of $C\ A\ D$ corresponds to the component $A\ B\ (= C\ D)$; the sine of $C\ D\ A\ (= D\ A\ B)$ corresponds to the component $A\ C$; and the sine of C (= sine of $C\ A\ B$) corresponds to the resultant $A\ D$. Each of the three forces is therefore represented by the sine of the angle between the other two.

45. Greatest and Least Values of the Resultant.—A change in the angle between the components alters the value of the resultant; as the angle increases from 0° to 180°, the resultant diminishes from the *sum* of the components to their *difference.* In Fig. 9, let $C\ A\ B$ and $D\ A\ B$ be two different angles between the same components $A\ C$ (or $A\ D$) and $A\ B$. As $C\ A\ B$ is less than $D\ A\ B$, its supplement $A\ B\ F$ is greater than $A\ B\ E$, the supplement of $D\ A\ B$; therefore $A\ F$ is greater than $A\ E$. When the angle $C\ A\ B$ is diminished to 0°, the sides $A\ B$, $B\ F$, become one straight line, and $A\ F$ equals their *sum;* when $D\ A\ B$ is enlarged to 180°, E falls on $A\ B$, and $A\ E$ equals the *difference* of $A\ B$ and $A\ C$. Between the sum and difference of the components, the resultant may have all possible values.

FIG. 9.



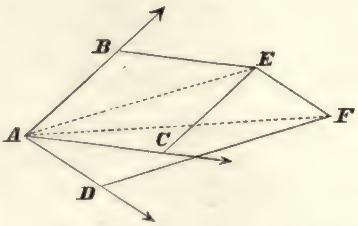
Two forces produce equilibrium when they are equal and act upon the same point in opposite directions.

Since two forces produce the least resultant when they act at an angle of 180° with each other, and the resultant then equals the *difference* of the forces, if the forces are equal, their difference

is zero, and the resultant vanishes; that is, the two forces produce equilibrium.

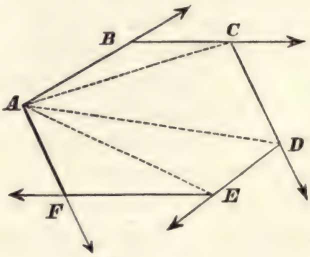
46. The Polygon of Forces.—All the sides of a polygon except one may represent so many forces acting at the same time on a body, and the remaining side will represent their resultant. In Fig. 10, suppose $A\ B$, $A\ C$, and $A\ D$, to represent three forces acting together on a body at A. The resultant of $A\ B$ and $A\ C$ is represented by the diagonal $A\ E$; and the resultant of $A\ E$ and $A\ D$ by the diagonal $A\ F$. As $A\ F$ is equivalent to $A\ E$ and $A\ D$, and $A\ E$ is equivalent to $A\ B$ and $A\ C$, therefore $A\ F$ is equivalent to the three, $A\ B$, $A\ C$, and $A\ D$. But if we substitute $B\ E$ for $A\ C$, and $E\ F$ for $A\ D$, then the three components are $A\ B$, $B\ E$, and $E\ F$, three sides of a polygon, and the resultant $A\ F$ is the fourth side of the same polygon.

FIG. 10.



So, in Fig. 11, $A\ B$, $B\ C$, $C\ D$, $D\ E$, and $E\ F$, may represent the directions and relative magnitudes of five forces, which act simultaneously on a body at A. The resultant of $A\ B$ and $B\ C$ is $A\ C$; the resultant of $A\ C$ and $C\ D$ is $A\ D$; the resultant of $A\ D$ and $D\ E$ is $A\ E$; and the resultant of $A\ E$ and $E\ F$ is $A\ F$; which last is therefore the resultant of all the forces, $A\ B$, $B\ C$, $C\ D$, $D\ E$, and $E\ F$; the components being represented by five sides, and their resultant by the sixth side, of a polygon of six sides.

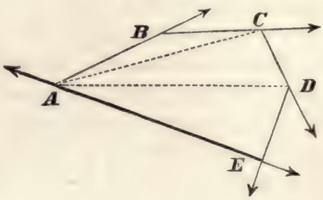
FIG. 11.



More than three forces in one plane will produce equilibrium when they can be represented by the sides of a polygon taken in order.

Since several forces acting on a body, are represented by all the sides of a polygon except one, their resultant is represented by the remaining side. Thus, the resultant of the forces $A\ B$, $B\ C$, $C\ D$, and $D\ E$ (Fig. 12), is $A\ E$. Now, the force $E\ A$, equal and opposite to $A\ E$, since it would be in equilibrium with $A\ E$, is therefore in equilibrium with all the

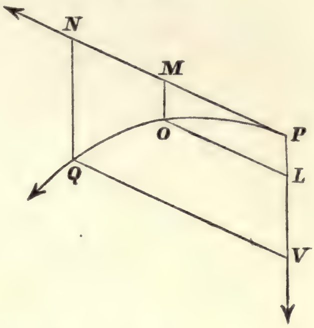
FIG. 12.



others. Hence the forces $A\,B$, $B\,C$, $C\,D$, $D\,E$, and $E\,A$, taken in order around the figure, are in equilibrium.

47. Curvilinear Motion.—Since, according to the first law of motion, a moving body proceeds in a straight line, if no force disturbs it, whenever we find a body describing a *curve*, it is certain that some force is continually deflecting it from a straight line. Besides the original impulse, therefore, which gave it motion in one direction, it is subject to the action of a continued force, which operates in another direction. A familiar example occurs in the path of a projectile. Suppose a body to be thrown from P (Fig. 13), with an impulse which would alone carry it to N, in the same time in which gravity alone would carry it to V. Complete the parallelogram $P\,Q$; then, as both motions coexist (2d law), the body at the end of the time will be found at Q. Let t be the time of describing $P\,N$ or $P\,V$; and let t' be the time of describing $P\,M$ by the impulse, or $P\,L$ by gravity. Then, at the end of the time t', the body will be at O. Now, as $P\,N$ is described uniformly, $P\,N : P\,M :: t : t'$; $\therefore P\,N^2 : PM^2 :: t^2 : t'^2$.

Fig. 13.



But (Art. 25), $P\,V : P\,L :: t^2 : t'^2$; $\therefore P\,V : P\,L :: P\,N^2 : P\,M^2$; or $Q\,V^2 : O\,L^2$.

Hence, the curve is such that $P\,V \propto Q\,V^2$; that is, the abscissa varies as the square of the ordinate, which is a property of the parabola. $P\,O\,Q$ is therefore a parabola, one of whose diameters is $P\,V$, and the parameter to that diameter is $\frac{Q\,V^2}{P\,V}$.

Owing to the resistance of the air, the curve deviates sensibly from a parabola, especially in swift motions.

48. Calculation of the Resultant of Two Impulsive Forces.—When two components and the angle between them are given, the resultant may be found both in direction and magnitude by trigonometry. The theorem required is that for solving a triangle, when two sides and the included angle are given; but the included angle is not that between the components, but its supplement (Art. 43). In Fig. 7, if $A\,B = 54$, and $A\,C = 22$, and $C\,A\,B = 75°$, then $A\,C\,D$ is the triangle for solution, in which $A\,C = 22$, $C\,D = 54$, and $A\,C\,D = 105°$. Performing the calculation, we find the resultant $A\,D = 63.363$, and the angle

$D\,A\,B$, which it makes with the greater force, $= 19°\ 35'\ 43''$. This method will apply in all cases.

1. A foot-ball received two blows at the same instant, one directly east at the rate of 71 feet per second, the other exactly northwest, at the rate of 48 feet per second; in what direction and with what velocity did it move?

Ans. N. 47° 30′ 52″ E. Vel. =50.253.

The process is of course abridged, if the forces act at a right angle with each other, as in the following example:

2. A balloon rises 1120 feet in one minute, and in the same time is borne by the wind 370 feet; what angle does its path make with the vertical, and what is its velocity per second?

Ans. 18° 16′ 53″; $v = 19.659$.

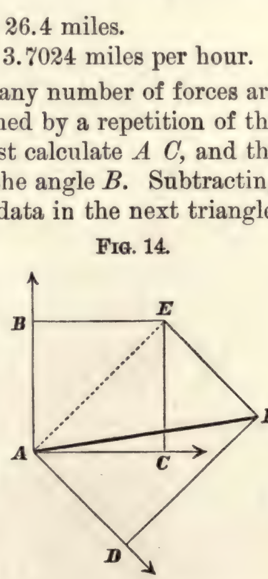
In the next example, one component and the angle which each component makes with the resultant, are given to find the resultant and the other component.

3. From an island in the Straits of Sunda, we sailed S. E. by S. (33° 45′) at the rate of 6 miles an hour; and being carried by a current, which was running toward the S. W. (making an angle with the meridian of 64° 12¼′), at the end of four hours we came to anchor on the coast of Java, and found the said island bearing due north; required *the length of the line* actually described by the ship, and *the velocity of the current?*

Ans. $s = 26.4$ miles.
$v = 3.7024$ miles per hour.

If the magnitudes and directions of any number of forces are given, the resultant of them all is obtained by a repetition of the same process as for two. In Fig. 11, first calculate $A\,C$, and the angle $A\,C\,B$, by means of $A\,B$, $B\,C$, and the angle B. Subtracting $A\,C\,B$ from $B\,C\,D$, we have the same data in the next triangle, to calculate $A\,D$, and thus proceed to the final resultant, $A\,F$.

FIG. 14.



As it is immaterial in what order the components are introduced into the calculation, it will diminish labor, to find first the resultant of any two *equal* components, or any two which make a *right angle* with each other; since it can be done by the solution of an isosceles, or a right-angled triangle.

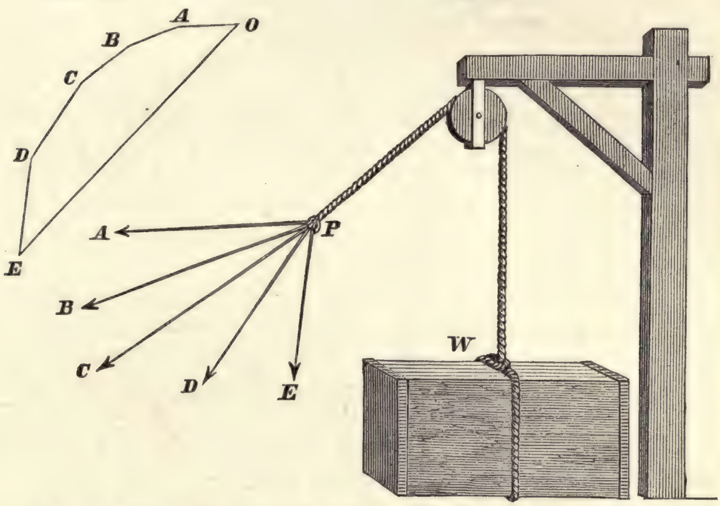
4. The particle A (Fig. 14) is urged by three equal forces $A\,B$, $A\,C$, and $A\,D$; the angle $B\,A\,C = 90°$, and $C\,A\,D = 45°$;

what is the direction of the resultant, and how many times AB? *Ans.* $BAF = 80° 16'$, and $AF : AB :: \sqrt{3} : 1$.

5. Five sailors raise a weight by means of five separate ropes, in the same plane, connected with the main rope that is fastened to the weight in the manner represented in Fig. 16. B pulls at an angle with A of 20°; C with B, 19°; D with C, 21° 30′; and E with D, 25°. A, B, and C, pull with equal forces, and D and E with forces one-half greater; required the magnitude and direction of the resultant.

Ans. Its angle with A is 46° 33′ 10″. Its magnitude is 5.1957 times the force of A.

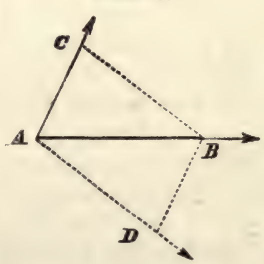
FIG. 15. FIG. 16.



If the polygon $O\,A\,B\,C\,D\,E$ (Fig. 15) be constructed for the above case, OB and CE are easily calculated in the isosceles triangles OAB and CDE, after which OC and then OE are to be obtained by the general theorem.

49. The Resultant and all Components, except one, being given, to Find that one Component.—If AB (Fig. 17) is the resultant to be produced, and there already exists the force AC, a second force can be found, which acting jointly with AC, will produce the motion required. Join CB, and draw AD equal and parallel to it, then AD is the force re-

FIG. 17.



quired; for $A B$ is equivalent to $A C$ and $C B$. Therefore $C B$ has the magnitude and direction of the required force; $A D$ is the line in which it must act.

Again, suppose that *several* forces act on A, and it is required to find the force which, in conjunction with them all, shall produce the resultant $A B$. Let the several forces be combined into one resultant, and let $A C$ represent that resultant. Then $A D$ may be found as before.

The trigonometrical process for finding a component is essentially the same as for finding a resultant.

1. A ferry-boat crosses a river $\frac{3}{4}$ of a mile broad in 45 minutes, the current running all the way at the rate of 3 miles an hour; at what angle with the direct course must the boat head up the stream in order to move perpendicularly across? *Ans.* 71° 34′.

2. A sloop is bound from the mainland of Africa to an island bearing W. by N. (78° 45′) distant 76 miles, a current setting N. N. W. (22° 30′) 3 miles an hour; what is the *course* to arrive at the island in the shortest time, supposing the sloop to sail at the rate of 6 knots per hour; and what *time* will she take?

Ans. Course, S. 76° 41′ 4″ W. Time, 10 h. 40 m. 7 sec.

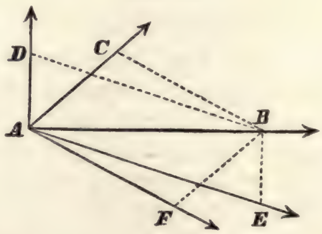
3. The resultant of two forces is 10; one of them is 8, and the direction of the other is inclined to the resultant at an angle of 36°. Find the angle between the two forces.

Ans. 47° 17′ 5″ or 132° 42′ 55″.

4. A ball receives two impulses: one of which would carry it N. 27 feet per second; the other N. 60° E. with the same velocity; what third impulse must be conjoined with them, to make the ball go E. with a velocity of 21 ft? *Ans.* S. 3° 22′ W. $v = 40.57$.

50. Resolution of Motion.—In the *composition* of motions or forces, the resultant of any given components is found; in the *resolution* of motion or force, the process is reversed; the resultant being given, the components are found, which are equivalent to that resultant.

FIG. 18.



If it be required to find what two components can produce the resultant $A B$ (Fig. 18), we have only to construct on $A B$, as a base, any triangle whatever, as $A B C$ or $A B D$ (Art. 43); then, if $A C$ is one component, the other is $A F$, equal and parallel to $C B$; or if $A D$ is one, the other is $A E$, equal and parallel to $D B$; and so for any triangle whatever on the base $A B$. The number of pairs is therefore infinite, whose resultant in each case is $A B$.

The *directions* of the components may be chosen at pleasure, provided the sum of the angles made with $A\ B$ is less than two right angles.

The *magnitude* and *direction* of one component may be fixed at pleasure.

The *magnitudes* of both components may be what we please, provided their difference is not greater, and their sum not less, than the given resultant.

These conditions are obvious from the properties of the triangle.

When a given force has been resolved into two others, each of those may again be resolved into two, each of those into two others still, and so on. Hence it appears that a given force may be resolved into any number of components whatever, with such limitations as to direction and magnitude as accord with the foregoing statements.

1. A motion of 153 toward the north is produced by the forces 100 and 125; how are they inclined to the meridian?

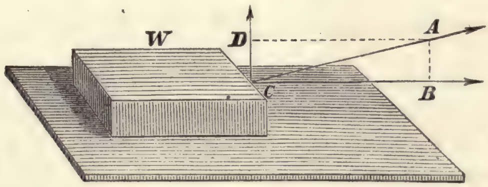
Ans. 54° 28′ and 40° 37′ 7″.

2. A resultant of 617 divides the angle between its components into 28° and 74°; what are the components?

Ans. 606.34 and 296.14.

51. Resolution of a Force, to Find its Efficiency in a Given Direction.—By the resolution of a force into two others acting at right angles with each other, it is ascertained how much efficiency it exerts to produce motion in any given direction. For example, a weight W (Fig. 19), lying on a hori-

FIG. 19.



zontal plane, and pulled by the oblique force $C\ A$, is prevented by gravity from moving in the line $C\ A$, and is compelled to remain on the plane. Resolve $C\ A$ into $C\ B$, *in* the plane, and $C\ D$ *perpendicular* to it: then the former represents the component which is efficient to cause motion along the plane; the latter has no influence to aid or hinder that motion; it simply diminishes pressure upon the plane. In like manner, if $A\ C$ is an oblique force, *pushing* the weight, its horizontal component, $B\ C$,

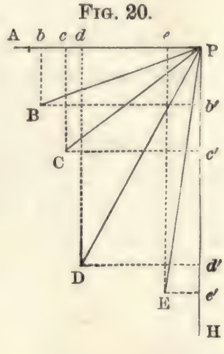
is alone efficient to move it; the other, $A\,B$, merely increasing the pressure. In either case, the whole force is to that component which is efficient to move the body along the plane, *as radius to the cosine of inclination.* Also, the whole force is to that component which increases or diminishes pressure on the plane, *as radius to the sine of inclination.*

If only 88 per cent. of the strength of a horse is efficient in moving a boat along a canal, what angle does the rope make with the line of the tow-path? *Ans.* $28° \, 21' \, 27''$.

52. Resultant found by means of Rectangular Axes.— When several forces act in one plane upon a body, their resultant may be conveniently found by the use of right-angled triangles alone. Select at pleasure two lines at right angles to each other, both of them lying in the plane of the forces, and passing through the point at which the forces are applied. These lines are called *axes.* The following example illustrates their use:

Let $P\,A$, $P\,B$, $P\,C$, $P\,D$, $P\,E$ (Fig. 20) represent the forces in Question 5 (Art. 48). Let one axis, for convenience, be chosen in the direction $P\,A$, and let $P\,H$ be drawn at right angles to it for the other axis. These axes are supposed to be of indefinite length. Then proceed as in Art. 51 to resolve each force into two components on these axes. As $P\,A$ acts in the direction of one axis, it does not need to be resolved. To resolve $P\,B$, say

FIG. 20.



$$R : \cos 20° :: P\,B : P\,b, \text{ and}$$
$$R : \sin 20° :: P\,B : P\,b';$$

again,

$$R : \cos 39° :: P\,C : P\,c, \text{ and}$$
$$R : \sin 39° :: P\,C : P\,c', \text{ \&c.}$$

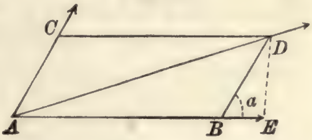
Suppose $P\,A$ produced so as to equal $P\,A + P\,b + P\,c + P\,d + P\,e = M$, and $P\,H$ produced so as to equal $P\,b' + P\,c' + P\,d' + P\,e' = N$. Now, as M acts in the line $P\,A$, and N at right angles to it, their resultant and the angle which it makes with $P\,A$ are found by the solution of another right-angled triangle. The resultant is 5.1957, and the angle is $46° \, 33' \, 10''$, as in Art. 48.

If any components of the resolved forces are opposite to $P\,A$ or $P\,H$, they are reckoned as negative quantities.

53. Analytical Expression for the Resultant.—Put $A\,C$ (Fig. 21) $= P$, $A\,B = P'$, $A\,D = R$, angle $C\,A\,B = a$; then in triangle $A\,B\,D$ we have, by Geometry, $\overline{A\,D}^2 = \overline{A\,B}^2 + \overline{B\,D}^2 +$

$2\,AB \times BE$, but $BE = BD \cos a = P \cos a$, and hence substituting as above $R^2 = P'^2 + P^2 + 2\,P'\,P \cos a$; whence

FIG. 21.



$$R = \sqrt{P'^2 + P^2 + 2\,P'\,P \cos a} \quad (1.)$$

Hence, *The resultant of any two forces, acting at the same point, is equal to the square root of the sum of the squares of the two forces, plus twice the product of the forces into the cosine of the included angle.*

If $a = 0$, its cosine will be 1, and (1) becomes

$$R = P + P'.$$

If $a = 90°$, its cosine will be 0, and we shall have

$$R = \sqrt{P^2 + P'^2}.$$

If $a = 180°$, its cosine will be -1, and we shall have

$$R = P - P'.$$

1. Two forces, P and P', are equal in intensity to 24 and 30, respectively, and the angle between them is 105°; what is the intensity of their resultant? *Ans.* 33.21.

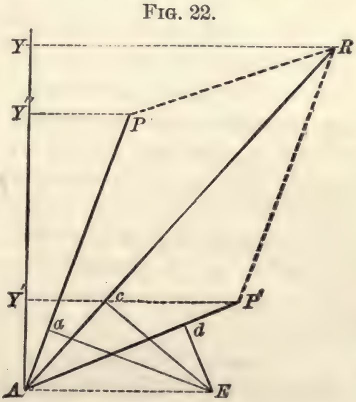
2. Two forces, P and P', whose intensities are, respectively, equal to 5 and 12, have a resultant whose intensity is 13; required the angle between them. *An.* 90°.

3. A boat is impelled by the current at the rate of 4 miles per hour, and by the wind at the rate of 7 miles per hour; what will be her rate per hour when the direction of the wind makes an angle of 45° with that of the current? *Ans.* 10.2 miles.

4. Two forces and their resultant are all equal; what is the value of the angle between the two forces? *Ans.* 120°.

54. Principle of Moments.—The *moment* of a force, with respect to a point, is the product of the force into the perpendicular let fall from the point to the line of direction of the force.

FIG. 22.



The fixed point is called the centre of *moments;* the perpendicular distance, the *lever-arm of the force;* and the *moment* measures the tendency of the force to produce rotation about the centre of moments.

Denote the forces (Fig. 22) by P, P' and their resultant by R. From E any point in the

plane of the forces let fall upon the directions of the forces the perpendiculars $E\,a$, $E\,c$, $E\,d$. Represent these by l, L, l'. Draw two rectangular axes of reference as in Art. 52, so that one of them may pass through A and E. The projection of the resultant R is equal to the sum of the projections of its components (Art. 52); hence,

$$A\,Y = A\,Y'' + A\,Y' \quad . \quad . \quad . \quad . \quad . \quad (1)$$

By similar triangles $A\,c\,E$ and $A\,R\,Y$, we have

$$A\,Y : E\,C = L :: A\,R = R : A\,E \quad \therefore A\,Y = \frac{L \times R}{A\,E};$$

by similar triangles $A\,a\,E$ and $A\,P\,Y''$, we have

$$A\,Y'' : E\,a = l :: A\,P = P : A\,E \quad \therefore A\,Y'' = \frac{l \times P}{A\,E};$$

and by similar triangles $A\,d\,E$ and $A\,P'\,Y'$, we have

$$A\,Y' : E\,d = l' :: A\,P' = P' : A\,E \quad \therefore A\,Y' = \frac{l' \times P'}{A\,E};$$

substituting these values in Eq. (1) and multiplying by $A\,E$, we have

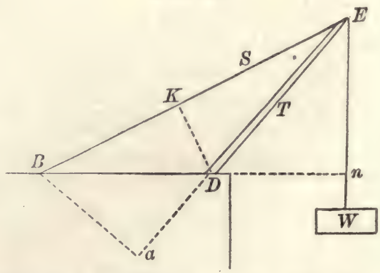
$$R \times L = P \times l + P' \times l'.$$

Hence, *the moment of the resultant of two forces with respect to any point is equal to the algebraic sum of the moments of the forces taken separately.*

By using the resultant as above and a third force the moment of the resultant of the three forces may be proved equal to the algebraic sum of the moments of the forces, and so on for any number of forces.

To illustrate the application of the principle of moments, suppose a weight W of 1000 lbs. to be suspended from the end of a spar $D\,E$ as in the figure; required the strain upon the stay $E\,B$.

FIG. 23.



We have three forces in equilibrium acting at E, viz., the weight W, the strain upon the rope S, and the upward thrust of the spar T. If we select the centre of moments upon the line of either force, the moment of that force will be zero. As the thrust T is equal and opposed to the resultant of S and W, we will take D as the centre of moments, and we have,

Moment of $T =$ moment of $S +$ moment of W; but $T \times 0 =$ moment of T, $S \times K\,D =$ moment of S, and $W \times D\,n =$ mo-

ment of W. But W tends to cause E to revolve towards the right about the point D, while S tends to cause revolution of E towards the left; hence one must be regarded as a positive and the other as a negative moment, and we have, finally, if we call $W \times D\,n$ positive, $0 = -S \times KD + W \times n\,D$, whence, $S = \dfrac{W \times n\,D}{K\,D}$.

If in the problem, $D\,E = 20$ ft, $B\,D = 20$ ft., and $E\,B\,D = 30°$, then will $K\,D = B\,D$ sine $30° = 10$, and $D\,n = D\,e$ sine $30° = 10$.

$$\therefore\ S = \frac{1000 \times 10}{10} = 1000 \text{ lbs.}$$

To find the thrust at D, take the point B as a centre of moments, then

$$T \times Ba = S \times 0 + W \times Bn\,;\ \therefore\ T = \frac{W \times Bn}{Ba}.$$

Now $B\,n = B\,D + D\,n = 30$; to find $B\,a$, we have

$$K\,E = \sqrt{\overline{DE}^2 - \overline{KD}^2} = \sqrt{300}, \text{ and } B\,E = 2\,K\,E = 2\sqrt{300}.$$

In similar triangles $K\,D\,E$ and $B\,E\,a$ we have

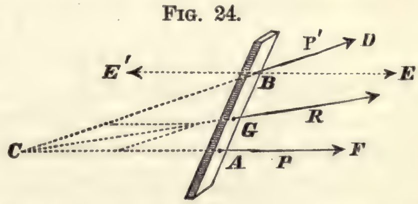
$$D\,E : B\,E :: K\,D : B\,a\,;$$

$$\therefore B\,a = \frac{2\sqrt{300} \times 10}{20} = \sqrt{300}\,; \text{ hence } T = \frac{1000 \times 30}{\sqrt{300}}.$$

Remember that when *three* forces, acting at the same point, are in equilibrium, one of the three being known, either of the other two can be found by taking the centre of moments on the line of the force not sought, and equating the moments of the two forces considered.

55. Forces Acting at Different Points. Parallel Forces.—We have thus far considered forces acting upon a single particle, or upon one point of a body. If, however, two forces P and P', in the same plane, act upon A and B, two different points of a rigid body, they may still have a resultant.

FIG. 24.



Let the lines of directions of the two forces $A\ F$ and $B\ D$ (Fig. 24) be produced to meet in C. The two forces may then be considered as acting at C, and thus compounded into a single force at that point, or at the point G of the body.

Calling the angle $B\,C\,G = \beta$ and $A\,C\,G = a$ we have, projecting P' and P upon the line of R,

$$R = P' \cos \beta + P \cos a \ldots (1).$$

When the forces become parallel, as $A\ F$ and $B\ E$, $\beta = 0$, and $a = 0$, and (1) becomes

$$R = P' + P \ldots (2).$$

If the parallel forces act in opposite directions, as $A\ F$ and $B\ E'$, then $a = 180°$, and $\beta = 0$, and (1) becomes

$$R = P' - P \ldots (3).$$ Hence,

The resultant of two parallel forces is in a direction parallel to them and equal to their algebraic sum.

56. Point of Application of the Resultant.—Let P and P' (Figs. 25, 26) be two parallel forces acting in the same or in

FIG. 25.

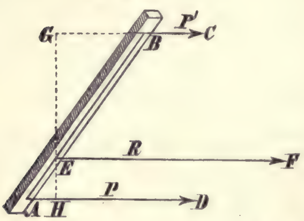
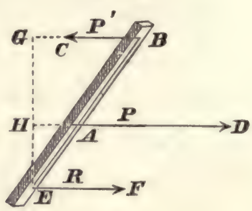


FIG. 26.



opposite directions, and let E be the point of application of the resultant. Assume this point as a centre of moments; then from Art. 54, since $L = 0$,

$P \times H\ E = P' \times G\ E$, or, in the form of a proportion,
$P' : P :: H\ E : G\ E$. But by similar triangles,
$H\ E : G\ E :: A\ E : E\ B$; $\therefore$
$P' : P :: A\ E : E\ B$.

That is, *the line of direction of the resultant of two parallel forces divides the line joining the points of application of the components, inversely as the components.*

By composition (Fig. 25) and division (Fig. 26) we obtain

$$P' + P : P :: A\ B : E\ B, \text{ and}$$
$$P - P' : P :: A\ B : E\ B.$$

That is, *if a straight line be drawn to meet the lines of two parallel forces and their resultant, each of the three forces will be proportional to that part of the line contained between the other two.*

When the forces act in the same direction, we have

$E\ B = \dfrac{P \times A\ B}{P' + P}$, and when they act in opposite directions,

$$E\ B = \frac{P \times A\ B}{P - P'}.$$

If, in the last case, $P = P'$, then $E\ B$ will be infinite. The two forces in this case constitute what is called a *couple*. Their effect is to produce rotation about a point between them.

Any number of parallel forces may be reduced to a single force

(or to a couple) by first finding the resultant of two forces, then the resultant of that and a third force, and so on to the last. And any single force may be resolved into two or any number of parallel forces by a method the reverse of this.

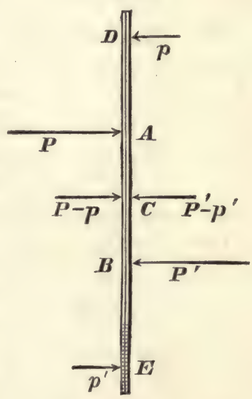
57. Equilibrium of Parallel Forces.—In order that a force may be in equilibrium with two parallel forces,

1. *It must be parallel to them.*
2. *It must be equal to their algebraic sum.*
3. *The distances of its line of action from the lines in which the two forces act, must be inversely as the forces.*

These three conditions belong to the *resultant* of two parallel forces, and therefore belong to that force which is in equilibrium with the resultant.

58. Equilibrium of Couples.—If two parallel forces are such as to constitute a couple, no *one* force can be in equilibrium with them. For the resultant of a couple is zero, and has its point of application at an infinite distance (Art. 56). But a couple can be held in equilibrium by another couple; and the second couple may be either larger or smaller than the given couple, or it may be equal to it.

FIG. 27.



Let the couple P and P' (Fig. 27) act on a body at the points A and B; they tend to produce rotation about the middle point C. If another couple, Q and Q', equal to P and P', should be applied to produce equilibrium, one must directly oppose P, and the other P'. Then A and B, being each held at rest, all the forces are in equilibrium.

But if the second couple is less than P and P', they must act at distances from C, which are as much greater as the forces are less; or, if the second couple is greater than the first, they must act at distances which are as much less. Thus, the couple p and p', acting at D and E, tend to produce rotation about C in one direction, and P and P' in the opposite; and these tendencies are equal when $D\ C : A\ C :: P : p$. For, since the opposite forces, P and p, are inversely as their distances from C, their resultant is at C, and is equal to $P - p$ (Art. 55). For the same reason, the resultant of P' and p' is at C, and equal to $P' - p'$. But $P-p = P' - p'$, and they act in opposite directions. Hence C is at rest, and therefore all the forces are in equilibrium.

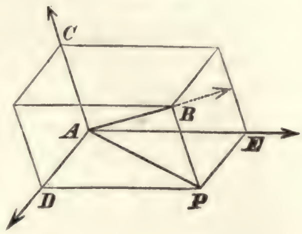
59. The Parallelopiped of Forces.—Hitherto forces have

been considered as acting in the same plane. But if forces act in different planes, the solution of every case may be reduced to the following principle, called the *parallelopiped of forces.*

Any three forces acting in different planes upon a body may be represented by the adjacent edges of a parallelopiped, and their resultant by the diagonal which passes through the intersection of those edges.

Let $A\ C$, $A\ D$, and $A\ E$ (Fig. 28), be three forces applied in different planes to the body at A. Construct the parallelopiped $C\ P$, having $A\ C$, $A\ D$, and $A\ E$, for its adjacent edges, and from A draw the diagonal $A\ B$. The section through the opposite edges $A\ C$ and $P\ B$ is a parallelogram, and therefore $A\ B$ is the resultant of $A\ C$ and $A\ P$, and $A\ P$ is the resultant of $A\ D$ and $A\ E$. Hence $A\ B$ is the resultant of $A\ C$, $A\ D$, and $A\ E$.

Fig. 28.



This process may obviously be reversed, and a given force may be resolved into three components in different planes along the edges of a parallelopiped, having such inclinations as we please.

60. Rectangular Axes.—The parallelopiped generally chosen is that whose sides are rectangles; the three adjacent edges of such a solid are called *rectangular axes.* All the forces which can possibly act on a body may be resolved into equivalent forces in the direction of three such axes. And since all forces which act in the direction of any one line may be reduced to a single force by taking their algebraic sum, therefore any number of forces acting through one point may be reduced to *three* in the direction of three axes chosen at pleasure.

Fig. 29.

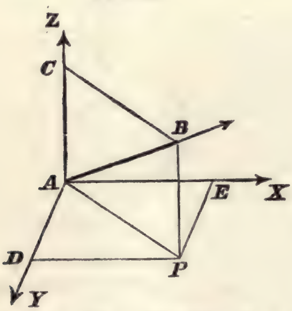
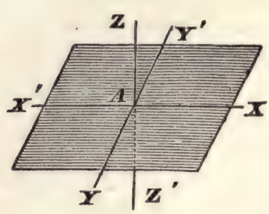


Fig. 30.



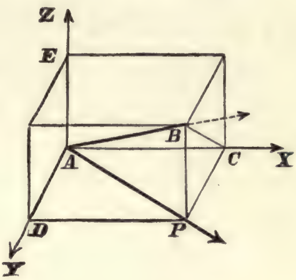
Let $A\ X$, $A\ Y$ (Fig. 29) be at right angles with each other, and $A\ Z$ perpendicular to the plane $A\ X$ and $A\ Y$. Let $A\ B$

represent a force acting on A. Resolve $A\,B$ into $A\,C$ on the axis $A\,Z$, and $A\,P$ in the plane of $A\,X$, $A\,Y$; then resolve $A\,P$ into $A\,D$ and $A\,E$ on the other two axes. Therefore, $A\,C$, $A\,D$, and $A\,E$ are three rectangular forces, whose resultant is $A\,B$.

Let the axes $A\,X$, $A\,Y$, $A\,Z$, be produced indefinitely (Fig. 30) to X', Y', Z'; then their planes will divide the angular space about A into eight solid right angles, namely: $A\text{-}X\,Y\,Z$, $A\text{-}X\,Y'\,Z$, $A\text{-}X'\,Y'\,Z$, $A\text{-}X'\,Y\,Z$, above the plane of X and Y, and $A\text{-}X\,Y\,Z'$, $A\text{-}X\,Y'\,Z'$, $A\text{-}X'\,Y'\,Z'$, $A\text{-}X'\,Y\,Z'$ below it.

61. Geometrical Relation of Components and Resultant.—A force acting on the body A may be situated in any one of the eight angles, and its value may be expressed in terms of the squares of its three components. Let $A\,B$ (Fig. 31) be resolved as before into the rectangular components $A\,C$, $A\,D$, and $A\,E$. Then, by the right-angled triangles, we find

FIG. 31.



$$A\,B^2 = B\,P^2 + A\,P^2 = A\,E^2 + A\,P^2;$$

and

$$A\,P^2 = A\,C^2 + C\,P^2 = A\,C^2 + A\,D^2;$$

$$\therefore A\,B^2 = A\,C^2 + A\,D^2 + A\,E^2;$$

$$\text{and } A\,B = \sqrt{A\,C^2 + A\,D^2 + A\,E^2}.$$

If $A\,B$ is in the plane of X and Y, the component on the axis of Z becomes zero, and $A\,B = \sqrt{A\,C^2 + A\,D^2}$, and similarly for the other planes.

62. Trigonometrical Relation of Components and Resultant.—Let the angles which $A\,B$ makes with the axes of X, Y, Z, respectively, be α, β, γ; that is, $B\,A\,C = \alpha$, $B\,A\,D = \beta$, $B\,A\,E = \gamma$. In the triangle $A\,B\,C$, right-angled at C, we have $A\,B : A\,C ::$ rad : $\cos \alpha$; therefore, making rad $= 1$,

$$A\,C = A\,B \,.\, \cos \alpha.$$

In like manner, $A\,D = A\,B \,.\, \cos \beta$;

and $A\,E = A\,B \,.\, \cos \gamma$.

And since $A\,B$ is the resultant of the forces $A\,C$, $A\,D$, and $A\,E$, it is the resultant of $A\,B \,.\, \cos \alpha$, $A\,B \,.\, \cos \beta$, $A\,B \,.\, \cos \gamma$. In general, the components of any force P, when resolved upon three rectangular axes, are $P \,.\, \cos \alpha$, $P \,.\, \cos \beta$, $P \,.\, \cos \gamma$.

63. Any Number of Forces Reduced to Three on Three Rectangular Axes.—Suppose the body at A to be acted upon by a second force P', whose direction makes with the axes the angles a', β', γ'; then, as before, P' is the resultant of $P' . \cos a'$, $P' . \cos \beta'$, $P' . \cos \gamma'$; and a third force P'', in like manner, has for its components $P'' . \cos a''$, $P'' . \cos \beta''$, $P'' . \cos \gamma''$; and so of any number of forces.

Now, all the components on one axis may be reduced to one force by adding them together. Hence, the whole force in the axis of $X = P . \cos a + P' . \cos a' + P'' . \cos a'' + P''' . \cos a''' +$ &c.; the whole in the axis of Y,

$$= P . \cos \beta + P' . \cos \beta' + P'' . \cos \beta'' + P''' . \cos \beta''' + \text{\&c.};$$

and that in the axis of Z,

$$= P . \cos \gamma + P' . \cos \gamma' + P'' . \cos \gamma'' + P''' . \cos \gamma''' + \text{\&c.}$$

If any component acts in a direction opposite to others in the same axis, it is affected by a contrary sign, so that the force in the direction of any axis is the algebraic sum of all the individual forces in that axis.

If the sum of the components in *one* axis is reduced to zero by contrary signs, the effect of all the forces is limited to the plane of the other axes, and is to be obtained as in Art. 52, where two axes were employed. If the sum of the components on each of *two* axes is reduced to zero, then the whole force is exerted in the direction of the remaining axis, and is therefore perpendicular to the plane of the other two.

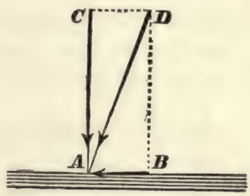
64. Equilibrium of Forces in Different Planes.—Since all the forces which can operate on a body may be reduced to three forces on rectangular axes, it is obvious that the whole system of forces cannot be in equilibrium till the sum of the components on each axis is reduced to zero. We must have, therefore, in Art. 63, as conditions of equilibrium, these three equations for the three axes, X, Y, Z:

$$P . \cos a + P' . \cos a' + P'' . \cos a'' +, \text{\&c.}, = 0;$$
$$P . \cos \beta + P' . \cos \beta' + P'' . \cos \beta'' +, \text{\&c.}, = 0;$$
$$P . \cos \gamma + P' . \cos \gamma' + P'' . \cos \gamma'' +, \text{\&c.}, = 0.$$

65. Forces Resisted by a Smooth Surface.—Whenever any forces cause pressure upon a surface without friction, and are held in equilibrium by its resistance, the resultant of those forces must be at right angles to the surface. Suppose that

$D A$ (Fig. 32) is either a single force or the resultant of two or more forces, and that it is held in equilibrium by the reaction of AB, a smooth surface. If $D A$ is not perpendicular to the surface, it can be resolved into two components, one perpendicular to the surface $A B$, the other parallel to it. The former, $C A$, is neutralized by the resistance of the surface; the latter, $B A$, is not resisted, and produces motion parallel to the surface, contrary to the supposition. Therefore $D A$, if held in equilibrium by the surface $A B$, must be perpendicular to it.

AIG. 32.



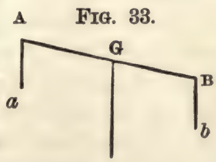
CHAPTER IV.

THE CENTRE OF GRAVITY.

66. The Centre of Gravity Defined.—In every body and in every system of bodies, there is a point so situated that all the parts acted on by the force of gravity balance each other about it in every position. That point is called the *centre of gravity.* The force of gravity acts in parallel lines on every particle of a body; the centre of gravity must therefore be the point through which the resultant of all these parallel forces is directed, in every position of the body. Hence, if the centre of gravity is supported, the body is supported. As to the support of the body, therefore, we may imagine all parts of it to be collected in its centre of gravity. When a system of bodies is considered, they are conceived to be united to each other by inflexible rods, which are without weight.

67. Centre of Gravity of Equal Bodies in a Straight Line.—The centre of gravity of two equal particles is in the middle point between them. Let A and B (Fig. 33), two equal particles, be joined by a straight line, and let $A\,a$ and $B\,b$ represent the forces of gravity. The resultant of these forces, since they are parallel and equal, will pass through the middle of $A\,B$ (Art. 56); G is therefore the centre of gravity. In like manner it is proved that the centre of gravity of two equal *bodies* is in the middle point between their respective centres of gravity.

FIG. 33.



Any number of equal particles or bodies, arranged at equal distances on a straight line, have their common centre of gravity in the middle; since the above reasoning applies to each pair, taken at equal distances from the extremes. Hence, the centre of gravity of a material straight line (e.g., a fine straight wire) is in the middle point of its length.

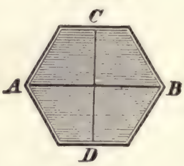
68. Centre of Gravity of Regular Figures.—In the discussion of the centre of gravity in relation to *form, bodies* are considered uniformly dense, and *surfaces* are regarded as thin laminæ of matter.

In plane figures the centre of gravity coincides with the centre of magnitude, when they have such a degree of regularity that there are two diameters, each of which divides the figure into equal and symmetrical parts.

The circle, the parallelogram, the regular polygon, and the ellipse, are examples.

For instance, the regular hexagon (Fig. 34) is divided symmetrically by $A\ B$, and also by $C\ D$. Conceive the figure to be composed of material lines parallel to $A\ B$. Each of these has its centre of gravity in its middle point, that is, in $C\ D$, which bisects them all (Art. 67). Hence, the centre of gravity of the whole figure is in $C\ D$. For the same reason it is in $A\ B$. It is, therefore, at their intersection, which is also the centre of magnitude.

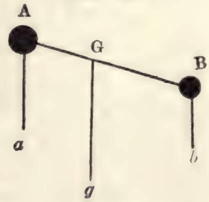
FIG. 34.



By a similar course of reasoning it is shown that in *solids* of uniform density, which are so far regular that they can be divided symmetrically by three different planes, the centres of gravity and magnitude coincide; e.g., the sphere, the parallelopiped, the cylinder, the regular prism, and the regular polyhedron.

69. Centre of Gravity between Two Unequal Bodies.—The centre of gravity of two unequal bodies is in a straight line joining their respective centres of gravity, and at the point which divides their distance in the inverse ratio of their weights. Let $A\ a$ and $B\ b$ (Fig. 35), passing through the centres of gravity of A and B, be proportional to their weights, and therefore represent the forces of gravity exerted upon them. By the laws of parallel forces, the resultant $G\ g = A\ a + B\ b$ (Art. 55), and $A\ a : B\ b :: B\ G : A\ G$. Therefore the centre of gravity must be at G, through which the resultant passes

FIG. 35.



(Art. 66). This obviously includes the case of *equal* weights (Art. 67).

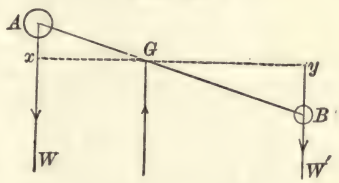
It appears from the foregoing that the whole pressure on a support at G is $A+B$, and that the system is kept in equilibrium by such support.

70. Equal Moments with Respect to the Centre of Gravity.—Applying the principle of moments we have, calling the weights W and W', and taking the centre of moments at G (Fig. 36)

$$W \times G\,x = W' \times G\,y;$$

but $G\,x : G\,y :: A\,G : G\,B$; $\therefore W \times A\,G = W' \times G\,B$, as was proved in Art. 56.

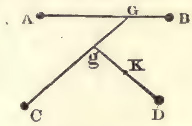
FIG. 36.



71. Centre of Gravity between Three or More Bodies.—The method of determining the centre of gravity of two bodies may be extended to any number.

Let A, B, C, D, &c. (Fig. 37), be the weights of the bodies, and let the centres of gravity of A and B be connected together by the inflexible line $A\,B$.

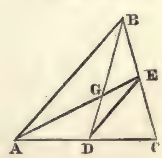
FIG. 37.



Divide $A\,B$ so that $A : B :: B\,G : A\,G$, or $A + B : B :: A\,B : A\,G$; then G is the centre of gravity of A and B. Join $C\,G$; and since $A + B$ may be considered as at the point G, divide $C\,G$ so that $A + B + C : C :: C\,G : G\,g$. In like manner, K, the centre of gravity of four bodies, is found by the proportion, $A + B + C + D : D :: D\,g : g\,K$. The same plan may be pursued for any number of bodies.

72. Centre of Gravity of a Triangle.—*The centre of gravity of a triangle is one-third of the distance from the middle of a side to the opposite angle.* Bisect $A\,C$ in D (Fig. 38), and $B\,C$ in E; join $A\,E$, $B\,D$, and $D\,E$. $B\,D$ bisects all lines across the triangle parallel to $A\,C$; therefore the centre of gravity of all those lines—that is, of the triangle—is in $B\,D$. For a like reason, it is in $A\,E$, and therefore at their intersection, G. Since $E\,C = \frac{1}{2}\,B\,C$, and $D\,C = \frac{1}{2}\,A\,C$, $\therefore\ E\,D = \frac{1}{2}\,A\,B$. But $E\,G\,D$ and $A\,G\,B$ are similar; $\therefore\ D\,G : B\,G :: D\,E : A\,B :: 1 : 2$; $\therefore D\,G = \frac{1}{2}\,B\,G = \frac{1}{3}\,B\,D$.

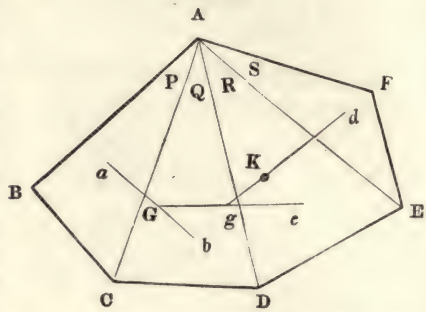
FIG. 38.



73. Centre of Gravity of an Irregular Polygon.—Divide the polygon into triangles by diagonals drawn through one of its

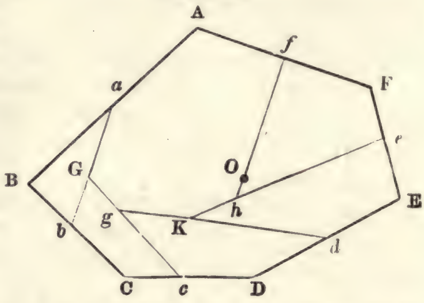
angles, and then proceed according to the methods already given. Let $A\,C\,E$ (Fig. 39) be an irregular polygon, whose centre of gravity is to be found. Divide it into the triangles P, Q, R, S, by diagonals through A, and find their centres of gravity a, b, c, d (Art. 72). Join $a\,b$, and divide it so that $a\,b : a\,G :: P + Q : Q$; then G is the centre of gravity of the quadrilateral $P + Q$. Then join $G\,c$, and make $G\,c : G\,g :: P+Q+R : R$. By proceeding in this manner till all the triangles are used, the centre of gravity of the polygon is found at the last point of division.

Fig. 39.



74. Centre of Gravity of the Perimeter of an Irregular Polygon.—Find the centre of gravity of each side, which is at its middle point, and then proceed as in Art. 71, the weight of each line being considered proportional to its length. Thus, let a, b, c, &c., be the centres of gravity of the sides, $A\,B$, $B\,C$, $C\,D$, &c. (Fig. 40); join $a\,b$, and divide it so that $a\,b : a\,G :: A\,B + B\,C : B\,C$; then G is the centre of gravity of $A\,B$ and $B\,C$. Next join $G\,c$, and make $G\,c : G\,g :: A\,B + B\,C + C\,D : C\,D$; then g is the centre of gravity of those three sides. Proceed in this manner till all the sides are used.

Fig. 40.

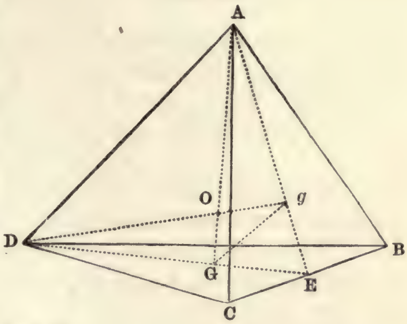


The perimeter of a polygon having the degree of regularity described in Art. 68, has its centre of gravity at the centre of the figure, as may be easily proved. If a polygon has a less degree of regularity than that, the centre of gravity both of its area and its perimeter may usually be found by methods more direct and simple than those given for polygons wholly irregular.

75. Centre of Gravity of a Pyramid.—*The centre of gravity of a triangular pyramid is in the line joining the vertex and the centre of gravity of the base, at one-fourth of the distance from the base to the vertex.*

Let G (Fig. 41) be the centre of gravity of the base $B\ D\ C$; and g that of the face $A\ B\ C$. The line $A\ G$ passes through the centre of gravity of every lamina parallel to $D\ B\ C$, on account of the similarity and similar position of all those laminæ; $\therefore$ the centre of gravity of the pyramid is in $A\ G$. For a similar reason, it is in $D\ g$; and therefore at their intersection, O. Now $EG = \frac{1}{3} E\ D$, and $E\ g = \frac{1}{3} E\ A$; hence, by similar triangles, $g\ G = \frac{1}{3} A\ D$. But $G\ g\ O$ and $A\ O\ D$ are also similar; $\therefore G\ O = \frac{1}{3} A\ O = \frac{1}{4} A\ G$.

FIG. 41.

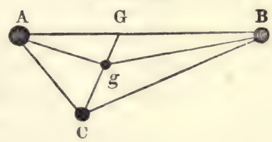


From this it is readily proved that the centre of gravity of *every* pyramid and cone is one-fourth of the distance from the centre of gravity of the base to the vertex.

76. Examples on the Centre of Gravity.—

1. A, B, and C (Fig. 42), weigh, respectively, 3, 2, and 1 pounds, $A\ B = 5$ ft., $B\ C = 4$ ft., and $C\ A = 2$ ft. Find the distance of their centre of gravity from C.

FIG. 42.



First, from the given sides of the triangle $A\ B\ C$, calculate the angles. A is found to be $49°\ 27\frac{1}{2}'$. Next find the place of G, the centre of gravity of A and B, by the proportion, $A + B : B :: A\ B : A\ G$; $A\ G$ is 2 ft., equal to $A\ C$. Calculate $C\ G$, the base of the isosceles triangle $A\ G\ C$. Its length is 1.673. Then find $C\ g$ by the proportion $C\ G : C\ g :: A + B + C : A + B$; therefore $C\ g = 1.394$.

2. $A = 5$ lbs., $B = 3$ lbs., and $C = 12$ lbs.; $A\ B = 8$ ft., $A\ C = 4$ ft., and the angle A is $90°$; find the distance of the centre of gravity of A, B, and C from C. *Ans.* 2 ft.

3. Three equal bodies are placed at the angles of any triangle whatever; show that the common centre of gravity of those bodies coincides with the centre of gravity of the triangle.

4. Find the centre of gravity of five equal heavy particles placed at five of the angular points of a regular hexagon.

Ans. It is one-fifth of the distance from the centre to the third particle.

5. A regular hexagon is bisected by a line joining two opposite angles; where is the centre of gravity of one-half?

Ans. Four-ninths of the distance from the centre to the middle of the second side.

6. A square is divided by its diagonals into four equal parts, one of which is removed; find the distance from the opposite side of the square to the centre of gravity of the remaining figure.

Ans. $\frac{7}{18}$ of the side of the square.

7. Two isosceles triangles are constructed on opposite sides of the same base, the altitude of the greater being h, and of the less, h'; where is the centre of gravity of the whole figure?

Ans. On the altitude of the greater triangle, at a distance from the common base equal to $\frac{1}{3}(h - h')$.

8. The base and the place of the centre of gravity of a triangle being given, required to construct the triangle.

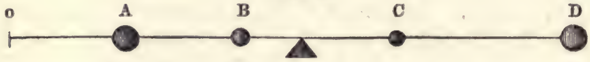
9. Given the base and altitude of a triangle; required to construct the triangle, when its centre of gravity is perpendicularly over one end of the base.

10. On a cubical block stands a square pyramid, whose base, volume, and mass are respectively equal to those of the cube; where is the centre of gravity of the figure?

Ans. One-eighth of the height of the cube above its upper surface.

77. Centre of Gravity of Bodies in a Straight Line referred to a Point in that Line.—If several bodies are in a straight line, their common centre of gravity may be referred to a point in that line; and its distance from that point is obtained by *multiplying each weight into its own distance from the same point, and dividing the sum of the products by the sum of the weights.* Let A, B, C, and D, represent the weights of several bodies, whose centres of gravity are in the straight line $o\,D$ (Fig. 43). Required

Fig. 43.



the distance of their common centre of gravity from any point o assumed in the same line. Let G be their common centre of gravity; then, calling R the resultant of the several weights A, B, C and D, which acts at the point G, we have from principle of moments,

$$R \times o\,G = A \times A\,o + B \times B\,o + C \times C\,o + D \times D\,o,$$

and since $R = A + B + C + D$ (Art 55), we have

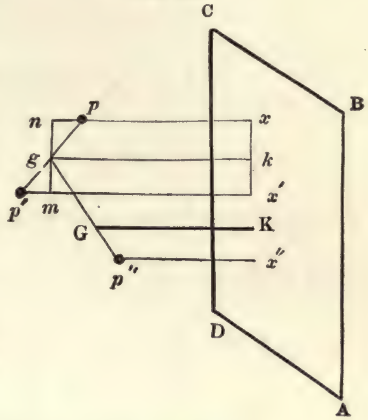
$$o\,G = \frac{A \times A\,o + B \times B\,o + C \times C\,o + D \times D\,o}{A + B + C + D}.$$

78. Centre of Gravity of a System referred to a Plane.—If the bodies are not in a straight line, they may be referred to a plane, which is assumed at pleasure. The distance of

their common centre of gravity from that plane is expressed as before : *multiply each weight into its own distance from the plane, and divide the sum of the products by the sum of the bodies.*

Let $p, p' p''$ (Fig. 44), represent the weights of several bodies, whose centres of gravity are at those points respectively, and let $A\ C$ be the plane of reference. Join $p\ p'$, and let g be the common centre of gravity of p and p'; draw $p\ x$, $g\ k$, $p'\ x'$ at right angles to the plane $A\ C$, and consequently parallel to each other; join $x\ x'$, and since the points p, g, p', are in a straight line, the points x, k, x' will also be in a straight line, and therefore $x\ x'$ will pass through k. Join $g\ p''$, and let G be the common centre of gravity of p, p', p''; draw $G\ K$, $p''\ x''$, perpendicular to the plane; and through g draw $m\ n$ parallel to $x\ x'$ meeting $p\ x$ produced in n.

FIG. 44.



Now $p : p' :: p'\ g : p\ g ::$ (by sim. triangles) $p'\ m : p\ n$;

$\therefore p \times p\ n = p' \times p'\ m$, or $p \times (n\ x - p\ x) = p' \times (p'\ x' - m\ x')$; but

$$n\ x = g\ k = m\ x',\ \therefore p \times (g\ k - p\ x) = p' \times (p'\ x' - g\ k),$$

and

$$(p + p') \times g\ k = p \times p\ x + p' \times p'\ x' \ \therefore g\ k = \frac{p \times p\ x + p' \times p'\ x'}{p + p'};$$

for the same reason, if $p + p'$ is placed at g, we have

$$G\ K = \frac{(p+p') \times g\ k + p'' \times p''\ x''}{(p+p') + p''} = \frac{p \times p\ x + p' \times p'x' + p'' \times p''x''}{p + p' + p''};$$

a formula which is applicable to any number of bodies.

Let the last equation be multiplied by the denominator of the fraction, and we have

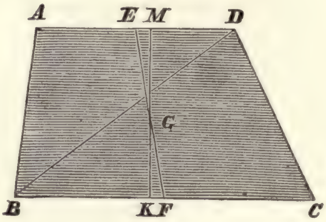
$$(p + p' + p'' + \text{\&c.})\ G\ K = p \times p\ x + p' \times p'\ x' + p'' \times p''\ x'' + \text{\&c.}:$$

that is, *the moment of any system of bodies with reference to a given plane, equals the sum of the moments of all the parts of the system with reference to the same plane.*

79. Centre of Gravity of a Trapezoid.—As an example of the foregoing principle, let it be proposed to find the centre of

gravity of a trapezoid, considered as composed of two triangles. The centre of gravity of the trapezoid $A\ C$ (Fig. 45) is in $E\ F$, which bisects all the lines of the figure parallel to $B\ C$. Suppose G to be the centre of gravity of the trapezoid; through G draw $K\ M$ perpendicular to the bases. Let $K M = h$, $B C = B$, $A D = b$, and join $B\ D$.

FIG. 45.



The moment of the trapezoid with reference to $B\ C$ is

$$(B + b)\frac{h}{2} \cdot G\,K.$$

The moment of the upper triangle is $\frac{b\,h}{2} \cdot \frac{2}{3} h$; the moment of the lower triangle is $\frac{B\,h}{2} \cdot \frac{h}{3}$;

$$\therefore (B + b)\frac{h}{2} \cdot G\,K = \frac{B\,h}{2} \cdot \frac{h}{3} + \frac{b\,h}{2} \cdot \frac{2}{3} h; \text{ whence}$$

$$G\,K = \frac{B + 2\,b}{B + b} \cdot \frac{h}{3}. \quad \text{But } G\,M = h - \frac{B + 2\,b}{B + b} \cdot \frac{h}{3} =$$

$$\frac{2\,B + b}{B + b} \cdot \frac{h}{3}; \quad \therefore G\,M : G\,K :: 2\,B + b : B + 2\,b.$$

By similar triangles

$$G\,M : G\,K :: E\,G : G\,F; \ \therefore E\,G : G\,F :: 2\,B + b : B + 2\,b; \text{ or}$$

the centre of gravity of a trapezoid is on the line which bisects the parallel bases, and divides it in the ratio of twice the longer plus the shorter to twice the shorter plus the longer.

1. Four bodies, A, B, C, D, weighing, respectively, 2, 3, 6, and 8 pounds are placed with their centres of gravity in a right line, at the distance of 3, 5, 7, and 9 feet from a given point; what is the distance of their common centre of gravity from that given point; and between which two of the bodies does it lie?

Ans. Between C and D; and its distance from the given point $7\frac{2}{19}$ feet.

2. There are five bodies, weighing, respectively, 1, 14, $21\frac{1}{2}$, 22, and $29\frac{1}{2}$ pounds; a plane is assumed passing through the last body, and the distances of the other four from the plane are, respectively, 21, 5, 6, and 10 feet; how far from the plane is the common centre of gravity of the five bodies? *Ans.* 5 feet.

80. Centrobaric Mensuration.—The properties of the centre of gravity furnish a very simple method of measuring

surfaces and solids of revolution. This method is comprehended in the two following propositions, known as the theorems of Guldinus:

1. *If any line revolve about a fixed axis, which is in the plane of that line, the* SURFACE *which it generates is equal to the product of the given line into the circumference described by its centre of gravity.*

Let any line, either straight or curved, revolve about a fixed axis which is in the plane of that line; and let f, f', f'', f''', etc., denote elementary portions of the line, d, d', d'', d''', &c., the distances of these portions, respectively, from the axis; then the surface generated by f, in one revolution, will be $2\pi d f$; hence the surface generated by the whole line will be

$$S = 2\pi (d f + d' f' + d'' f'' + d''' f''' + \text{\&c.}) \ldots (1).$$

Put $L =$ the length of the revolving line, and $G =$ the distance from the axis to the centre of gravity of the line; then (Art. 78)

$$G L = d f + d' f' + d'' f'' + d''' f''' + \text{\&c.} \ldots (2).$$

Combining (1) and (2), we have

$$S = 2\pi G L \ldots\ldots\ldots (3).$$

2. *If a plane surface, of any form whatever, revolve about a fixed axis which is in its own plane, the* VOLUME *generated is equal to the product of that surface into the circumference described by its centre of gravity.*

Let any plane surface revolve about an axis which is in the plane of that surface; and let f, f', f'', f''', &c., denote elementary portions of the surface, d, d', d'', d''', &c., the distances of these portions, respectively, from the axis; then the volume generated by f in one revolution will be $2\pi d f$; hence the volume generated by the whole surface will be

$$V = 2\pi (d f + d' f' + d'' f'' + d''' f''' + \text{\&c.}) \ldots (4).$$

Put $A =$ the area of the revolving surface, and $G =$ the distance from the axis to the centre of gravity of that surface; then (Art. 78)

$$A G = d f + d' f' + d'' f'' + d''' f''' + \text{\&c.}, \ldots (5).$$

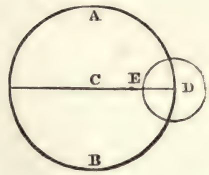
Substituting in (4), we have

$$V = 2\pi A G \ldots\ldots\ldots (6).$$

As an illustration of the first theorem, the straight line $C D$ (Fig. 46), revolving about the center C, describes a circle whose

surface is equal to CD into the circumference of the circle described by its centre of gravity, E. This is evident also from the consideration that, since E is the centre of the line CD, the circumference described by it will be half the length of the circumference ADB; and the area of a circle is equal to the product of the radius into half the circumference.

Fig. 46.



The second theorem is illustrated by the volume of a cylinder, whose height $= h$, and the radius of whose base $= r$.

Common method; base$=\pi r^2$; height$=h$; $\therefore$ vol. $= \pi r^2 h$.

Centrobaric method; revolving area $= r h$; circumference described by the centre of gravity $= \frac{1}{2} r \times 2\pi$; $\therefore$ vol. $= r h \,.\, \frac{1}{2} r \,.\, 2\pi = \pi r^2 h$.

81. Examples.—

1. Suppose the small circle (Fig. 46) to be placed with its plane perpendicular to the plane of the paper, and revolved about C, the point D describing the line DBA; required the content of the solid ring. If $CD = R$, and $ED = r$, then the area revolved $= \pi r^2$, and the circumference $DBA = 2\pi R$; $\therefore$ the ring $= 2\pi^2 R r^2$. It is equal to a cylinder whose base is the circle ED, and whose height equals the line DBA.

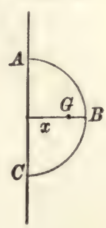
2. Find the convex surface of a cone; slant height $= s$; and rad. of base $= r$. The line revolved being s, and the distance from the axis to its centre of gravity, $\frac{1}{2} r$, the surface is $\pi r s$.

3. A square, whose side is one foot, is revolved about an axis which passes through one of its angles, and is parallel to a diagonal; required the volume of the figure thus formed.

Ans. $\pi\sqrt{2}$, or 4.4429 cubic ft.

4. Find the centre of gravity of a semi-circumference. In this case the revolving semi-circumference ABC (Fig. 47) generates the surface of a sphere; hence, taking the diameter AC as an axis, calling the distance of the centre of gravity G from the axis x and radius r, we have

Fig. 47.



$$4\pi r^2 = 2\pi x \times \pi r; \therefore x = \frac{2r}{\pi}.$$

Hence the distance of the centre of gravity of the semi-circumference from the centre of the circle is

$$\frac{2r}{\pi} = .637\, r.$$

5. Find the centre of gravity of a semicircle. The revolving

area generates a sphere, and hence, as in the preceding problem, we have

$$\tfrac{4}{3}\pi r^3 = 2\pi x \times \tfrac{1}{2}\pi r^2; \ \therefore x = \frac{4}{3}\frac{r}{\pi} = .424\, r.$$

In any case, when a simple expression for the surface generated by a revolving line can be found, it is easy to find the centre of gravity of the line by this method, and the centre of gravity of an area may be readily found from the expression of the volume generated.

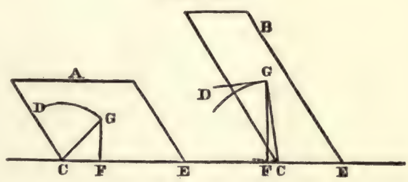
82. Support of a Body.—A body cannot rest on a smooth plane unless it is horizontal; for the pressure on a plane (Art. 65) cannot be balanced by the resistance of that plane, except when perpendicular to it; therefore, as the force of gravity is vertical, the resisting plane must be horizontal.

The *base of support* is that area on the horizontal plane which is comprehended by lines joining the extreme points of contact.

If there are *three* points of contact, the base is a triangle; if *four*, a quadrilateral, &c.

When the vertical through the centre of gravity (called *the line of direction*) falls within the base, the body is supported; if without, it is not supported. In the body A (Fig. 48) the force of gravity acts in the line $G\ F$, and there are lines of resistance on both sides of $G\ F$, as $G\ C$ and $G\ E$, so that the body cannot turn on the edge of the base, without *rising* in an arc whose radius is $G\ C$ or $G\ E$.

Fig. 48.



But, in the body B, there is resistance only on one side; and therefore, if the force of gravity be resolved on $G\ C$ and a perpendicular to it, the body is not prevented from moving in the direction of the latter, that is, in the arc whose radius is $G\ C$.

If the line of direction fall at the edge of the base, the least force will overturn it.

83. Different Kinds of Equilibrium.—If the base is reduced to a line or point, then, though there may be support, there is no *firmness* of support; the body will be moved by the least force. But it is affected very differently in different cases.

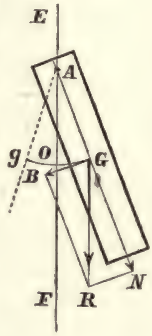
When it is moved from its position of support and left, it will in some cases return to it, pass by, and return again, and continue thus to vibrate till it settles in its place of support by friction and other resistances. This condition is called *stable equilibrium.*

In other cases, when moved from its position of support and left, it will depart further from it, and never recover that position again. This is called *unstable equilibrium.*

In other cases still, the body, when moved from its place of support and left, will remain, neither returning to it nor departing further from it. This is called *neutral equilibrium.*

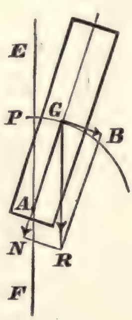
84. Stable Equilibrium.—Let the body (Fig. 49) be suspended on the pivot *A*. This is its base of support. While the centre of gravity is below *A*, the line of direction *E O F* passes through the base, and the body is supported. Let it be moved aside, and the centre of gravity be left at *G*. Let *G R* represent the force of gravity, and resolve it into *G N* on the line *A G*, and *N R*, or *G B*, perpendicular to *A G*. *G N* is resisted by the strength of *A*, and *G B* moves the centre of gravity in the arc whose radius is *A G*. Hence the body swings with accelerated motion till the centre of gravity reaches *O*, where the force *G B* becomes zero. But by its inertia, the body passes beyond that position, and ascends on the other side, till the retarding force of gravity stops it at *g*, as far from *O* as *G* is. It then descends again, and would never cease to oscillate were there no obstructions.

Fig. 49.



85. Unstable Equilibrium.—Next, let the body be turned on the pivot till the centre of gravity *G* is at *P*, above *A* (Fig. 50). Then, as well as when *G* is below *A*, the body is supported, because the line of direction *E P F* passes through the base *A*. But if turned and left in the slightest degree out of that position, it cannot recover it again, but will depart further and further from it. Let *G R* represent the force of gravity, and let it be resolved into *G N*, acting through *A*, and *G B* perpendicular to it. The former is resisted by *A*; the latter moves *G* away from *P*, the place of support. If the body is free to revolve about *A*, without falling from it, the centre of gravity will, by friction and other resistances, finally settle below *A*, as in the case of stable equilibrium.

Fig. 50.



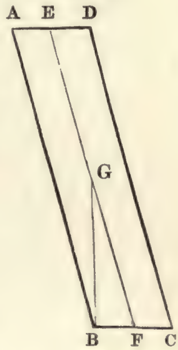
86. Neutral Equilibrium.—Once more, suppose the pivot supporting the body to be at *G*, the centre of gravity; then, in

whatever situation the body is left, the line of direction passes through the base, and the body rests indifferently in any position.

These three kinds of equilibrium may be illustrated also by bodies resting by curved surfaces on a horizontal plane. Thus, if a cylinder is uniformly dense, it will always have a *neutral* equilibrium, remaining wherever it is placed. But if, on account of unequal density, its centre of gravity is not in the axis, then its equilibrium is *stable*, when the centre of gravity is below the axis, and *unstable* when above it.

In general, there is stable equilibrium when the centre of gravity, on being disturbed in either direction, begins to *rise;* unstable when, if disturbed either way, it begins to *descend;* and neutral when the disturbance neither raises nor lowers the centre of gravity.

FIG. 51.



87. Questions on the Centre of Gravity.—

1. A frame 20 feet high, and 4 feet in diameter, is racked into an oblique form (Fig. 51), till it is on the point of falling; what is its inclination to the horizon? *Ans.* 78° 27′ 47″.

2. A stone tower, of the same dimensions as the former, is inclined till it is about to fall, but preserves its rectangular form; what is its inclination? *Ans.* 78° 41′ 24″.

3. A cube of uniform density lies on an inclined plane, and is prevented by friction from sliding down; to what inclination must the plane be tipped, that the cube may just begin to roll down? *Ans.* 45°.

4. What must be the inclination of a plane, in order that a regular prism of any given number of sides may be on the point of rolling down, if friction prevents sliding?

Ans. Equal to half the angle at the centre of the prism, subtended by one side.

5. A body weighing 83 lbs. is suspended, and drawn aside from the vertical 9° by a horizontal force; what pressure is there on the point of support, and what force urges it down the arc?

Ans. Pressure on the support, 81.978 lbs.
Moving force, 12.984 lbs.

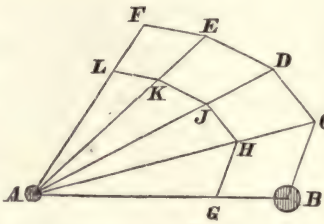
88. Motion of the Centre of Gravity of a System when one of the Bodies is Moved.—

When one body of a system is moved, the centre of gravity of the system moves in a similar path, and its velocity is to that of

the moving body as the mass of that body is to the mass of the whole system.

If the system contains but two bodies, A and B (Fig. 52), suppose A to remain at rest, while B describes the straight lines $B\ C$, $C\ D$, &c., the centre of gravity G will in the same time describe the similar series, $G\ H$, $H\ J$, &c. When B is in the position B, and the centre of gravity at G, $A\ G : A\ B :: B : A + B$; when B is at C, $A\ H : A\ C :: B : A + B$; $\therefore A\ G : A\ B :: A\ H : A\ C$. Hence $G\ H$ is parallel to $B\ C$, and $G\ H : B\ C :: B : A + B$. In like manner, $H\ J : C\ D :: B : A + B$, &c. Thus all the parts of one path are parallel to the corresponding parts of the other, and have a constant ratio to them. Therefore the paths are similar. As the corresponding parts are described in equal times, their lengths are as the velocities. But the lengths are as $B : A + B$; therefore the velocity of the common centre of gravity is to that of the moving body as the mass of the moving body is to the mass of both. The same reasoning is applicable when the body moves in a curve.

FIG. 52.



If the system contain any number of bodies, and the centre of gravity of the whole be at G, then the centre of gravity of all except B must be in the line $B\ G$ beyond G. Suppose it to be at A, and to remain at rest, while B moves; then it is proved in the same manner as before, that G, the centre of gravity of the whole system, moves in a path parallel to the path of B, and with a velocity which is to B's velocity as the mass of B to the mass of the entire system.

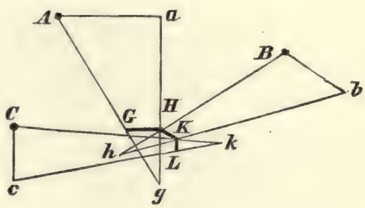
89. Motion of the Centre of Gravity of a System when Several of the Bodies are Moved.—

When any or all of the bodies of a system are moved, the centre of gravity moves in the same manner as if all the system were collected there, and acted on by the forces which act on the separate bodies.

Let A, B, C, &c. (Fig. 53), belong to a system containing any number of bodies, and let M be the mass of the system. Let A be moved over $A\ a$, B over $B\ b$, C over $C\ c$, &c. And first suppose the motions to be made in equal successive times. If the centre of gravity of the system is first at G, then that of all the bodies except A is in $A\ G$ produced, as at g. While A moves to a, G moves in a parallel line to H (Art. 88), and $G\ H : A\ a :: A : M$. In like manner, when B describes $B\ b$, the centre of gravity of the other bodies being at h, the centre of gravity of the system de-

scribes the parallel line, $H K$, and $H K : B b :: B : M$; and when C moves, $K L : C c :: C : M$, &c. Now, $A a$ and $G H$ represent the respective velocities of the body A, and the system M; therefore, if we convert the proportion $G H : A a :: A : M$ into an equation, we have $A \times A a = M \times G H$; that is, the momentum of the body A equals the momentum of the system M. It therefore requires the same force to move A over $A a$ as to move the system M over $G H$. The same is true of the other bodies. If then the several forces which move the bodies, limiting the number to three, for the present, were applied successively to the system collected at G, they would move it over $G H$, $H K$, $K L$. But if applied at once, they would move it over $G L$, the remaining side of the polygon. If, therefore, the forces, instead of acting successively on the bodies, were to move A over $A a$, B over $B b$, and C over $C c$, at the same time, the centre of gravity of the system would describe $G L$ in the same time. In the same way it may be proved, that whatever forces are applied to the several bodies of a system, the centre of gravity of the system is moved in the same manner as a body equal to the whole system would be moved, if all the same forces were applied to it.

Fig. 53.



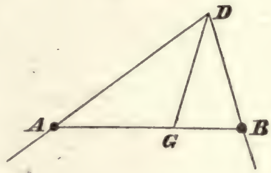
It is possible that the centre of gravity of a system should remain at rest, while all the bodies in it are in motion. For, suppose all the forces acting on the bodies to be such that they might be represented in direction and intensity by all the sides of a polygon, then, since a single body acted on by them would be in equilibrium, therefore the centre of gravity of the system would remain at rest, though the bodies composing it are in motion.

90. Mutual Action among the Bodies of a System.— The forces which have been supposed to act on the several bodies of a system are from without, and not forces which some of the bodies within the system exert on others. If the bodies of a system mutually attract or repel each other, such action cannot affect the centre of gravity of the whole system. For action and reaction are always opposite and equal. Whatever force one body exerts on any other to move it, that other exerts an equal force on the first, and the two actions produce equal and opposite effects on the centre of gravity between them. Therefore the centre of gravity of a system remains at rest, if the bodies which compose it are acted on only by their mutual attractions or repulsions.

91. Examples on the Motion of the Centre of Gravity.—

1. Two bodies, A and B, of given weights, start together from D (Fig. 54), and move uniformly with given velocities in the directions $D\ A$ and $D\ B$; required the direction and velocity of their centre of gravity.

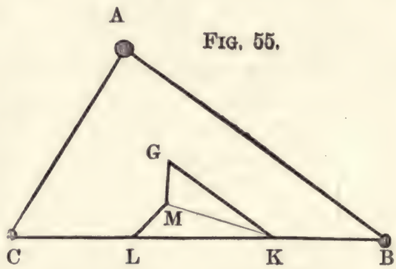
FIG. 54.



As the directions of $D\ A$ and $D\ B$ are given, we know the angle $A\ D\ B$; from the given velocities, we also know the lines $D\ A$ and $D\ B$, described in a certain time. Calculate the side $A\ B$, and the angles A and B. Find the place of the centre of gravity G between the bodies at A and B. Then, in the triangle $D\ B\ G$, $D\ B$, $B\ G$, and angle B are known, by which may be found the distance $D\ G$ passed over by the centre of gravity in the time, and $B\ D\ G$ the angle which its path makes with that of the body B.

2. Three bodies of given weight, A, B, C, in the same time and in the same order, describe with uniform velocity the three sides of the given triangle $A\ B\ C$ (Fig. 55); required the path of their centre of gravity.

FIG. 55.

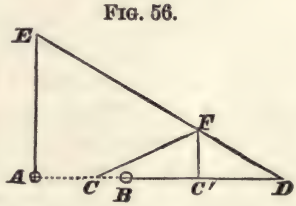


Let G be their centre of gravity before they move. If they move successively, G describes $G\ K$, $K\ L$, $L\ M$, parallel to the sides of the triangle, and having to them respectively the same ratios as the corresponding moving bodies have to the sum of the bodies (Art. 89). Thus, three sides of the polygon are known; and the angle $K = B$, and $L = C$. These data are sufficient for calculating the fourth side, $G\ M$, which the centre of gravity describes, when the bodies move together.

3. Show that when the three bodies in Example 2 are equal, the centre of gravity will remain at rest.

4. A (Fig. 56) weighs *one* pound; B weighs *two* pounds, and lies directly east of A; they move simultaneously, A northward, and B eastward, at the same uniform rate of 40 feet per second; required the direction and velocity of their centre of gravity.

FIG. 56.



Ans. Velocity is 29.814 feet per second.
Direction is E. 26° 33′ 54″ N.

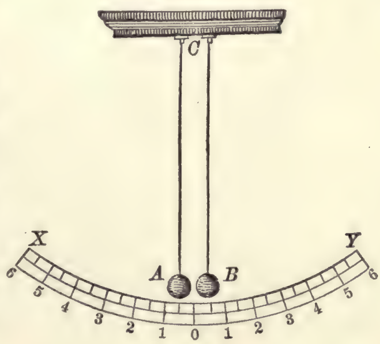
CHAPTER V.

THE COLLISION OF BODIES.

92. Elastic and Inelastic Bodies.—*Elastic* bodies are those which, when compressed, or in any way altered in form, tend to return to their original state. Those which show no such tendency are called *inelastic* or *non-elastic.* No substance is known which is entirely destitute of the property of elasticity; but some have it in so small a degree, that they are called inelastic, such as lead and clay. Elasticity is *perfect* when the *restoring* force, whether great or small, is equal to the *compressing* force. Air, and the gases generally, seem to be almost perfectly elastic; ivory, glass, and tempered steel, are imperfectly, though highly, elastic; and in different substances, the property exists in all conceivable degrees between the above-named limits.

93. Mode of Experimenting.—Experiments on collision may be made with balls of the same density suspended by long threads, so as to move in the line which joins their centres of gravity. If the arcs through which they swing are short compared with their radii, the balls, let fall from different heights, will reach the bottom sensibly at the same time, and will impinge with velocities which are very nearly proportional to the arcs. Thus A (Fig. 57), falling from 6, and B from 3, will come into collision at 0, with velocities which are as 2 : 1.

FIG. 57.



94. Collision of Inelastic Bodies.—Such bodies, after impact, move together as one mass.

The velocity of two inelastic bodies after collision is equal to the algebraic sum of their momenta, divided by the sum of the bodies.

Let A, B, represent the masses of the two bodies, and a, b, their respective velocities. Considering a as positive, if B moves

in the opposite direction, its velocity must be called $-b$. Let v be the common velocity after impact, and suppose the bodies to be moving in the same direction, the momentum of A is $A\,a$; that of B is $B\,b$; and the momentum of both after collision is $(A+B)\,v$. According to the third law of motion (Art. 13), whatever momentum A loses, B gains, so that the whole momentum is the same after collision as before; therefore

$$A\,a + B\,b = (A+B)\,v\,;\ \therefore v = \frac{A\,a + B\,b}{A+B.}.$$

To find the gain or loss of velocity of either body subtract the resulting velocity from the original velocity; a negative result indicates motion in a direction opposite to the original motion.

95. Questions on Inelastic Bodies.—

1. A, weighing 3 oz., and moving 10 feet per second, overtakes B, weighing 2 oz., and moving 3 feet per second; what is the common velocity after impact? *Ans.* $7\frac{1}{5}$ feet per second.

2. A weight of 7 oz., moving 11 feet per second, strikes upon another at rest weighing 15 oz.; required the velocity after impact? *Ans.* $3\frac{1}{2}$ feet per second.

3. A weighs 4 and B 2 pounds; they meet in opposite directions, A with a velocity of 9, and B with one of 5 feet per second; what is the common velocity after impact? *Ans.* $4\frac{1}{3}$ feet per second.

4. $A = 7$ pounds, $B = 4$ pounds; they move in the same direction, with velocities of 9 and 2 feet per second; required the velocity lost by A and gained by B? *Ans.* $A\ 2\frac{6}{11}$, $B\ 4\frac{5}{11}$.

5. A body moving 7 feet per second, meets another moving 3 feet per second, and thus loses half its momentum; what are the relative masses of the two bodies? *Ans.* $A : B :: 13 : 7$.

6. A weighs 6 pounds and B 5; B is moving 7 feet per second in the same direction as A; by collision B's velocity is doubled; what was A's velocity before impact? *Ans.* $19\frac{5}{6}$ feet per second.

7. A body weighing 100 lbs., and having velocity 40 feet per second meets another weighing 20 lbs., and having velocity of 200 feet per second; what will be the velocity after impact? *Ans.* 0.

96. Collision of Elastic Bodies.—Elastic bodies after collision do not move together, but each has its own velocity. These velocities are found by *doubling* the *loss* and *gain* of inelastic bodies. When the elastic body A impinges on B, it loses velocity

while it is becoming compressed, and again, while recovering its form, it loses as much more, because the restoring force is equal to the compressing force. For a like reason, B gains as much velocity while recovering its form as it gained while being compressed by the action of A. Hence, doubling the expressions for loss and gain found by Art. 94, and applying them to the original velocities, we find the velocity of each body after collision, on the supposition of perfect elasticity.

Two equal elastic bodies, A and B, weighing 50 lbs. each, moving with velocities, $A = 40$ ft., and $B = 20$ ft. per second, meet; what will be the velocity of each after impact? First we must find the gain and loss of velocity on the supposition that the bodies are inelastic, and then double such gain or loss; therefore, according to Art. 94, calling the velocity after impact v, we have $(50 + 50)\, v = 50 \times 40 - 50 \times 20$, calling the velocity of B negative as the bodies move in opposite directions,—

$$\text{whence } v = \frac{1000}{100} = 10.$$

A loses $40 - 10 = 30$ ft. per second, and B loses $-20 - 10 = -30$ feet per second; that is to say, B loses all its motion in its original direction, and moves backward with velocity 10.

Now as these are elastic we must double the gain and loss, and we have A's loss $= 60$ ft. and B's $= -60$ ft.; therefore A must move with velocity $40 - 60 = -20$, and B with velocity $-20 - (-60) = 40$, hence A must now move in a direction opposite to the first, with velocity 20, and B also in direction opposite to its previous motion, with velocity 40. *Each body takes the velocity of the other when the bodies are equal.*

97. Questions on Elastic Bodies.—

1. A, weighing 10 lbs. and moving 8 feet per second, impinges on B, weighing 6 lbs. and moving in the same direction, 5 feet per second; what are the velocities of A and B after impact?

Ans. A's $= 5\frac{3}{4}$, B's $= 8\frac{3}{4}$.

2. $A : B :: 4 : 3$; directions the same; velocities $5 : 4$; what is the ratio of their velocities after impact? *Ans.* $29 : 36$.

3. A, weighing 4 lbs., velocity 6, meets B, weighing 8 lbs., velocity 4; required their respective directions and velocities after collision? *Ans.* A is reflected back with a velocity of $7\frac{1}{3}$, and B with a velocity of $2\frac{2}{3}$.

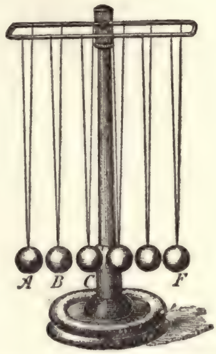
4. A and B move in opposite directions; A equals $4\,B$, and $b = 2\,a$; how do the bodies move after collision?

Ans. A returns with $\frac{1}{5}$, B with $1\frac{2}{5}$ its original velocity.

98. Series of Elastic Bodies.—

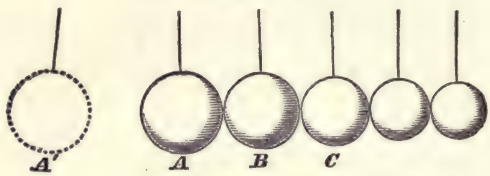
1. *Equal bodies.*—Let a row of equal elastic bodies, $A, B, C \ldots F$ (Fig. 58) be suspended in contact; then (Art. 96), if A be drawn back and left to fall against B, it will rest after impact, and B will tend to move on with A's velocity; after the impact of B on C, B will remain, and C tend to move with the same velocity; and so the motion will be transmitted through the series, and F will move away, while all the others remain at rest.

FIG. 58.



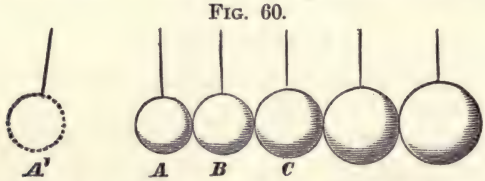
2. *Decreasing series.*—If the bodies decrease, as A, B, C, &c. (Fig. 59), and A be drawn back to A', and allowed to fall against B, then A still moves forward, while B receives a greater velocity than A had, C still greater, &c. The last of the series, therefore, moves with the greatest velocity, and each one with a greater velocity than that which impinged on it.

FIG. 59.



3. *Increasing series.*—If the bodies increase, as A, B, C, &c. (Fig. 60), then, when A falls from A' against B, it imparts to B a

FIG. 60.



less velocity than it had itself, and rebounds; in like manner B rebounds from C, and so on; while the last of the series goes forward with less velocity than any previous one would have had if it had been the last.

1. There are ten bodies whose masses increase geometrically by the constant ratio 3, and the first impinges on the second with the velocity of 5 feet per second; required the motion of the last body? *Ans.* The last body would move with the velocity of $\frac{5}{512}$ feet per second.

99. Living Force lost in the collision of Inelastic Bodies.—The amount of living force (Art. 37) before collision is $A\,a^2+B\,b^2$; and after collision it is $(A+B)\times\frac{(A\,a+B\,b)^2}{(A+B)^2}=\frac{(A\,a\,+\,B\,b)^2}{A\,+\,B}$. Subtract the latter from the former, and call the remainder d. Then $d=A\,a^2+B\,b^2-\frac{(A\,a\,+\,B\,b)^2}{A\,+\,B}$. Expanding and uniting terms, $d=\frac{AB\,(a-b)^2}{A+B}$. This value of d is positive, because $(a-b)^2$ is necessarily positive, as well as A and B. Therefore there is always a loss of living force in the collision of inelastic bodies.

This motion, or visible energy, seemingly lost, is transformed into heat by the impact.

100. Living Force Preserved in the Collision of Elastic Bodies.—The living force of A before collision is $A\,a^2$; after collision, it is $A\times\frac{\{(A-B)\,a+2\,B\,b\}^2}{(A+B)^2}$. Subtracting the latter from the former, the loss (supposing there is loss) is

$$\frac{(A+B)^2 A\,a^2-(A-B)^2 A\,a^2-4\,(A-B)\,A\,B\,a\,b-4\,A\,B^2 b^2}{(A+B)^2}\quad . \;(1.)$$

The living force of B before collision is $B\,b^2$; after collision, it is $B\times\frac{\{(B-A)\,b+2\,A\,a\}^2}{(A+B)^2}$; and the expression for loss is

$$\frac{(A+B)^2 B\,b^2-(B-A)^2 B\,b^2-4\,(B-A)\,A\,B\,a\,b-4\,A^2\,B\,a^2}{(A+B)^2}\quad . \;(2.)$$

Therefore the total loss of living force is the sum of the expressions (1.) and (2.).

Reducing the two first terms in each fraction to one, the fractions become

$$\frac{4\,A^2\,B\,a^2-4\,(A-B)\;A\;B\,a\,b-4\;A\;B^2\,b^2}{(A+B)^2}\quad .\;.\;.\;.\;.\;(3.)$$

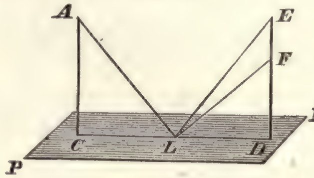
and

$$\frac{4\;A\;B^2\,b^2-4\,(B-A)\;A\;B\,a\,b-4\;A^2\;B\,a^2}{(A+B)^2}\quad .\;.\;.\;.\;.\;(4.)$$

If the fractions (3) and (4) be added, it is evident that the numerators cancel each other, and therefore the sum of the fractions is zero. Hence, there is no loss of living force in the collision of elastic bodies.

101. Impact on an Immovable Plane.—If an inelastic body strikes a plane perpendicularly, its motion is simply *destroyed;* in strictness, however, it imparts an infinitely small velocity to the body called immovable. If it strikes obliquely, and the plane is smooth, it slides along the plane with a diminished velocity. Let *A L* (Fig. 61) represent the motion of the body before impact on the plane *P N,* and resolve it into *A C,* perpendicular, and *C L,* parallel to the plane. Then *A C,* as before, is destroyed, but *C L* is not affected; hence the former velocity is to its velocity on the plane, as *A L* : *C L* :: radius : cosine of the inclination.

Fig. 61.



If a perfectly elastic body impinges perpendicularly upon a plane, then, after its motion is destroyed, the force by which it resumes its form causes an equal motion in the opposite direction; that is, the body rebounds in its own path as swiftly as it struck. But if the impact is oblique, the body rebounds at an equal angle on the opposite side of the perpendicular. For, resolve *A L,* as before, into *A C, C L*; the latter continues uniformly; but, instead of the component *A C,* there is an equal motion in the opposite direction. Therefore, if *L D* is made equal to *C L,* and *D E* equal to *A C,* the resultant of *L D* and *D E* is *L E,* which is equal to *A L,* and has the same inclination to the plane. Hence, the angles of incidence and reflection are equal, and on opposite sides of the perpendicular to the surface at the place of impact.

102. Imperfect Elasticity.—The formulæ for the velocity of bodies after collision, and the statements of the preceding article, are correct only on the supposition that bodies are, on the one hand, entirely destitute of elasticity, or on the other perfectly elastic. As no solid bodies are known, which are strictly of either class, these deductions are found to be only near approximations to the results of experiment. In all practical cases of the impact of movable bodies, the loss and gain of velocity are *greater* than if they were inelastic, and *less* than if perfectly elastic. And in cases of impact on a plane, there is always *some* velocity of rebound, but less than the previous velocity; and therefore, if the collision is oblique, the body has less velocity, and makes a smaller angle with the plane than before. For, making *D F* less than *A C,* the resultant *L F* is less than *A L,* and the angle *D L F* is smaller than *D L E,* or *A L C.*

CHAPTER VI.

SIMPLE MACHINES.

103. Classification of Machines.—In the preceding chapters, the motion of bodies has been supposed to arise from the immediate action of one or more forces. But a force may produce effects *indirectly*, by means of something which is interposed for the purpose of changing the mode of action. These intervening bodies are called, in general, *machines;* though the names, *tools, instruments, engines,* &c., are used to designate particular classes of them. The elements of machinery are called *simple machines.* The following list embraces those in most common use :

1. The lever.
2. The wheel and axle.
3. The pulley.
4. The rope machine.
5. The inclined plane.
6. The wedge.
7. The screw.
8. The knee-joint.

In respect to principle, these eight, and all others, may be reduced to three.

1. The law of *equal moments,* applicable in those cases in which the machine turns on a pivot or axis, as in the lever and the wheel and axle.

2. The principle of *transmitted tension,* to be applied wherever the force is exerted through a flexible cord, as in the pulley or rope machine.

3. The principle of *oblique action,* applicable to all the other machines, the force being employed to balance or overcome one component only of the resistance.

The force which ordinarily puts a machine in motion is called the *power;* the force which resists the power, and is balanced or overcome by it, is called the *resistance, or weight.*

A *compound* machine is one in which two or more simple machines are so connected that the weight of the first constitutes the power of the second, the weight of the second the power of the third, &c.

I. The Lever.

104. The Three Orders of Straight Lever.—The lever is a bar of any form, free to turn on a fixed point, which is called the *fulcrum.* In the *first* order of lever, the *fulcrum* is between the power and weight (Fig. 62); in the *second,* the *weight* is between the power and fulcrum (Fig. 63); in the *third,* the *power* is between the weight and fulcrum (Fig. 64).

Fig. 62.

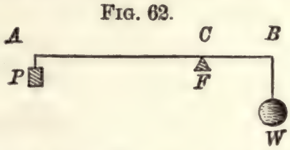


Fig. 63.

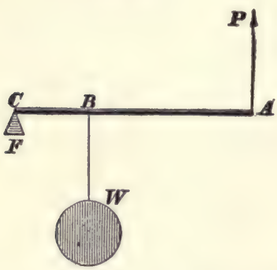
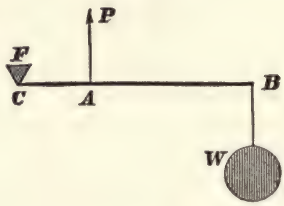


Fig. 64.



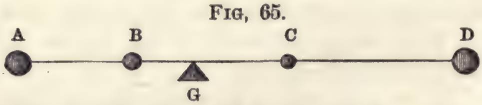
105. Equal Moments in Relation to the Fulcrum.—According to the principle of moments, we find for each order of the lever, $P \times A\ C = W \times B\ C$; that is,

The power and weight have equal moments in relation to the fulcrum.

The *moment* of either force is the measure of its efficiency to turn the lever; for, since the lever is in equilibrium, the efficiency of the power to turn it in one direction must equal the efficiency of the weight to turn it in the opposite direction. We may therefore use $P \times A\ C$ to represent the former, and $W \times B\ C$, the latter.

If *several* forces, as in Fig. 65, are in equilibrium, some tending

Fig, 65.



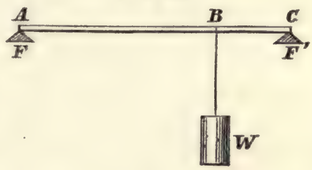
to turn the bar in one direction, and others in the opposite, then A and B must have the same efficiency to produce one motion as C and D have to produce the opposite; that is, $A \times A\ G + B \times B\ G = C \times C\ G + D \times D\ G$; or,

The *sum of the moments* of A and B equals the *sum of the moments* of C and D.

In order to allow for the influence of the weight of the lever itself, consider it to be collected at its centre of gravity, and add its moment to that of the power or weight, according as it aids the one or the other. In Fig. 62, let the weight of the lever $= w$, and the distance of its centre from C on the side of $P = m$; then $P \times A\,C + m\,w = W \times B\,C$. In the 2d and 3d orders, the moment of the lever necessarily aids the weight; and hence, in each case, $P \times A\,C = W \times B\,C + m\,w$.

If a weight hangs on a bar between two supports, as in Fig. 66, it may be regarded as a lever of the 2d order, the reaction of either support being considered as a power. Let F denote the reaction at A, and F' at C; then by the theorems of parallel forces, we have the pressures at A and C inversely as their distances from B, and $W = F + F'$.

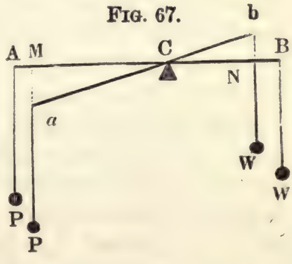
FIG. 66.



Resuming the equation $P \times A\,C = W \times B\,C$, we derive the proportion $P : W :: B\,C : A\,C$; hence, in each order of the straight lever, when the forces act in parallel lines, *The power and weight are inversely as the lengths of the arms on which they act.*

106. The Acting Distance.—In the three orders, as above described, the equilibrium is not destroyed by inclining the lever to any angle whatever with the horizon, provided the lever is symmetrical with respect to its longer axis and the centre of motion C is on this axis and not *above* or *below* it, and provided the directions of the forces remain vertical. For by the principle of parallel forces *any* straight line intersecting the lines of the forces is divided by the line of the resultant into parts which are inversely as the forces; therefore (Fig. 67) $b\,C : a\,C :: P : W$. Hence, the resultant of P and W remains at C, in every position of the lever. By similar triangles, $b\,C : a\,C :: C\,N : C\,M$; $\therefore P : W :: C\,N : C\,M$; $\therefore P \times C\,M = W \times C\,N$. The lines $C\,M$ and $C\,N$, which are drawn from the fulcrum perpendicular to the lines in which the forces act, are called the *acting distances* or the *lever arms* of the power and weight, respectively. And as they may be employed in levers of irregular form, the moments of power and weight are usually measured by the products, $P \times C\,M$ and $W \times C\,N$; therefore, *the power multiplied*

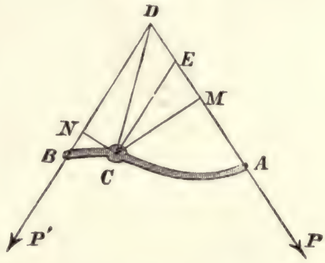
FIG. 67.



by its acting distance equals the weight multiplied by its acting distance; or, more briefly, the moment of the power equals the moment of the weight, as in Art. 105. In Figs. 62, 63, and 64, the acting distances are in each case identical with the arms of the lever.

107. Lever not Straight, and Forces not Parallel.—Let $A\ C\ B$ (Fig. 68) be a lever of any form, and let it be in equilibrium by the forces P and P', acting in any oblique directions in the same plane. Produce $P\ A$ and $P'\ B$ till they meet in D; then, if the fulcrum is at C, the resultant must be in the direction $D\ C$; otherwise the reaction of the fulcrum cannot keep the system in equilibrium (Art. 43). Therefore (Art 44),

FIG. 68.



$$P : P' :: \sin B\ D\ C : \sin A\ D\ C.$$

Draw $C\ M$ perpendicular to $A\ D$, and $C\ N$ to $B\ D$, and they are the sines of $A\ D\ C$ and $B\ D\ C$, to the same radius $D\ C$.

$$\therefore\ P : P' :: C\ N : C\ M;\ \text{and}\ P \times C\ M = P' \times C\ N.$$

The lines $C\ M$ and $C\ N$ are the acting distances of P and P'; therefore the law of the lever in all cases is the same, namely:

The moment of the power equals the moment of the weight.

When the forces act obliquely, the pressure on the fulcrum is less than the sum of the forces; for, if $C\ E$ is parallel to $B\ D$, then $D\ E$, $E\ C$, and $C\ D$, represent the three forces which are in equilibrium. But $C\ D$ is less than the sum of $D\ E$ and $E\ C$.

108. The Compound Lever.—When a lever acts on a second, that on a third, &c., the machine is called a *compound lever.* The law of equilibrium is—

The continued product of the power and acting distances on the side of the power is equal to the continued product of the weight and acting distances on the side of the weight.

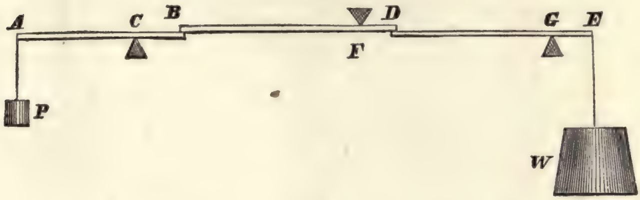
Let the force exerted by $A\ B$ on $B\ D$ (Fig. 69) be called x, and that of $B\ D$ on $D\ E$ be called y; then

$$P \times A\ C = x \times B\ C;$$
$$x \times B\ F = y \times D\ F;$$
$$y \times D\ G = W \times G\ E.$$

Multiply these equations together and omit common factors, and we have

$$P \times A\,C \times B\,F \times D\,G = W \times B\,C \times D\,F \times G\,E.$$

FIG. 69.



If the levers were of irregular forms, the acting distances might not be identical with the arms, as they are in the figure.

109. The Balance.—This is a common and valuable instrument for weighing. It is a straight lever with equal arms, having scale-pans, either suspended at the ends, or standing upon them, one to contain the poises, and the other the substance to be weighed. For scientific purposes, particularly for chemical analysis, great care is bestowed on the construction of the balance.

The arms of the balance, measured from the fulcrum to the points of suspension, must be precisely equal.

The knife-edges forming the fulcrum, and the points of suspension, are made of hardened steel, and arranged exactly in a straight line.

The centre of gravity of the beam is *below* the fulcrum, so that there may be a stable equilibrium ; and yet below it by an exceedingly small distance, in order that the balance may be very sensitive.

To preserve the edge of the fulcrum from injury, the beam is raised by supports called *Y's*, when not in use.

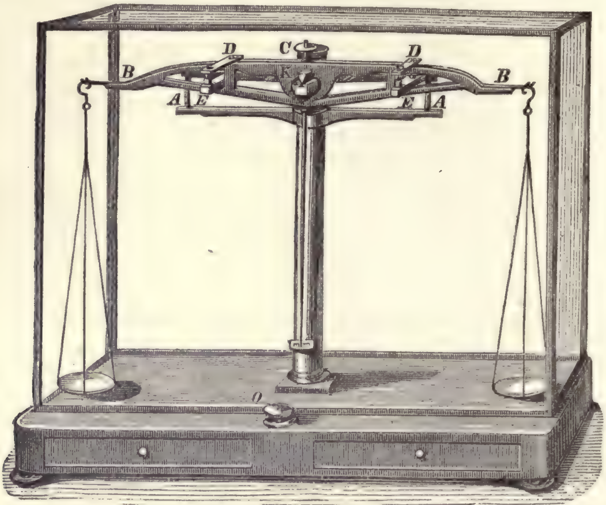
A long index at right angles to the beam, points to zero on a scale when the beam is horizontal.

To protect the instrument from dust and moisture at all times, and from air-currents while weighing, the balance is in a glass case, whose front can be raised or lowered at pleasure.

A balance for chemical analysis is shown in Fig. 70. By turning the knob *O*, the beam can be raised on the *Y's A A* from the surface on which the fulcrum *K* rests. The screw *C* raises and lowers the fulcrum in relation to the centre of gravity of the beam, in order to increase or diminish the sensitiveness of the instrument. In the most carefully made balances, the index will make

a perceptible change, by adding to the scale *one* millionth of the poise.

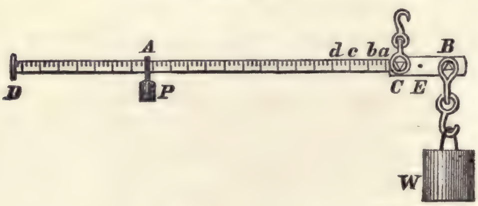
FIG. 70.



For commercial purposes, it is convenient to have the scale-pans above the beam. This is done by the use of additional bars, which with the beam form parallelograms, whose upright sides are rods, projecting upward and supporting the scales. Such contrivances necessarily increase friction ; but balances so constructed are sufficiently sensitive for ordinary weighing.

110. The Steelyard.—This is a weighing instrument, having a graduated arm, along which a poise may be moved, in order to balance various weights on the short arm. While the moment of the article weighed is changed by increasing or diminishing its quantity, that of the poise is changed by altering its acting distance. Since $P \times A\ C = W \times B\ C$ (Fig. 71), and P is constant,

FIG. 71.



and also the distance $B\ C$ constant, $A\ C \propto W$; hence, if W is successively 1 lb., 2 lbs., 3 lbs., &c., the distances of the notches,

a, b, c, &c., are as 1, 2, 3, &c.; in other words, the bar $C\,D$ is divided into equal parts. In this case, the graduation begins from the fulcrum C as the zero point.

But suppose, what is often true, that the centre of gravity of the steelyard is on the long arm, and that P placed at E would balance it; then the moment of the instrument itself is on the side $C\,D$, and equals $P \times C\,E$. Hence, the equation becomes

$$P \times A\,C + P \times C\,E = W \times B\,C;\ \text{or}$$
$$P \times A\,E = W \times B\,C.$$

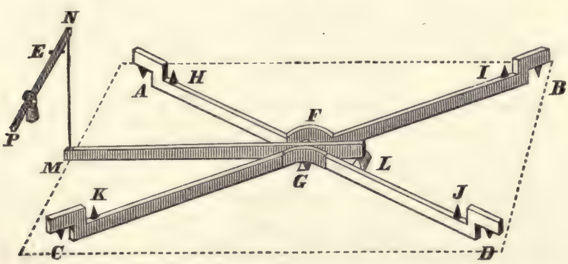
$\therefore W \propto A\,E$; and the graduation must be considered as commencing at E for the zero point. Such a steelyard cannot weigh below a certain limit, corresponding to the first notch a.

To find the length of the divisions on the bar, divide $A\,E$, the distance of the poise from the zero point, by W, the number of units balanced by P, when at that distance.

The steelyard often has *two* fulcrums, one for less and the other for greater weights.

111. Platform Scales.—This name is given to machines arranged for weighing heavy and bulky articles of merchandise. The largest, for cattle, loaded wagons, &c., are constructed with the platform at the surface of the ground. In order that the platform may stand firmly beneath its load, it rests by four feet on as many levers of the second order, whose arms have equal ratios. $A\,F$, $B\,F$, $C\,G$, $D\,G$ (Fig. 72), are four such levers, resting on the

Fig. 72.



fulcrums, A, B, C, D, while the other ends meet on the knife-edge, $F\,G$, of another lever, $L\,M$. This fifth lever has its fulcrum at L, and its outer extremity is attached by a vertical rod, $M\,N$, to a steelyard, whose fulcrum is E, and poise P. The five levers are arranged in a square cavity just below the surface of the ground. The dotted line shows the outline of the cavity. On the bearing points of the four levers, H, I, J, K, rest the feet of the platform (not represented), which is firmly built of plank, and just

fits into the top of the cavity without touching the sides. The machine is a compound lever of three parts ; for the four levers act as one at $F\ G$, and are used to give steadiness to the platform which rests upon them.

A construction quite similar to the above is made of portable size, and used in all mercantile establishments for weighing heavy goods.

112. Questions on the Lever.—1. $A\ B$ (Fig. 73) is a uniform bar, 2 feet long, and weighs 4 oz.; where must the fulcrum be put, that the bar may be balanced by P, weighing 5 lbs. ?

FIG. 73.

Ans. $\frac{4}{7}$ of an inch from A.

2. A lever of the second order is 25 feet long ; at what distance from the fulcrum must a weight of 125 pounds be placed, so that it may be supported by a power able to sustain 60 pounds, acting at the extremity of the lever.

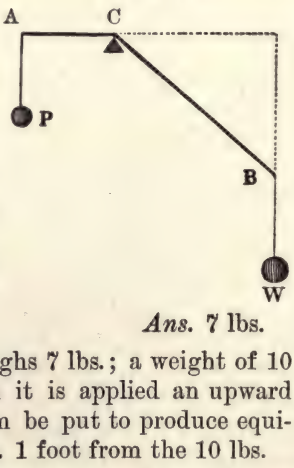
Ans. 12 feet.

3. A and B are of the same height, and sustain upon their shoulders a weight of 150 pounds, placed on a pole $9\frac{1}{3}$ feet long ; the weight is placed $6\frac{2}{3}$ feet from A ; what is the weight sustained by each person ?

Ans. A sustains $42\frac{6}{7}$ lbs., and B sustains $107\frac{1}{7}$ lbs.

4. A *bent lever*, $A\ C\ B$ (Fig. 74), has the arm $A\ C = 3$ feet, $C\ B = 8$ feet, $P = 5$ lbs., and the angle $A\ C\ B = 140°$; what weight, W, must be attached at B, in order to keep $A\ C$ horizontal ?

FIG. 74.



Ans. 2.4476 lbs.

5. A cylindrical straight lever is 14 feet long, and weighs 6 lbs. 5 oz.; its longer arm is 9, and its shorter 5 feet; at the extremity of its shorter arm a weight of 15 lbs. 2 oz. is suspended ; what weight must be placed at the extremity of the longer arm to keep it in equilibrium ?

Ans. 7 lbs.

6. A uniform bar, 12 feet long, weighs 7 lbs.; a weight of 10 lbs. hangs on one end, and 2 feet from it is applied an upward force of 25 lbs., where must the fulcrum be put to produce equilibrium ? *Ans.* 1 foot from the 10 lbs.

7. The lengths of the arms of a balance are a and b. When p ounces are hung on a, they balance a certain body ; but it re-

quires q ounces to balance the same body, when placed in the other scale. What is the true weight of the body? According to the first weighing, $a\,p = b\,x$; according to the second, $b\,q = a\,x$. $\therefore a\,b\,p\,q : = a\,b\,x^2$, and $x = \sqrt{p\,q}$. Hence, the true weight is a geometrical mean between the apparent weights.

8. On one arm of a false balance a body weighs 11 lbs., on the other, 17 lbs. 3 oz.; what is the true weight?

Ans. 13 lbs. 12 oz.

9. Four weights of 1, 3, 5, 7 lbs., respectively, are suspended from points of a straight lever, eight inches apart; how far from the point of suspension of the first weight must the fulcrum be placed, that the weights may be in equilibrium?

Ans. 17 inches.

10. Two weights keep a horizontal lever at rest, the pressure on the fulcrum being 10 lbs., the difference of the weights 4 lbs., and the difference of the lever arms 9 inches; what are the weights and their lever arms?

Ans. Weights, 7 lbs. and 3 lbs.; arms, $6\frac{3}{4}$ in. and $15\frac{3}{4}$ in.

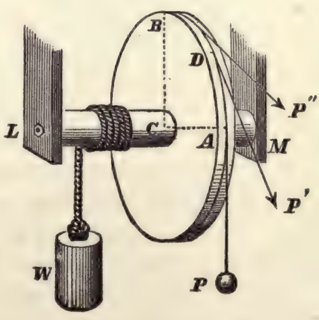
II. The Wheel and Axle.

113. Description and Law of the Machine.—The wheel and axle consists of a cylinder and a wheel, firmly united, and free to revolve on a common axis. The power acts at the circumference of the wheel in the direction of a tangent, and the weight in the same manner, at the circumference of the cylinder or axle; so that the acting distances are the radii at the two points of contact. As the system revolves, the radii successively take the place of acting distances, without altering at all the relation of the forces to each other. The wheel and axle is therefore a kind of endless lever.

Let W (Fig. 75) be the weight suspended from the axle, tending to revolve it on the line $L\,M$; and P, the power acting on the wheel, tending to revolve the system in the opposite direction. It is plain that the acting distances are the radius of the axle, and $A\,C$ the radius of the wheel. In case of equilibrium, the moment of W equals the moment of P. Calling the radius of the axle r, and the radius of the wheel R, then $W \times r = P \times R$; or

$$P : W :: r : R.$$

Fig. 75.

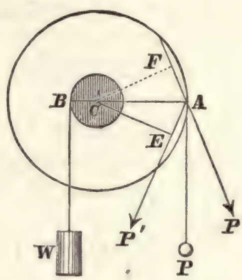


If, instead of the weight P, suspended on the wheel, the rope be drawn by any force in the direction P' or P'', it is still tangent to the circumference, and therefore its acting distance, CD or CB, the same as before. In general, the law of equilibrium for this machine is,

The moment of the Power is equal to the moment of the Weight.

If the rope on the wheel, being fastened at A (Fig. 76) is drawn by the side of the wheel, as $A\ P'$, the acting distance of the power is diminished from CA to CE, and therefore its efficiency is diminished in the same ratio. Were the rope drawn away from the wheel, as $A\ P''$, making an equal angle on the other side of $A\ P$, the same effect is produced, the acting distance now becoming CF.

Fig. 76.



The radius of the wheel and the radius of the axle should each be reckoned from the axis of rotation to the centre of the rope; that is, half of the thickness of the rope should be added to the radius of the circle on which it is coiled. Calling t the half thickness of the rope on the axle, and t' that of the rope on the wheel, the equation of equilibrium is

$$P \times (R + t') = W \times (r + t).$$

In considering the wheel and axle no account has been taken of the stiffness of the rope, which acts as a constant resistance, opposing motion in winding upon a drum or wheel, and also in unwinding.

114. Differential Pulley.—A modification of the wheel and axle, called a *differential pulley*, is of great use in raising very heavy weights through short distances.

The pulley consists of a *solid* wheel A (Fig. 77), one half of which, b, is of less diameter than the other half, a, suspended in a block in the usual manner.

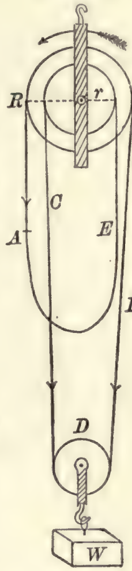
Fig. 77.

A continuous *chain* is used, which we may trace from the point A (Fig. 78), upward, over the larger of the two circumferences, then downward through B to the movable pulley D, thence upward through C around the smaller circumference of the wheel, thence down through E and back to the point of beginning at A.

Call the radii R and r as indicated in the figure, and suppose

a downward force P to be applied to the chain at A, then $P \times R + \frac{1}{2} W \times r = \frac{1}{2} W \times R$, in which equation no account is taken of the weight of the chain.

FIG. 78.



Transposing, we obtain,

$$P \times R = \tfrac{1}{2} W (R-r) \text{ or}$$
$$P : \tfrac{1}{2} W :: R-r : R.$$

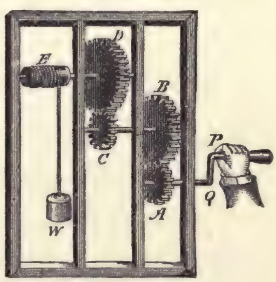
Now $R-r$ may be made as small as we please, and hence the power also may be made small as compared with the weight. The weight of the chain and the friction act as resistances to motion, and are sufficient to prevent the downward run of D after the hand is removed from A, even when W is very great. This pulley may be found in any large foundry, or machine shop.

115. The Compound Wheel and Axle.—When a train of wheels, like that in Fig. 79, is put in motion, those which *communicate* motion by the circumference are called *driving wheels*, as A and C; those which *receive* motion by the circumference are called *driven wheels.* And the law of equilibrium is,

The continued product of the power and radii of the driven wheels is equal to the continued product of the weight and radii of the driving wheels.

The crank $P\,Q$ is to be reckoned among driven wheels; the axle E among driving wheels.

FIG. 79.



Let the radius of B be called R; of D, R'; of A, r; of C, r'; of E, r''. Call the force exerted by A on B, x; that of C on D, y. Then

$$P \times P\,Q = r \times x;$$
$$x \times R = r' \times y;$$
$$y \times R' = r'' \times W.$$

Multiply and omit common factors, and we have

$$P \times P\,Q \times R \times R' = W \times r'' \times r' \times r.$$

If the driving wheels are equal to each other, and also the driven wheels, and the number of each is n, then

$$P \times R^n = W r^n.$$

116. Direction and Rate of Revolution.—When two wheels are geared together by teeth, they necessarily revolve in contrary directions. Hence, in a train of wheels, the alternate axles revolve the same way.

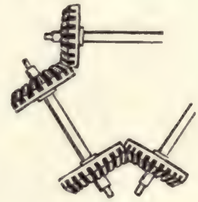
The circumferences of two wheels which are in gear move with

the same velocity ; hence the number of revolutions will be reciprocally as the radii of the wheels.

Since teeth which gear together are of the same size, the relative *number of teeth* is a measure of the relative circumferences, and therefore of the relative radii of the wheels. If the wheel *A* (Fig. 79) has 20 teeth, and *B* has 40, and again if *C* has 15, and *D* 45, then for every revolution of *B, A* revolves twice, and for every revolution of *D, C* revolves three times. Therefore, six turns of the crank are necessary to give one revolution to the axle *E*.

By cutting the teeth of wheels on a conical instead of a cylindrical surface, the axles may be placed at any angle with each other, as represented in Fig. 80.

Fig. 80.



Whether axles are parallel or not, *bands* instead of teeth may be used for transmitting rotary motion. But as bands are liable to slip more or less, they cannot be employed in cases requiring exact relations of velocity.

117. Questions on the Wheel and Axle.—

1. A power of 12 lbs. balances a weight of 100 lbs. by a wheel and axle; the radius of the axle is 6 inches; what is the *diameter* of the wheel?

Ans. 8 ft. 4 in.

2. $W=500$ lbs.; $R = 4$ ft.; $r = 8$ in.; the weight hangs by a rope 1 inch thick, but the power acts at the circumference of the wheel without a rope; what power will sustain the weight?

Ans. 88.54 lbs.

3. In Fig. 79, *A* and *C* have each 15 teeth, *B* and *D* each 40 teeth; the radius of the axle *E* is 4 inches; the rope on it 1 inch in diameter; and the radius of the crank *P Q* is 18 inches; what is the ratio of power to weight in equilibrium? *Ans.* 1 : $28\frac{4}{9}$.

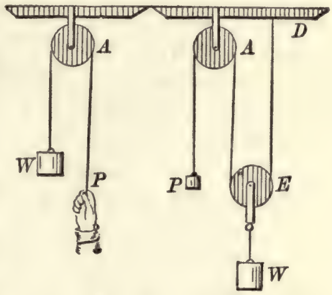
118. The Pulley Described.—The pulley consists of one or more wheels or rollers, with a rope passing over the edge in which a groove is sunk to keep the rope in place. The axis of the roller is in a *block*, which is sometimes fixed, and sometimes rises and falls with the weight; and the pulley is accordingly called a *fixed pulley* or a *movable pulley*. The principle which explains the relation of power and weight in every form of pulley is this:

Whatever strain or tension is applied to one end of a cord, is transmitted through its whole length, if it does not branch, however much its direction is changed.

In the pulley, the sustaining portions of the rope are assumed to be parallel to each other.

119. The Fixed Pulley.—In the fixed pulley, A (Fig. 81), the force P, produces a tension in the string, which is transmitted through its whole length, and which can be balanced only when W equals P. Hence, in the fixed pulley, *the power and weight are equal.* This machine is useful for changing the *direction* in which the force is applied to the weight; and if the power only acts in the plane of the groove of the wheel, it is immaterial what is its direction, horizontal, vertical, or oblique.

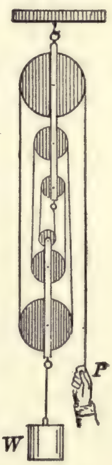
FIG. 81. FIG. 82.



120. The Movable Pulley.—In Fig. 82, the tension produced by P, is transmitted from A down to the wheel E, and thence up to D; therefore W is sustained by *two* portions of the rope, each of which exerts a force equal to P.

$$\therefore W = 2P; \text{ or } P : W :: 1 : 2.$$

FIG. 83.



The same reasoning applies, where the rope passes between the upper and lower blocks any number of times, as in Fig. 83. The force causes a tension in the rope, which is transmitted to every portion of it. If n is the number of portions which sustain the lower block, then W is upheld by $n\,P$; and if there is equilibrium, $P : W :: 1 : n$. In the figure, the weight equals *six times* the power. The law of equilibrium, therefore, for the movable pulley with one rope, is this, *The power is to the weight as one to the number of the sustaining portions of the rope.*

121. The Compound Pulley.—Wherever a system of pulleys has separate ropes the machine is to be regarded as compound, and its efficiency is calculated accordingly. Figures 84 and 85 are examples. In Fig. 84 call the weight sustained by F, x, and that sustained by D, y. Then (Art. 120),

$$P : x :: 1 : 2;$$
$$x : y :: 1 : 2:$$
$$y : W :: 1 : 2.$$
$$\therefore P : W :: 1 : 2^3 :: 1 : 8.$$

And if n is the number of ropes, $P : W :: 1 : 2^n$.

In Fig. 85 the tension P is transmitted over A directly to the weight at G; the wheel A is loaded, therefore, with $2\ P$, and a tension of $2\ P$ comes upon the second rope, which is transmitted over B to the weight at F. In like manner, a tension of $4\ P$ is transmitted over C to E. The sum of all these being applied to the weight, it must therefore be equal to that sum in case of equilibrium. Therefore, $P : W :: 1 : 1 + 2 + 4 + \&c.$ Now the sum of this geometrical series to n terms is $2^n - 1$, $\therefore P : W :: 1 : 2^n - 1$. This combination is therefore a little less efficient than the preceding.

Fig. 84.

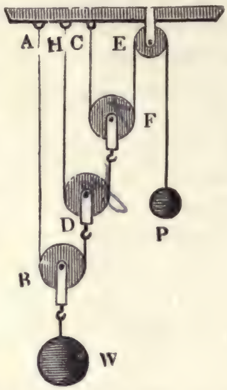
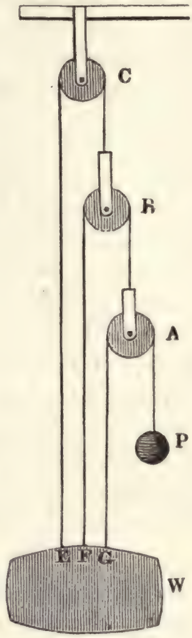


Fig. 85.



Since the several ropes have different tensions, the weight cannot be balanced upon them, unless those of greatest tension are nearest the line of direction of the body. For example, if the rope F is directed toward the centre of gravity of the weight, the rope G should be attached four times as far from it as the rope E, in order to prevent the weight from tipping.

The pulley owes its efficiency as a machine to the fact, that the tension produced by the power is applied *repeatedly* to the weight. The only use of the wheels is to diminish friction. Were it not for friction, the rope might pass round fixed pins in the blocks, and the ratio of power to weight would still be in every case the same as has been shown.

IV. The Rope Machine.

122. Definition and Law of this Machine.—

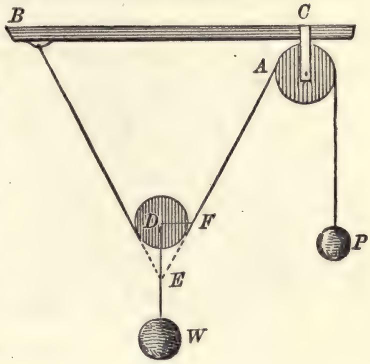
The rope machine is one in which the power and weight are in equilibrium by the tension of one or more ropes.

According to this definition the pulley is included. It is that particular form of the rope machine in which the sustaining parts of the ropes are parallel; and it is treated as a separate machine,

because its theory is very simple, and because it is used far more extensively than any other forms.

If the two portions of rope which sustain the weight are inclined, as in Fig. 86, then W is no longer equal to the sum of their tensions, as it is in the pulley, but is always less than that, according to the following law:

FIG. 86.



The power is to the weight as the sine of ½ the angle is to the sine of the whole angle between the parts of the rope.

Put $A\ E\ B = 2\ a$; then $F\ E\ D = a$, and since $\sin B\ E\ W = \sin B\ E\ D = \sin a$, we shall have (Art. 44) $P : W :: \sin a : \sin 2\ a$.

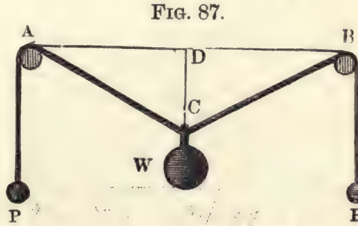
If in Fig. 87, the end of the cord, instead of being attached to the beam, is carried over another fixed pulley, and a weight equal to P is hung upon it, the equilibrium will be preserved, because all parts of the rope have a tension equal to P; therefore, as before,

$$P : W :: \sin a : \sin 2\ a.$$

123. Change in the Ratio of Power and Weight.—If P is given, all the possible values of W are included between $W = 0$, and $W = 2\ P$.

When the rope is straight from A to B (Fig. 87), so that $C\ D = 0$, then, by the above proportion, $W = 0$. As W is increased from zero, the point C descends; and when $D\ C = \frac{1}{2}\ B\ C$, then, by the proportion, $W = P$. In that case $D\ C\ B = 60°$, and the angles $A\ C\ B$, $A\ C\ W$, and $B\ C\ W$ are equal (each being 120°), as they should be, because each of the equal forces, P, P, and W, is as the sine of the angle between the directions of the other two.

FIG. 87.



But when W has increased to $2\ P$, it descends to an infinite distance; for then, by the proportion, $C\ D = B\ C$, that is, the side of a right-angled triangle is equal to the hypothenuse. Thus, the extreme values of W are 0 and $2\ P$.

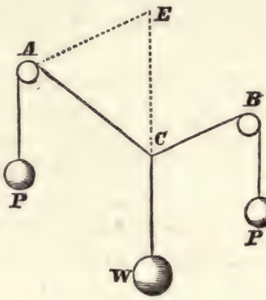
It appears from the foregoing, that a perfectly *flexible rope*

having weight cannot be drawn into a straight horizontal line, by any force however great; for C cannot coincide with D, except when $W = 0$.

124. The Branching Rope.—When C, where the weight is suspended, is a *fixed* point of the rope, we have a branching rope, and the principle of transmitted tension does not apply beyond the point of division.

Let P, P' and W (Fig. 88), be given, and C a fixed point of the rope. Produce $W\,C$, and let $A\,E$, drawn parallel to $C\,B$, intersect it in E. The sides of $A\,C\,E$ are proportional to the given forces; therefore its angles can be found, and the inclinations of $A\,C$ and $B\,C$ to the vertical $C\,W$ are known.

FIG. 88.



V. THE INCLINED PLANE.

125. Relation of Power, Weight, and Pressure on the Plane.—The mechanical efficiency of the inclined plane is explained on the principle of *oblique action;* that is, it enables us to apply the power to balance or overcome only *one component* of the weight, instead of the whole. Let the weight of the body G, lying on the inclined plane $A\,C$ (Fig. 89), be represented by W; and resolve it into F parallel, and N perpendicular to the plane. N represents the perpendicular pressure, and is equal to the reaction of the plane; F is the force by which the body tends to move down the plane.

Let $a =$ the angle C, the inclination of the plane; therefore $W\,D\,N = a$. Then $F = W \,.\, \sin a$; and $N = W \,.\, \cos a$.

FIG. 89.

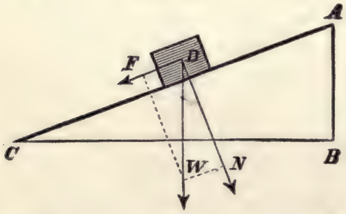
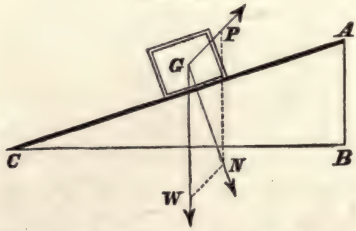


FIG. 90.



Now suppose a force P is applied at G (Fig. 90), which keeps the body at rest. Then the resultant of W and P must be N, which is resisted by the plane; therefore,

$$P : W :: \sin G\,N\,P, \text{ or } \sin a : \sin P\,G\,N.$$

When the power acts parallel to the plane, $P\,G\,N = 90°$, and we have $P : W :: \sin a : \sin 90° :: A\,B : A\,C$. Hence, when the power acts in a line parallel to the inclined plane, which is the most common direction,

The power is to the weight as the height to the length of the inclined plane.

When the power acts in a line parallel to the base of the inclined plane, $P\,G\,N = 90° - a$, and we have $P : W :: \sin a : \cos a :: A\,B : B\,C$. Hence, when the power acts in a line parallel to the base of the inclined plane,

The power is to the weight as the height is to the base of the inclined plane.

126. Power most Efficient when Acting Parallel to the Plane.—From the proportion

$$P : W :: \sin a : \sin P\,G\,N, \text{ we derive}$$

$$W = \frac{P \,.\, \sin P\,G\,N}{\sin a}.$$

Now as P and $\sin a$ are given, W varies as $\sin P\,G\,N$, which is the greatest possible when $P\,G\,N = 90°$; that is, when the power acts in a line parallel to the plane.

Whether the angle $P\,G\,N$ diminishes or increases from $90°$, its sine diminishes, and becomes zero, when $P\,G\,N = 0°$, or $180°$. Therefore $W = 0$, or no weight can be sustained, when the power acts in the line $G\,N$, perpendicular to the plane, either toward the plane or from it.

127. Expression for Perpendicular Pressure.—From the triangle $P\,G\,N$ we obtain

$$N : W :: \sin G\,P\,N : \sin P\,G\,N,$$

$$\text{or } N : W :: \sin P\,G\,W : \sin P\,G\,N;$$

$$\therefore\ N = W\frac{\sin P\,G\,W}{\sin P\,G\,N}.$$

If the power acts in a line parallel to the inclined plane, $P\,G\,W = 90° + a$, $P\,G\,N = 90°$, and $N = W\dfrac{\sin (90° + a)}{\sin 90°} = W \cos a$.

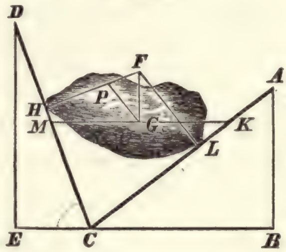
If the power acts in a line parallel to the base of the inclined plane, $P\,G\,W = 90°$, $P\,G\,N = 90° - a$, and $N = W\dfrac{1}{\cos a} = W \sec a$.

If the power acts in a line perpendicular to the inclined plane,

$$P\,G\,W = a,\ P\,G\,N = 0°, \text{ and } N = W\frac{\sin a}{0} = \propto.$$

128. Equilibrium between Two Inclined Planes.—If a body rests, as represented in Fig. 91, between two inclined planes, the three forces which retain it are its weight, and the resistances of the planes. Draw $H\,F$ and $L\,F$ perpendicular to the planes through the points of contact, and $G\,F$ vertically through the centre of gravity of the body. Since the body is in equilibrium, these three lines will pass through the same point (Art. 43). Let that point be F, and draw $G\,P$ parallel to $L\,F$, and $M\,K$ parallel to the horizon. $G\,P\,F$ is similar to $K\,C\,M$. Therefore (since Pressure on $A\,C$: Pr. on $D\,C$ :: $P\,G : F\,P$),

FIG. 91.



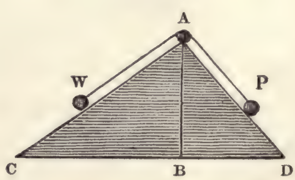
$$\begin{aligned} \text{Pressure on } A\,C : \text{Pr. on } D\,C &:: K\,C : M\,C, \\ &:: \sin M : \sin K, \\ &:: \sin D\,C\,E : \sin A\,C\,B. \end{aligned}$$

That is, when a body rests between two planes, it exerts pressures on them which are inversely as the sines of their inclinations to the horizon.

If, therefore, one of the planes is horizontal, none of the pressure can be exerted on any other plane. *It is friction alone which renders it possible for a body on a horizontal surface to lean against a vertical wall.*

129. Bodies Balanced on Two Planes by a Cord passing over the Ridge.—Let P and W balance each other on the planes $A\,D$ and $A\,C$ (Fig. 92), which have the common height $A\,B$, by means of a cord passing over the fixed pulley A. The tension of the cord is the common power which prevents each body from descending; and as the cord is parallel to each plane, we have (calling the tension t),

FIG. 92.



$$\begin{aligned} t : P &:: A\,B : A\,D; \\ \text{and } t : W &:: A\,B : A\,C; \\ \therefore P : W &:: A\,D : A\,C; \end{aligned}$$

that is, the weights, in case of equilibrium, are directly as the lengths of the planes.

130. Questions on the Inclined Plane.—

1. If a horse is able to raise a weight of 440 lbs. perpendicularly, what weight can he raise on a railway having a slope of five degrees? *Ans.* 5048.5 lbs.

2. The grade of a railroad is 20 feet in a mile; what power must be exerted to sustain any given weight upon it?

Ans. 1 lb. for every 264 lbs.

3. What force is requisite to hold a body on an inclined plane, by pressing perpendicularly against the plane?

Ans. An infinite force.

4. A certain power was able to sustain 500 tons on a plane of $7\frac{1}{2}°$; but on another plane, it could sustain only 400 tons; what was the inclination of the latter? *Ans.* $9° 23' 25''$.

5. Equilibrium on an inclined plane is produced when the power, weight, and perpendicular pressure are, respectively, 9, 13, and 6 lbs.; what is the inclination of the plane, and what angle does the power make with the plane?

Ans. $a = 37° 21' 26''$. Inclination of power to plane $= 28° 46' 54''$.

6. A power of 10 lbs., acting parallel to the plane, supports a certain weight; but it requires a power of 12 lbs. parallel to the base to support it. What is the weight of the body, and what is the inclination of the plane?

Ans. $W = 18.09$ lbs. $a = 33° 33' 25''$.

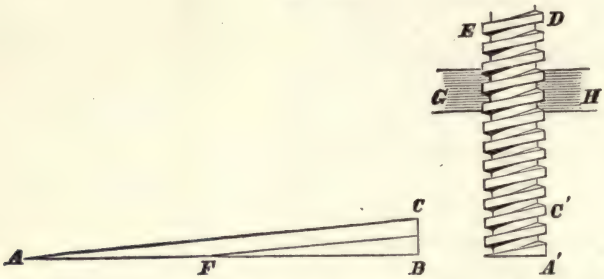
7. To support a weight of 500 lbs. upon an inclined plane of 50° inclination to the horizon, a lifting force is applied whose direction makes an angle of 75° with the horizon. What is the magnitude of this force, and the pressure of the weight against the plane? *Ans.* $P = 422.6$ lbs. $N = 142.8$ lbs.

VI. The Screw.

131. Reducible to the Inclined Plane.—The screw is a cylinder having a spiral ridge or thread around it, which cuts at a constant oblique angle all the lines of the surface parallel to the axis of the cylinder. A hollow cylinder, called a *nut*, having a similar spiral within it, is fitted to move freely upon the thread of the solid cylinder. In Fig. 93, let the base $A\ B$ of the inclined plane $A\ C$ be equal to twice the circumference of the cylinder $A'\ E$; then let the plane be wrapped about the cylinder, bringing the points A, F, and B, to the point A'; then will $A\ C$ describe two revolutions of the thread from A' to C'. Therefore the me-

chanical relations of the screw are the same as of the inclined plane.

Fig. 93.



If a weight be laid on the thread of the screw, and a force be applied to it horizontally in the direction of a tangent to the cylinder, the case is exactly analogous to that of a body moved on an inclined plane by a force parallel to the base. Let r be the radius of the cylinder, then $2\pi r$ is the circumference; also let d be the distance between the threads, (that is, from any point of one revolution to the corresponding point of the next,) measured parallel to the axis of the cylinder; then $2\pi r$ is the base of an inclined plane, and d its height. Therefore (Art. 125),

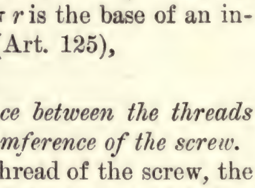
$$P : W :: d : 2\pi r; \text{ or,}$$

The power is to the weight as the distance between the threads measured parallel to the axis, is to the circumference of the screw.

If instead of moving the weight on the thread of the screw, the force is employed to turn the screw itself, while the weight is free to move in a vertical direction, the law is the same. Thus, whether the screw $A'\ E$ is allowed to rise and fall in the fixed nut $G\ H$, or whether the nut rises and falls on the thread of the screw, while the latter is revolved, without moving longitudinally, in each case, $P : W :: d : 2\pi r$.

132. The Screw and Lever Combined.—The screw is so generally combined with the lever in practical mechanics, that it is important to present the law of the compound machine. Let $A\ F$ (Fig. 94) be the section of a screw, and suppose $B\ C$, a lever of the second order, to be applied to turn it. The fulcrum is at C, the power acts at B, and the effect produced by the lever is at A, the surface of the cylinder. Call that effect x, and let $d =$ the distance between the threads; then,

Fig. 94.



$$P : x :: A\ C : B\ C,$$
$$\text{and } x : W :: d : 2\ \pi\ A\ C;$$

compounding and reducing, we have

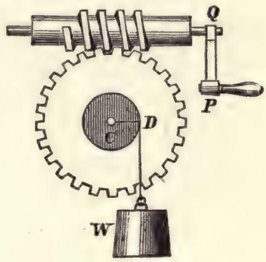
$$P : W :: d : 2\ \pi\ B\ C; \text{ that is,}$$

The power is to the weight as the distance between the threads, measured parallel to the axis, to the circumference described by the power.

The law as thus stated is applicable to the screw when used with the lever or without it.

133. The Endless Screw.—The screw is so called, when its thread moves between the teeth of a wheel, thus causing it to revolve. It is much used for diminishing very greatly the velocity of the weight.

FIG. 95.



Let $P\ Q$ (Fig. 95) be the radius of the crank to which the power is applied; d, the distance between the threads; R, the radius of the wheel; r, the radius of the axle; and call the force exerted by the thread upon the teeth, x. Then,

$$P : x :: d : 2\ \pi \times P\ Q,$$
$$\text{and } x : W :: r : R;$$
$$\therefore P : W :: d\ r : 2\ \pi \times R \times P\ Q.$$

If, for example, $P\ Q = 30$ inches, $d = 1$ in., $R = 18$ in.; $r + t = 2$ in.; then $W = 1696\ P$, and moves with 1696 times less velocity than P.

134. The Right and Left Hand Screw.—The common form of screw is called the *right-hand* screw, and may be described thus: *if the thread in its progress along the length of the cylinder, passes from the left over to the right, it is called a right-hand screw.* Hence, a person in driving a screw forward turns it from his left *over* (not *under*) to his right, and in drawing it back he reverses this movement. Fig. 93 represents a right-hand screw.

The *left-hand* screw is one whose thread is coiled in the opposite direction,—that is, it advances by passing from right over to left. This kind is used only when there is special reason for it. For example, the screws which are cut upon the left-hand ends of carriage axles are left-hand screws; otherwise there would be danger that the friction of the hub against the nut might turn the nut off from the axle. Also, when two pipes for conveying gas or steam are to be drawn together by a nut, one must have a right-hand, and the other a left-hand screw.

135. Questions on the Screw.—

1. The distance between the threads of a screw is one inch, the bar is two feet long from the axis, and the power is 30 lbs.; what is the weight or pressure? *Ans.* 4523.89 lbs.

2. The bar is three feet long, reckoned from the axis, $P = 60$ lbs., $W = 2240$ lbs.; what is the distance between the threads? *Ans.* 6.058 inches.

3. A compound machine consists of a crank, an endless screw, a wheel and axle, a pulley, and an inclined plane. The radius of the crank is 18 inches; the distance between the threads of the screw, one inch; the radius of the wheel on which the screw acts, two feet; the radius of the axle, 6 inches; the pulley block has two movable pulleys with one rope, the power exerted by the pulley being parallel to the plane, and the inclination of the plane to the horizon is 30°. What weight on the plane will be balanced by a power of 100 lbs. applied to the crank? *Ans.* 361911.474 lbs.

VII. The Wedge.

136. Definition of the Wedge, and the Mode of Using.—The usual form of the wedge is a triangular prism, two of whose sides meet at a very acute angle. This machine is used to raise a weight by being driven as an inclined plane underneath it, or to separate the parts of a body by being driven between them. When it is used by itself, and does not form part of a compound machine, force is usually applied by a blow, which produces an intense pressure for a short time, sufficient to overcome a great resistance.

137. Law of Equilibrium.—Whatever be the direction of the blow or force, we may suppose it to be resolved into two components, one perpendicular to the back of the wedge, and the other parallel to it. The latter produces no effect. The same is true of the resistances; we need to consider only those components of them which are perpendicular to the sides of the wedge.

Fig. 96.

Let $M\ N\ O$ (Fig. 96) represent a section of the wedge perpendicular to its faces; then $P\ A$, $Q\ A$, and $R\ A$, drawn perpendicular to the faces severally, show the directions of the forces which hold the wedge in equilibrium. Taking $A\ B$ to represent the power, draw $B\ C$ parallel to $R\ A$, and we have the triangle $A\ B\ C$, whose sides represent these forces. But $A\ B\ C$ is similar to $M\ N\ O$, as their

sides are respectively perpendicular to each other. Hence, calling the forces P, Q, and R, respectively,

$$P : Q :: M N : M O :$$

$$\text{and } P : R :: M N : N O ;$$

that is, there is equilibrium in a wedge, when

The power is to the resistances as the back of the wedge to the sides on which the resistances respectively act.

If the triangle is isosceles, the two resistances are equal, as the proportions show; and P is to either resistance, R, as the breadth of the back to the length of the side.

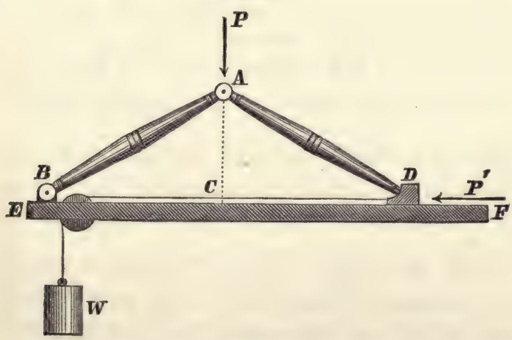
If the resisting surfaces touch the sides of the wedge only in one point each, then $Q A$ and $R A$, drawn through the points of contact, must meet $A P$ in the same point (Art. 43); otherwise the wedge will roll, till one face rests against the resisting body in two or more points.

The efficiency of the wedge is usually very much increased by combining its own action with that of the lever, since the point where it acts generally lies at a distance from the point where the effect is to be produced. Thus, in splitting a log of wood, the resistance to be overcome is the cohesion of the fibers; and this force is exerted at a distance from the wedge, while the fulcrum is a little further forward in the solid wood.

VIII. The Knee-Joint.

138. Description and Law of Equilibrium.—The knee-joint consists of two bars, usually equal, hinged together at one end, while the others are at liberty to separate in a straight line. The power is applied at the hinge, tending to thrust the bars into a straight line; the weight is the force which opposes the separation.

Fig. 97.

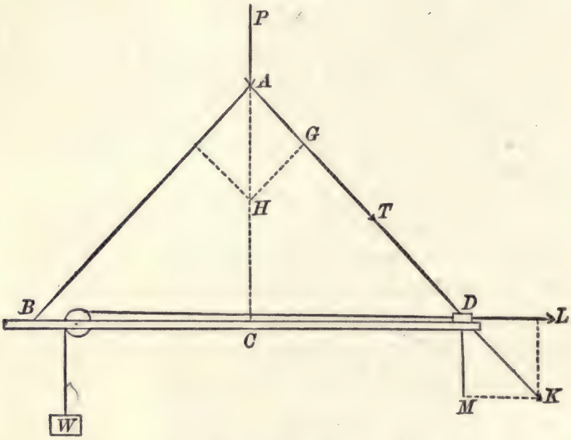


Suppose that $A B$ and $A D$ (Fig. 97) are equal bars, hinged together at A; and that the bar $A B$ is free only to revolve about

the axis B, while the end D of the other bar can move parallel to the base $E F$. If P urges A toward the base, it tends to move D further from the fixed point B. The force P', which opposes that motion, is represented in the figure by the weight W. The law of equilibrium is,

The power is to the weight as twice the height of the joint to half the distance between the ends of the bars.

FIG. 98.



Resolving the force P in the direction of $A B$ and $A D$, we have, Fig. 98,

$$P : T :: A H : A G :: 2 A C : A D,$$

in which T stands for the component of P in the direction $A G$, called the *thrust*.

This component T acts at D and must be again resolved in the directions $D L$ and $D M$, of which $D L$ is equal and opposed to W, and $D M$ is equal and opposed to the upward resistance of the plane on which the block D slides, giving the proportion

$$T : W :: D K : D L \text{ or } M K :: A D : C D.$$

Multiplying like terms of the two proportions and omitting common factors, we have,

$$P : W :: 2 A C : C D.$$

139. Ratio of Power and Weight Variable.—It is obvious that the ratio between power and weight is different for different positions of the bars. As A is raised higher $C D$ diminishes, and when the bars are parallel, we have

$$P : W :: 2 A C : 0;$$

that is to say, the power has no efficiency. But as A approaches the base $A\ C$ diminishes, and at last we have, when $B\ A$ and $A\ D$ are in the same line,

$$P : W :: 0 : B\ A.$$

Hence the weight or resistance in such case is infinite as compared with the power applied. The indefinite increase of efficiency in the power, which occurs during a single movement, renders this machine one of the most useful for many purposes, as printing and coining.

Questions on the knee-joint.—

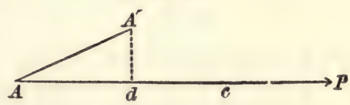
1. A power of 50 lbs. is exerted on the joint A (Fig. 97); compare the weight which will balance it, when $B\ A\ D$ is 90°, and when it is 160°. *Ans.* 25 lbs. and 141.78 lbs.

2. When the angle between the bars is 110°, a certain power just overcomes a weight of 65 lbs.; what must be the angle, in order that the weight overcome may be five times as great? *Ans.* 164° 3′ 22″.

Principle of Virtual Velocities.

140. Definition.—The *virtual velocity* of a point, with respect to any force, is the product of its actual velocity by the cosine of the angle which its actual path makes with the direction of the force. Thus, let a point A (Fig. 99) be acted upon by a force P in the direction $A\ c$, and because of some other external force or resistance suppose the point to be constrained to move in the line $A\ A'$ to A' in any unit of time: then $A\ d$, the projection of $A\ A'$ upon $A\ c$, is the virtual velocity of the point A with reference to the force P.

FIG. 99.



141. The Point of Application Moving in the Line of the Force.—It can be shown, in every case, that *the velocities, when reckoned in the direction in which the forces act, are inversely as the forces.*

Some examples are first given in which the point of application moves in the line in which the force acts.

In the *straight lever* (Fig. 100), which is in equilibrium by the weights P and W, suppose a slight motion to exist; then the velocity of each will be as the arc described in the same time; but the arcs are similar, since they subtend

FIG. 100.

equal angles. Therefore, if V = velocity of P, and v = velocity of W.

$$V : v :: A\,P : B\,W :: A\,C : B\,C;$$

but it has been shown (Art. 106) that

$$P : W :: B\,C : A\,C;$$
$$\therefore\ V : v :: W : P;$$

that is, the velocity of the power is to the velocity of the weight as the weight to the power. Hence, $P \times$ its velocity $= W \times$ its velocity; that is, the momentum of the power equals the momentum of the weight.

In the *wheel and axle*, let R and r be the radii, and suppose the machine to be revolved; then while P descends a distance equal to the circumference of the wheel $= 2\,\pi\,R$, the weight ascends a distance equal to the circumference of the axle $= 2\,\pi\,r$. Therefore,

$$V : v :: 2\,\pi\,R : 2\,\pi\,r :: R : r;$$

but (Art. 113), $P : W :: r : R;$

$$\therefore\ V : v :: W : P;$$

or, the velocities are inversely as the weights; and $P \times V = W \times v$, the momentum of the power equals the momentum of the weight.

In the *fixed pulley* the velocities are obviously equal; and we have before seen that the power and weight are equal; therefore the proportion holds true, $V : v :: W : P$; and the momenta are equal.

In the *movable pulley*, if n is the number of sustaining parts of the cord, when W rises any distance $= x$, each portion of cord is shortened by the distance x, and all these n portions pass over to P, which therefore descends a distance $= n\,x$.

Hence,

$$V : v :: n\,x : x :: n : 1;$$

but (Art. 120), $P : W :: 1 : n;$

$$\therefore\ V : v :: W : P;$$

as in all the preceding cases.

In the screw (Fig. 94), while the power describes the circumference $= 2\,\pi \times B\,C$, the weight moves only the distance $= d$; therefore,

$$V : v :: 2\,\pi \times B\,C : d;$$

but (Art. 131), $P : W :: d : 2\,\pi \times B\,C;$

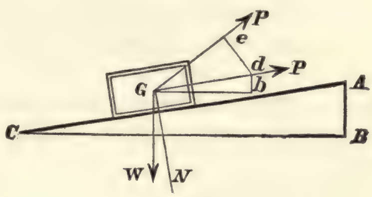
$$\therefore\ V : v :: W : P;$$

therefore the momentum of the power equals the momentum of the weight, as before.

142. The Point of Application Moving in a Different Line from that in which the Force Acts.—The cases thus far noticed are the most obvious ones, because the points of application of power and weight *actually move* in the directions in which their force is exerted. The case of the inclined plane will illustrate the principle, when the point of application does not move in the direction of the force.

First, let P (Fig. 101) act parallel to the plane, and suppose the body to be moved either up or down the plane a distance equal to $G\ d$. That is the velocity of the *power*. But in the direction of the *weight* (force of gravity) the body moves only the distance $b\ d$. Therefore the velocity of the power is to the velocity of the weight (each being reckoned in the line of its action) as $G\ d$ to $b\ d$.

Fig. 101.



By similar triangles, $G\ d : b\ d :: A\ C : A\ B$;

or $V : v :: A\ C : A\ B$.

But (Art. 125), $P : W :: A\ B : A\ C$;

$\therefore\ V : v :: W : P$.

Again, let the power act in any oblique direction, as $G\ e$. If the body moves over $G\ d$, draw $d\ e$ perpendicular to $G\ e$; then $G\ e$ is the distance passed over in the direction of the power, and $b\ d$ in the direction of the weight. $G\ d$ being taken as radius, $G\ e$ is $\cos d\ G\ e = \cos (P\ G\ N - 90°) = \sin P\ G\ N$; and $b\ d = \sin a$. Therefore, the virtual velocity of the power is to the virtual velocity of the weight as $\sin P\ G\ N$ to $\sin a$;

or $V : v :: \sin P\ G\ N : \sin a$.

But (Art. 126), $P : W :: \sin a : \sin P\ G\ N$;

$\therefore\ V : v :: W : P$.

We learn from the foregoing principle, that a machine does not enable us to obtain any *greater* effect than the power could produce without its aid, but only to produce an effect in a *different form*. A given power, for instance, may move a much *greater quantity* of matter by the aid of a machine, but it will move it as much more slowly. On the other hand, a power, by means of a machine, may produce a far *greater velocity* than would be possible without such aid; but the quantity moved, or the intensity of the force exerted, would be proportionally less. **By machines, therefore, we do not increase the effects of a power, but only modify them.**

Friction in Machinery.

143. The Power and Weight not the only Forces in a Machine.—For each machine a certain proportion has been given, which ensures equilibrium. And it is implied that if either the power or the weight be altered, the equilibrium will be destroyed. But practically this is not true; the power or weight may be considerably changed, or possibly one of them may be entirely removed, and the machine still remain at rest. The obstruction which prevents motion in such cases, and which always exists in a greater or less degree, arises from *friction;* and friction is caused by roughness in the surfaces which rub against each other. The minute elevations of one surface fall in between those of the other, and directly interfere with the motion of either, while they remain in contact. Polishing diminishes the friction, but can never remove it, for it never removes all roughness.

The *coefficient* of friction *is the fraction whose numerator is the force required to overcome the friction, and its denominator the normal pressure between the bodies.*

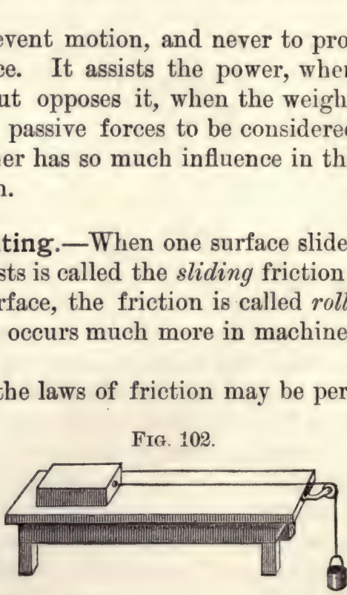
In the case of bodies whose surfaces of contact are horizontal the denominator in the coefficient is the weight of the pressing body.

As friction always tends to prevent motion, and never to produce it, it is called a *passive* force. It assists the power, when the weight is to be kept at rest, but opposes it, when the weight is to be moved. There are other passive forces to be considered in the study of science, but no other has so much influence in the operations of machinery as friction.

144. Modes of Experimenting.—When one surface slides on another, the friction which exists is called the *sliding* friction; but when a wheel rolls along a surface, the friction is called *rolling* friction. The sliding friction occurs much more in machines than the rolling friction.

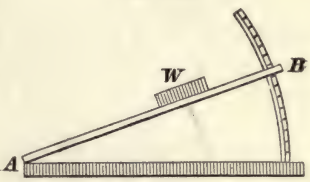
Experiments for ascertaining the laws of friction may be performed by placing on a table a block of three different dimensions, and measuring its friction under different circumstances by weights acting on the block by means of a cord and pulley, as represented in Fig. 102. This was the method by which Coulomb first ascertained the laws of friction.

Fig. 102.



Another mode is to place the block on an inclined plane, whose angle can be varied, and then find the relative friction in different cases, by the largest inclination at which it will prevent the block from sliding. For, when W on the inclined plane $A\ B$ (Fig. 103) is on the point of sliding down, friction is the power which, acting parallel to the plane, is in equilibrium with that component of the weight which tends to move the block down the plane.

FIG. 103.



This component parallel to the plane, is $W \sin A$, and the normal pressure $W \cos A$; hence, calling the coefficient of friction u, we have (Art. 143) $u = \dfrac{W \sin A}{W \cos A} = \tan A$, or *the coefficient of friction is equal to the tangent of the angle of inclination of the plane.*

For example, suppose a block of cast iron to rest upon an oak plank, and that the end of the plank is raised so that the block slides with uniform motion down the plane; then the angle A will be found by actual measurement to be about 26°, the natural tangent of which is .48773; hence, in pounds, 49 per cent. of the weight will represent the force in pounds required to overcome the friction.

145. Laws of Sliding Friction.—The laws of sliding friction on which experimenters are generally agreed are the following:

1. *Friction varies as the pressure.*—If weights are put upon the block, it is found that a double weight requires a double force to move it, a triple weight a triple force, &c.

2. *It is the same, however great or small the surface on which the body rests.*—If the block be drawn, first on its broadest side, then on the others in succession, the force required to overcome friction is found in each case to be the same. Extremes of size are, however, to be excepted. If the loaded block were to rest on three or four very small surfaces, the obstruction might be greatly increased by the indentations thus occasioned in the surface beneath them.

3. *Friction is a uniformly retarding force.*—That is, it destroys equal amounts of motion in equal times, whatever may be the velocity, like gravity on an ascending body.

4. *Friction at the first moment of contact is less than after contact has continued for a time.*—And the time during which friction increases, varies in different materials. The friction of wood on wood reaches its maximum in three or four minutes; of metal

on metal, in a second or two; of metal on wood, it increases for several days. As any jar or vibration changes at once the friction of rest to that of motion, the coefficients to be considered in determining the stability of any structure should be those of motion.

5. *Friction is less between substances of different kinds than between those of the same kind.*—Hence, in watches, steel pivots are made to revolve in sockets of brass or of jewels, rather than of steel.

146. Friction of Axes.—In machinery, the most common case of friction is that of an axis revolving in a hollow cylinder, or the reverse, a hollow cylinder revolving on an axis. These are cases of sliding friction, in which the power that overcomes the friction, usually acts at the circumference of a wheel, and therefore at a mechanical advantage. Thus, the friction on an axis, whose coefficient is as high as 20 per cent., requires a power of only *two* per cent. to overcome it, provided the power acts at the circumference of a wheel whose diameter is ten times that of the axis.

147. Rolling Friction.—This form of friction is very much less than the sliding, since the projecting points of the surfaces do not directly encounter each other, but those of the rolling wheel are lifted up from among those of the other surface, as the wheel advances.

By the use of the apparatus described in Art. 144, the laws of the rolling are found to be the same as those of the sliding friction. But on account of the manner in which this form of friction is overcome, there is this additional law:

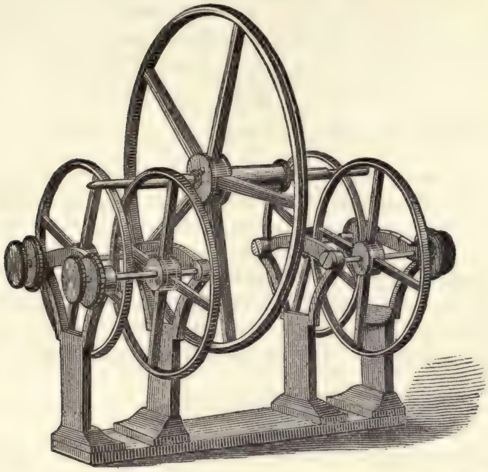
The force required to roll the wheel varies inversely as the diameter.

For the power, acting at the centre of the wheel to turn it on its lowest point as a momentary fulcrum, has the advantage of greater acting distance as the diameter increases.

It is the rolling friction which gives value to *friction wheels*, as they are called. When it is desirable that a wheel should revolve with the least possible friction, each end of its axis is made to rest in the angle between two other wheels placed side by side, as shown in Fig. 104. The wheel is obstructed only by the rolling friction on the surfaces of the four wheels, and the retarding effect of the sliding friction at the pivots of the latter is greatly reduced on the principle of the wheel and axle.

The sliding friction is diminished by lubricating the surface, the rolling friction is not.

FIG, 104.

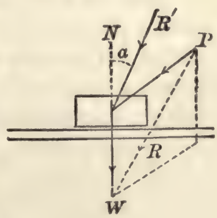


148. Advantages of Friction.—Friction in machinery is generally regarded as an evil, since more power is on this account required to do the work for which the machine is made. But it is easy to see, that in general friction is of incalculable value, or rather, that nothing could be accomplished without it. Objects stand firmly in their places by friction ; and the heavier they are, the more firmly they stand, because friction increases with the pressure. All fastening by nails, bolts, and screws, is due to friction. The fibers of cotton, wool or silk, when intertwined with each other, form strong threads or cords, only because of the power of friction. Without friction, it would be impossible to walk or even to stand, or to hold anything by grasping it with the hand.

149. Limiting Angle of Friction.—If two surfaces are in contact and forces be applied, oblique to the surface of contact, no motion will result so long as the resultant of all forces makes with the normal an angle whose tangent is less than the coefficient of friction, no matter how great this resultant force may be.

Thus (Fig. 105) let a block of iron rest upon a surface of oak, as in the case heretofore considered, and let the force P be applied. In this case the forces are P and W, and their resultant is R (or R') which may be considered in their stead. The component of R', which tends to produce motion, is R' sine a, and the total normal pressure is R' cos a. If $\frac{R' \text{ sine } a}{R' \cos a} =$ tan a, is less than the coefficient of friction no

FIG. 105.



motion can result; that is to say, if a is less than the inclination of the plane, in Art. 144, there will be stable equilibrium.

The greatest angle a which the resultant of all the forces can make with the normal without producing sliding motion of the surfaces is called the *limiting angle of friction*, and its tangent is equal to the coefficient of friction.

[NOTE.—The laws of sliding friction have claimed the attention of modern experimenters, and the results obtained modify very essentially the laws given in the text. Prof. A. S. Kimball, of Worcester, gives the following conclusions drawn from his numerous experiments: "The coefficient of friction at very low velocities is small; it increases rapidly at first, then more gradually as the velocity increases, until at a certain rate, which depends upon the nature of the surfaces in contact and the intensity of the pressure, a maximum coefficient is reached. As the velocity increases beyond this point the coefficient decreases."

"For a considerable range of velocities in the vicinity of the maximum coefficient the coefficient is sensibly constant."

Prof. R. H. Thurston, of Hoboken, draws the following conclusions from his experiments upon friction of lubricated journals: "Studying table A we see that the coefficient rapidly diminishes with increase of pressure, until a pressure of 500 lbs. per square inch is attained; the coefficient, after passing a pressure of probably 600 to 800 lbs. per square inch, increases, and, at 1000 lbs., becomes about equal to that obtained at 100 lbs."

"The coefficients of quiescence increase with the pressure, instead of diminishing as do the coefficients of friction of motion; and at the highest pressures, their values become from ten to forty times the corresponding values of the latter."

The results of all the more modern experiments must be collated before the *laws of friction* can be given satisfactorily.]

CHAPTER VII.

MOTION ON INCLINED PLANES.—THE PENDULUM.

150. The Force which Moves a Body Down an Inclined Plane.—It was shown (Art. 125) that when the power acts in a line parallel to the inclined plane, $P : W :: A\,B : A\,C$. If, therefore, P ceases to act, the body descends the plane only with a force equal to P.

Let g (the velocity acquired in a second in falling freely) = the force of gravity, f = the force acting down the plane, h = the height, l = the length; then by substitution,

$$f : g :: h : l, \text{ and}$$

$$f = \frac{h}{l} g.$$

Therefore, the force which moves a body down an inclined plane is equal to that fraction of gravity which is expressed by the height divided by the length. This is evidently a constant force on any given plane, and produces uniformly accelerated motion. Therefore the motion on an inclined plane does not differ from that of free fall in kind, but only in degree. Hence the formulæ for time, space, and velocity on an inclined plane are like those relating to free fall, if the value of f be substituted for g.

151. Formulæ for the Inclined Plane.—The formulæ for free fall (Art. 27) are here repeated, and against them the corresponding formulæ for descent on an inclined plane.

	Free fall.	*Descent on an inclined plane.*
1.	$s = \frac{1}{2} g t^2$	$s = \frac{g h t^2}{2 l}$.
2.	$t = \sqrt{\frac{2 s}{g}}$	$t = \sqrt{\frac{2 l s}{g h}}$.
3.	$s = \frac{v^2}{2 g}$	$s = \frac{l v^2}{2 g h}$.
4.	$v = \sqrt{2 g s}$	$v = \sqrt{\frac{2 g h s}{l}}$.
5.	$t = \frac{v}{g}$.	$t = \frac{l v}{g h}$.
6.	$v = g t$	$v = \frac{g h t}{l}$.

By formula 1, $s \propto t^2$, and by formula 3, $s \propto v^2$. It follows that in equal successive times the spaces of descent are as the odd numbers, 1, 3, 5, &c., and of ascent as these numbers inverted; also, that with the acquired velocity continued uniformly, a body moves twice as far as it must descend to acquire that velocity. If a body be projected up an inclined plane, it will ascend as far as it must descend in order to acquire the velocity of projection. The distance passed over in the time t by a body projected with the velocity v, down or up an inclined plane, equals $t\ v \pm \frac{g h t^2}{2 l}$.

152. Formulæ for the whole Length of a Plane.—

1. *The velocity acquired in descending a plane is the same as that acquired in falling down its height.*

For now $s = l$; hence (formula 4), $v = \left(\frac{2 g h s}{l}\right)^{\frac{1}{2}} = (2 g h)^{\frac{1}{2}}$,

which is the formula for free fall through h, the height of the plane.

On different planes, therefore, $v \propto h^{\frac{1}{2}}$.

2. *The time of descending a plane is to the time of falling down its height as the length to the height.*

For (formula 2) $t = \left(\frac{2\ l\ s}{g\ h}\right)^{\frac{1}{2}} = l\left(\frac{2}{g\ h}\right)^{\frac{1}{2}}$. But the time of fall down the height is $\left(\frac{2\ h}{g}\right)^{\frac{1}{2}}$. Therefore,

$$t \text{ down plane} : t \text{ down height} :: l\left(\frac{2}{g\ h}\right)^{\frac{1}{2}} : \left(\frac{2\ h}{g}\right)^{\frac{1}{2}};$$

$$:: l\left(\frac{2}{g}\right)^{\frac{1}{2}} : h\left(\frac{2}{g}\right)^{\frac{1}{2}};$$

$$:: l : h.$$

On different planes, $t \propto \frac{l}{\sqrt{h}}$.

It follows that if several planes have the same height, the velocities acquired in descending them are equal, and the times of descent are as the lengths of the planes. For, let $A\ C$, $A\ D$, $A\ E$, (Fig. 106) have the same height $A\ B$; then, since $v \propto h^{\frac{1}{2}}$, and h is the same for all, v is the same. And since $t \propto \frac{l}{\sqrt{h}}$, and h is the same for all the planes, $t \propto l$.

FIG. 106.

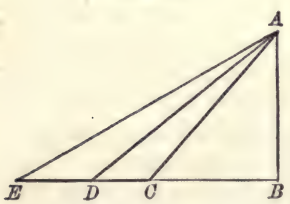
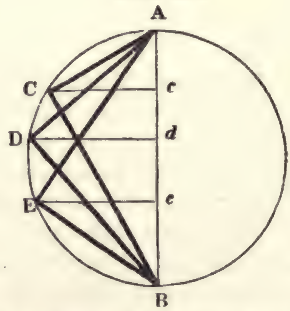


FIG. 107.



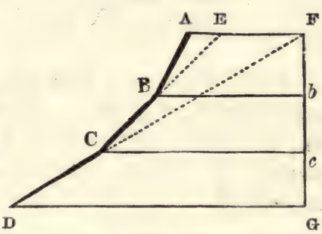
153. Descent on the Chords of a Circle.—In descending the chords of a circle which terminate at the ends of the vertical diameter, *the acquired velocities are as the lengths, and the times of descent are equal to each other and to the time of falling through the diameter.*

For (Art. 152) the velocity acquired on $A\ C$ (Fig. 107) = $(2\ g \cdot A\ c)^{\frac{1}{2}} = \left(2\ g \cdot \frac{A\ C^2}{A\ B}\right)^{\frac{1}{2}} = A\ C\left(\frac{2\ g}{A\ B}\right)^{\frac{1}{2}}$, which, since $\left(\frac{2\ g}{A\ B}\right)^{\frac{1}{2}}$ is constant, varies as $A\ C$, the length.

Again (Art. 152), the time down $A\ C = \left(\frac{2\ A\ C^2}{g \cdot A\ c}\right)^{\frac{1}{2}} = \left(\frac{2\ A\ B\ .\ A\ c}{g\ .\ A\ c}\right)^{\frac{1}{2}} = \left(\frac{2\ A\ B}{g}\right)^{\frac{1}{2}}$, which is equal to the time of falling freely through $A\ B$, the diameter.

154. Velocity Acquired on a Series of Planes.—If no velocity be lost in passing from one plane to another, the velocity acquired in descending a series of planes is equal to that acquired in falling through their perpendicular height. For, in Fig. 108, the velocity at B is the same, whether the body comes down $A\ B$ or $E\ B$, as they are of the same height, $F\ b$. If, therefore, the body enters on $B\ C$ with the acquired velocity, then it is immaterial whether the descent is on $A\ B$ and $B\ C$ or on $E\ C$; in either case, the velocity at C is equal to that acquired in falling $F c$. In like manner, if the body can change from $B\ C$ to $C\ D$ without loss of velocity, then the velocity at D is the same, whether acquired on $A\ B$, $B\ C$, and $C\ D$, or on $F\ D$, which is the same as down $F\ G$.

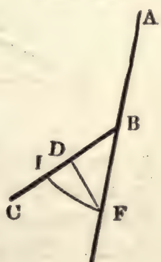
FIG. 108.



155. The Loss in Passing from one Plane to Another.—The condition named in the foregoing article is not fulfilled. A body *does* lose velocity in passing from one plane to another. And the *loss* is to the *whole previous velocity* as the *versed sine of the angle* between the planes to *radius.*

Let $B\ F$ (Fig. 109) represent the velocity which the body has at B. Resolve it into $B\ D$ on the second plane, and $D\ F$ perpendicular to it. $B\ D$ is the initial velocity on $B\ C$; and, if $B\ I = B\ F$, $D\ I$ is the loss. But $D\ I$ is the versed sine of the angle $F\ B\ D$, to the radius $B\ F$; and $\therefore$ the loss is to the velocity at B as $D\ I : B\ F ::$ ver. sin B : rad.

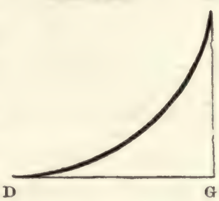
FIG. 109.



156. No Loss on a Curve.—Suppose now the number of planes in a system to be infinite; then it becomes a curve (Fig. 110). As the angle between two successive elements of the curve is infinitely small, its chord is also infinitely small; but its versed sine is *infinitely smaller still,* i. e., an infinitesimal of the *second* order; for diam. : chord :: chord : ver. sin. Therefore, although the sum of all the infinitely small angles

is a finite angle $180° - A\ G\ D$, yet, as the loss of velocity at each point is an infinitesimal of the *second* order, the *entire loss* (which is the sum of the losses at all points of the curve) is an infinitesimal of the *first* order.

FIG. 110.



Hence, a body loses no velocity on a curve, and therefore acquires at the bottom the same velocity as in falling freely through its height.

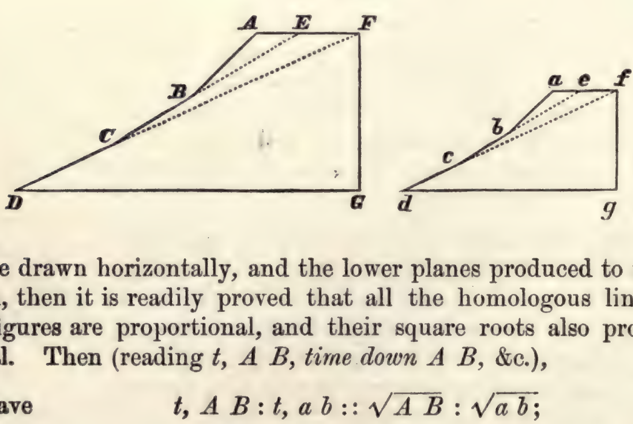
It appears, therefore, that whether a body descends *vertically*, or on an *inclined plane*, or on a *curve* of any kind, the *acquired velocity is the same*, if the height is the same.

157. Times of Descending Similar Systems of Planes and Similar Curves.—If planes are equally inclined to the horizon, the *times* of describing them are as *the square roots of their lengths.* For, if the height and base of each plane be drawn, similar triangles are formed, and $h : l$ is a constant ratio for the several planes. By Art. 152, $t \propto \frac{l}{\sqrt{h}} \propto \frac{l}{\sqrt{l}} \propto \sqrt{l}$; that is, the time varies as the square root of the length.

If two *systems* of planes are similar, i. e., if the corresponding parts are proportional and equally inclined to the horizon, it is still true that *the times of descending them are as the square roots of their lengths.*

Let $A\ B\ C\ D$ and $a\ b\ c\ d$ (Fig. 111) be similar, and let $A\ F$ and

FIG. 111.



$a\ f$ be drawn horizontally, and the lower planes produced to meet them, then it is readily proved that all the homologous lines of the figures are proportional, and their square roots also proportional. Then (reading *t, A B, time down A B*, &c.),

we have $$t, A\ B : t, a\ b :: \sqrt{A\ B} : \sqrt{a\ b};$$

$$t, E\ B : t, e\ b :: \sqrt{E\ B} : \sqrt{e\ b} :: \sqrt{A\ B} : \sqrt{a b};$$

and $\quad t, EC : t, ec :: \sqrt{EC} : \sqrt{ec} :: \sqrt{AB} : \sqrt{ab};$

$\therefore$ (by subtraction) $t, BC : t, bc :: \sqrt{AB} : \sqrt{ab}.$

In like manner, $t, CD : t, cd :: \sqrt{AB} : \sqrt{ab}.$

$\therefore$ (by addition)

$$t, (AB + BC + CD) : t, (ab + bc + cd) :: \sqrt{AB} : \sqrt{ab}$$
$$:: \sqrt{(AB + BC + CD)} : \sqrt{(ab + bc + cd)}.$$

Though there is a loss of velocity in passing from one plane to another, the proposition is still true; because, the angles being equal, the losses are proportional to the acquired velocities; and therefore the initial velocities on the next planes are still in the same ratio as before the losses; hence the ratio of times is not changed.

The reasoning is applicable when the number of planes in each system is infinitely increased, so that they become *curves*, similar, and similarly inclined to the horizon. Suppose these curves to be *circular* arcs; then, as they are similar, they are proportional to their radii. Hence, the times of descending similar circular arcs are as the square roots of the radii of those arcs.

158. Questions on the Motions of Bodies on Inclined Planes.—

1. How long will it take a body to descend 100 feet on a plane whose length is 150 feet, and whose height is 60 feet?

Ans. 3.9 sec.

2. There is an inclined railroad track, $2\frac{1}{2}$ miles long, whose inclination is 1 in 35. What velocity will a car acquire, in running the whole length of the road by its own weight?

Ans. 106.2 miles per hour.

3. A body weighing 5 lbs. descends vertically, and draws a weight of 6 lbs. up a plane whose inclination is 45°. How far will the first body descend in 10 seconds? *Ans.* 110.74 feet.

159. The Pendulum.—A pendulum is a weight attached by an inflexible rod to a horizontal axis of suspension, so as to be free to vibrate by the force of gravity. If it is drawn aside from its position of rest, it descends, and by the momentum acquired, rises on the opposite side to the same height, when gravity again causes its descent as before. If unobstructed, its vibrations would never cease.

A *single vibration* is the motion from the highest point on one side to the highest point on the other side. The motion from the highest point on one side to the same point again is called a *double vibration*.

The *axis of the pendulum* is a line drawn through its centre of gravity perpendicular to the horizontal axis about which the pendulum vibrates.

The *centre of oscillation* of a pendulum is that point of its axis at which, if the entire mass were collected, its time of vibration would be unchanged.

The *length* of a pendulum is that part of its axis which is included between the axis of suspension and the centre of oscillation.

All the particles of a pendulum may be conceived to be collected in points lying in the axis. Those which are *above* the centre of oscillation tend to vibrate quicker (Art. 157), and therefore accelerate it; those which are *below* tend to vibrate slower, and therefore retard it. But, according to the definition of the centre of oscillation, these accelerations and retardations exactly balance each other at that point.

160. Calculation of the Length of a Pendulum.—Let $C\,q$ (Fig. 112) be the axis of a pendulum in which all its weight is collected, C the point of suspension, G the centre of gravity, O the centre of oscillation, a b, &c., particles above O, which accelerate it, p, q, &c., particles below O, which retard it. $C\,O = l$, is the length of the pendulum required. Denote the masses concentrated in $a, b \ldots . p, q$, by $m, m' \ldots . m'', m'''$, and their distances from C by $r, r' \ldots, . r'', r'''$; and denote the distance from C to G by k. Denote the angular velocity, that is, the velocity at unit's distance from the centre, at any instant by θ; then the velocity of m will be $r\,\theta$ and its momentum will be $m\,r\,\theta$.

Fig. 112.

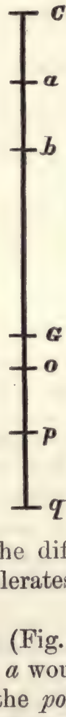
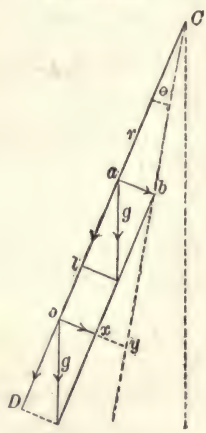


Fig. 113.



If m had been placed at O, the momentum would have been $m\,l\,\theta$. The difference $m\,(l - r)\,\theta$, is that portion of the force which accelerates the motion of the system.

For suppose a material particle m (Fig. 113) to act upon a pendulum $C\,D$ without weight; m at a would, under the action of the component of gravity $a\,b$, move the *point* a to b and swing the pendulum through the angle θ; m if transferred to o would,

gravity being the same, move the *point* o to x, and swing the pendulum through an angle less than θ. Thus m at a swings the pendulum through a greater angle in a given time than it would if at o, or accelerates the pendulum, by a force which would carry m over $x\,y$ in the given time, or by $m\,(l - r)\,\theta$; for, calling $C\,a = r$ and $C\,o = l$, $a\,b = r\,\theta$, $o\,x = a\,b = r\,\theta$, $o\,y = l\,\theta$; then $o\,y - o\,x = x\,y = l\,\theta - r\,\theta = (l - r)\,\theta$, and m moving with velocity $x\,y$, or $(l - r)\,\theta$, gives momentum $m\,(l - r)\,\theta$.

The moment of this force with respect to C is $m\,(l - r)\,r\,\theta$.

In like manner the moment of m' is $m'\,(l - r')\,r'\,\theta$, and so on for all the particles between C and O.

The moments of the forces tending to retard the system applied at the points p, q, &c., are

$$m''\,(r'' - l)\,r''\,\theta,\ m'''\,(r''' - l)\,r'''\,\theta,\ \text{\&c.}$$

But since these forces are to balance each other, we have

$$m\,(l - r)\,r\,\theta + m'\,(l - r')\,r'\,\theta + \text{\&c.} = m''\,(r'' - l)\,r''\,\theta + m'''\,(r''' - l)\,r'''\,\theta + \text{\&c.};$$

whence
$$l = \frac{m\,r^2 + m'\,r'^2 + m''\,r''^2 + \text{\&c.}}{m\,r + m'\,r' + m''\,r'' + \text{\&c.}}$$

Or $l = \dfrac{S\,(m\,r^2)}{S\,(m\,r)}$, where S denotes the sum of all the terms similar to that which follows it.

The numerator of this expression is called the *moment of inertia* of the body with respect to the axis of suspension, and the denominator is called the *moment of the mass*, with respect to the axis of suspension.

By the principle of moments $m\,r + m'\,r' + \text{\&c.}$, or $S\,(m\,r) = M\,k$, where M denotes the entire mass of the pendulum; hence, by substitution, $l = \dfrac{S\,(m\,r^2)}{M\,k}$.

That is, *the distance from the axis of suspension to the centre of oscillation is found by dividing the moment of inertia, with respect to that axis, by the moment of the mass with respect to the same axis.*

161. The Point of Suspension and the Centre of Oscillation Interchangeable.—Let the pendulum now be suspended from an axis passing through O, and denote by l' the distance from O to the new centre of oscillation. The distances of $a, b \ldots p, q$, from O, will be $l - r$, $l - r'$, &c., and the distance $G\,O$ will be $l - k$.

Hence, from the principle just established, we have

$$l' = \frac{S\,[m\,(l-r)^2]}{M\,(l-k)} = \frac{S\,(m\,l^2 - 2\,m\,r\,l + m\,r^2)}{M\,(l-k)}$$

$$= \frac{S\,(m\,l^2) - 2\,S\,(m\,r\,l) + S\,(m\,r^2)}{M\,(l-k)}.$$

But from the preceding paragraph $l = \frac{S\,(m\,r^2)}{M\,k}$, whence $S(m\,r^2) = M\,k\,l$; and since l is constant, $S\,(m\,l^2) = S\,(m + m' + m'' + \&c.)\,l^2 = M\,l^2$, which values substituted above give

$$l' = \frac{M\,l^2 - 2\,l\,S\,(m\,r) + M\,k\,l}{M\,(l-k)} = \frac{M\,l^2 - 2\,M\,k\,l + M\,k\,l}{M\,(l-k)}$$

$$= \frac{M\,(l-k)\,l}{M\,(l-k)} = l.$$

This last equation shows that the centre of oscillation and the point of suspension are interchangeable; that is, if the pendulum were suspended from O, it would vibrate in the same time as when suspended from C.

This fact is taken advantage of in determining the length of the second's pendulum at any place. A solid bar $A\,B$ (Fig. 114), is furnished with two knife-edge axes C and D, and a sliding weight H. By adjusting this weight the bar can be made to oscillate in the same time when suspended upon either axis. The distance between the knife-edges C and D is the length of an equivalent simple pendulum, and by comparing the time of oscillation with that of a pendulum beating seconds, the time of one oscillation of this reversible pendulum is obtained; from these data the length of the second's pendulum is readily computed.

Fig. 114.

162. Calculation of the Time of Oscillation.—

Let the length of the pendulum $A\,B$ (Fig. 115) be represented by l, and the height of the arc of oscillation by $B\,D$. Suppose the pendulum to have moved from C to a; its acquired velocity will be $v = \sqrt{2\,g \times D\,H}$. (Art. 156).

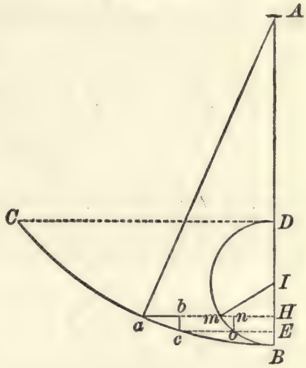
During the succeeding infinitely small interval of time t' it will describe the element of its arc $a\,c$ with the velocity v; hence

$$t' = \frac{a\,c}{v} = \frac{a\,c}{\sqrt{2\,g \times D\,H}}.$$

Describe upon $D\,B$ a semi-circumference; $m\,o$ is the elemen-

tary arc of the semi-circumference having the same altitude $E\,H$ as the element $a\,c$. As these arcs are elements of the curves, they may be regarded as straight lines, and $a\,b\,c$ and $m\,n\,o$ become triangles. $A\,H\,a$ and $a\,b\,c$ are similar triangles, their corresponding sides being perpendicular two and two, and give the equation $\frac{a\,c}{c\,b} = \frac{A\,a}{a\,H}$, and because the triangles $m\,n\,o$ and $I\,m\,H$ are similar, for the reason assigned above, $\frac{m\,o}{n\,o} = \frac{I\,m}{m\,H}$.

FIG. 115.



Divide the first of these equations by the second, and we get $\frac{a\,c}{m\,o} = \frac{A\,a \times m\,H}{a\,H \times I\,m}$.

But $\overline{a\,H}^2 = B\,H\,(2\,A\,B - B\,H) = B\,H\,(2\,l - B\,H)$ and $\overline{m\,H}^2 = B\,H \times H\,D$, whence,

$$\frac{a\,c}{m\,o} = \frac{l\sqrt{B\,H \times H\,D}}{I\,m\sqrt{B\,H(2\,l - B\,H)}} = \frac{l\sqrt{H\,D}}{I\,m\sqrt{2\,l - B\,H}}, \text{ or since}$$

$$I\,m = \tfrac{1}{2}\,B\,D \text{ and } B\,H = B\,D - D\,H,$$

$$a\,c = \frac{l\sqrt{H\,D}}{\frac{1}{2}\,B\,D\sqrt{2\,l - (B\,D - D\,H)}} \times m\,o,$$

and this substituted in the value of t', gives

$$t' = \frac{l\sqrt{H\,D}}{\frac{1}{2}\,B\,D\sqrt{2\,l - (B\,D - D\,H)}} \times \frac{m\,o}{\sqrt{2\,g \times D\,H}}$$

$$= \frac{2\,l \times m\,o}{B\,D\sqrt{2\,g\,[2\,l - (B\,D - D\,H)]}};$$

or dividing both terms by $2\sqrt{l}$, we obtain

$$t' = \frac{\sqrt{l} \times m\,o}{B\,D \times \sqrt{g}\sqrt{1 - \frac{B\,D - D\,H}{2\,l}}}.$$

When the amplitude is small, the double arc $B\,C$ not being over 5 degrees, $\frac{B\,D}{2\,l}$, and $\frac{D\,H}{2\,l}$ are very small, and their difference, $\frac{B\,D - D\,H}{2\,l}$, is smaller still, and may be neglected, giving as a

result $t' = \sqrt{\frac{l}{g}} \times \frac{m\,o}{B\,D}$. The time required to describe $C\,B$ is the sum of the times of describing the elements of $C\,B$, or calling this time $\frac{t}{2}$, we have $\frac{t}{2} = \sqrt{\frac{l}{g}} \times \frac{1}{B\,D} \times$ (sum of the elementary arcs $m\,o$).

But the sum of the elements of $D\,m\,B$ corresponding to the elements of $C\,B$ is the semicircle $D\,m\,B$ itself, or $\pi\,\frac{B\,D}{2}$; whence $\frac{t}{2} = \frac{\pi}{2}\sqrt{\frac{l}{g}}$ = time of semi-oscillation, or calling t the time of a complete oscillation, we have

$$t = \pi\sqrt{\frac{l}{g}}.$$

163. Applications of the Formula.—From the equation $t = \pi\sqrt{\frac{l}{g}}$, we get $l = \frac{g\,t^2}{\pi^2}$. Therefore, the length of a pendulum being known, the time of one vibration is found; and on the other hand, if the time of a vibration is known, the length of the pendulum is obtained from it.

From the same formulæ, we find that $t \propto \sqrt{l}$, or

The time in which a pendulum makes a vibration varies as the square root of the length.

As $t \propto \sqrt{l}$, $\therefore\ l \propto t^2$; hence, if the length of a seconds pendulum equals l, then a pendulum which vibrates *once in two seconds* equals $4\,l$, and one which beats *half seconds* $= \frac{1}{4}\,l$, &c.

Again, by observing the *length* of a pendulum which vibrates in a given *time*, the *force of gravity*, g, may be found. For, as $l = \frac{g\,t^2}{\pi^2}$, $\therefore g = \frac{\pi^2\,l}{t^2}$. And if g varies, as it does in different latitudes and at different altitudes, then $l = \frac{g\,t^2}{\pi^2} \propto g\,t^2$; and if the time is constant (as, for example, *one second*), then $l \propto g$. Hence,

The length of a pendulum for beating seconds varies as the force of gravity.

Also, $t \propto \left(\frac{l}{g}\right)^{\frac{1}{2}}$; that is, the time of a vibration varies directly as the square root of the length, and inversely as the square root of the force of gravity.

Since the *number*, n, of vibrations in a given time varies inversely as the *time* of one vibration, therefore $n \propto \left(\frac{g}{l}\right)^{\frac{1}{2}}$, and

$g \propto l\,n^2$. Hence, if the time and the length of a pendulum are given,

The force of gravity varies as the square of the number of vibrations.

1. What is the length of a pendulum to beat seconds, at the place where a body falls $16\frac{1}{12}$ ft. in the first second?

Ans. 39.11 inches, nearly.

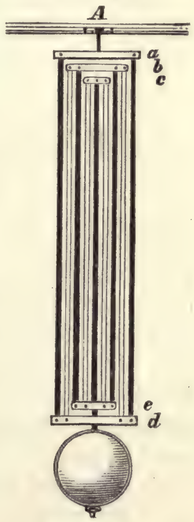
2. If 39.11 inches is taken as the length of the seconds pendulum, how long must a pendulum be to beat 10 times in a minute?

Ans. $117\frac{1}{3}$ feet.

3. In London, the length of a seconds pendulum is 39.1386 inches; what velocity is acquired by a body falling one second in that place? *Ans.* 32.19 feet.

164. The Compensation Pendulum.—This name is given to a pendulum which is so constructed that its length does not vary by changes of temperature. As all substances expand by heat and contract by cold, therefore a pendulum will vibrate more slowly in warm than in cold weather. This difficulty is overcome in several ways, but always by employing two substances whose rates of expansion and contraction are unequal. One of the most common is the *gridiron pendulum*, represented in Fig. 116. It consists of alternate rods of steel and brass, connected by cross-pieces at top and bottom. The rate of longitudinal expansion and contraction of brass to that of steel is about as 100 to 61; so that *two* lengths of brass will increase and diminish more than *three* equal lengths of steel. Therefore, while there are three expansions of steel downward, two upward expansions of brass can be made to neutralize them. In the figure the dark rods represent steel, the white ones brass. Suppose the temperature to rise, the two outer steel rods (acting as one) let down the cross-bar d; the two brass rods standing on d raise the bar b; the steel rods suspended from b let down the bar e, on which the inner brass rods stand, and raise the short bar c; and finally, the centre steel rod, passing freely through d and e, lets down the disk of the pendulum. These lengths (counting each pair as a single rod) are adjusted so as to be in the ratio of 100 for the steel to 61 for the brass; in which case the upward expansions just equal those which are downward, and therefore

FIG. 116.



the centre of oscillation remains at the same distance from the point of suspension.

If the temperature falls, the two contractions of brass are equal to the three of steel, so that the pendulum is not shortened by cold.

The *mercurial pendulum* consists of a steel rod terminating at the bottom with a rectangular frame in which is a tall narrow jar containing mercury, which is the weight of the pendulum. It requires only 6.31 inches of mercury to neutralize the expansions and contractions of 42 inches of steel. See Appendix for calculations of the place of the centre of oscillation.

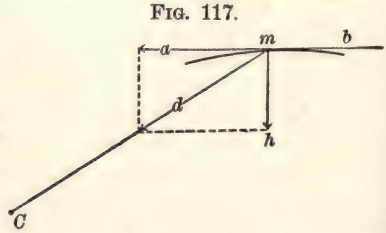
CHAPTER VIII.

CENTRAL FORCES.

165. Central Forces Described.—Motion in a curve is always the effect of two forces; one an impulse, which alone would cause uniform motion in a straight line; the other a continued force, which urges the body toward some point out of the original line of motion. The first is called the *projectile* force, the second the *centripetal* force.

Suppose a point m (Fig 117) to be acted upon by an impulse, in direction and intensity represented by $b\ m$, and also by a constant force, $m\ d$. This centripetal force $m\ d$ may be resolved into two components; one $m\ a$ in the direction of the tangent, the other, $m\ h$, perpendicular to it. The *tangential* component will accelerate or retard the motion in the curved path according as it acts with the projectile force, or in opposition to it, while the component at right angles to this tends to deflect the body from a rectilinear path, and therefore determines the character of the curve at any instant.

FIG. 117.



When the body moves in the circumference of a circle, the *tangential* component of the centripetal force is 0, and hence the motion is uniform.

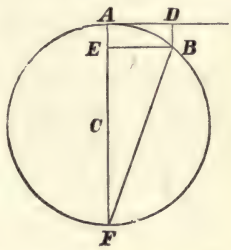
If the centripetal force should cease to act at any instant, the

body, by its inertia, would immediately begin to move in a straight line tangent to the curve at the point where the body was when the force ceased to act.

Since the body, by its inertia, *tends* to move in a tangent, there is a continued resistance to deflection into a curved path, equal and opposed to the component $m\ h$, in the direction of the radius of curvature at the instant; this is called the *centrifugal* force.

166. Expressions for the Centrifugal Force in Circular Motion.—

Fig. 118.



1. Let r = the radius of the circle, v = the velocity of the body, c = the distance through which the centrifugal force causes the body to move in one second, and let $A\ B$ (Fig. 118) be the arc described in the infinitely small time t; then $A\ B = v\ t$, and, by a method similar to that employed in the discussion of the force of gravity, it may be shown that $B\ D = c\ t^2$.

But $A\ B$, being a very small arc, may be considered as equal to its chord, which is a mean proportional between $A\ E$ and the diameter $2\ r$. Hence $c\ t^2 = \frac{v^2\ t^2}{2\ r}$, or

$$c = \frac{v^2}{2\ r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If this be doubled, then (Art. 26) $\frac{v^2}{r}$ is the velocity which the centrifugal force is capable of generating in one second, and this is sometimes taken as the measure of the centrifugal force.

From (1) it follows, that *in equal circles the centrifugal force varies as the square of the velocity.*

2. The value of c may be expressed in a different form. Let t' = the time of a complete revolution; then $2\ \pi\ r = v\ t'$; whence $v = \frac{2\ \pi\ r}{t'}$. This substituted in (1) gives

$$c = \frac{2\ \pi^2\ r}{t'^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Hence, *The centrifugal force varies directly as the radius of the circle, and inversely as the square of the time of revolution.*

3. Let w = the weight of the revolving body, and c' = the centrifugal force expressed in pounds; then

$$w : c' :: \tfrac{1}{2} g : \frac{v^2}{2\ r}\text{; whence } c' = \frac{w\ v^2}{r\ g} \quad . \quad . \quad (3)$$

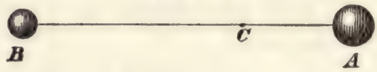
Let n = the number of revolutions per second; then

$$v = 2\,\pi\,r\,n, \text{ and (3) becomes}$$

$$c' = \frac{4\,\pi^2}{g} \,.\, w \,.\, r \,.\, n^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

167. Two Bodies Revolving about their Centre of Gravity.—Let A and B (Fig. 119) be two bodies connected by a rod, and let them be made to revolve about the centre of gravity C; then by (4) the centrifugal force of A will be

Fig. 119.

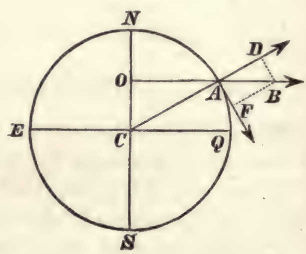


$\frac{4\,\pi^2}{g} \,.\, A \,.\, A\,C \,.\, n^2$, and of B, $\frac{4\,\pi^2}{g} \,.\, B \,.\, B\,C \,.\, n^2$.

But C being the centre of gravity of the two bodies, $A \,.\, A\,C = B \,.\, B\,C$; $\therefore$ the centrifugal force of A equals that of B. Hence, *If two bodies revolve in the same time about an axis passing through their centre of gravity, there will be no strain upon that axis.*

168. Centrifugal Force on the Earth's Surface.—As the earth revolves upon its axis, all free particles upon it are influenced by the centrifugal force. Let $N\,S$ (Fig. 120) be the axis, and A a particle describing a circumference with the radius $A\,O$. Put $r = C\,Q$, $r' = A\,O$, l = the angle $A\,C\,Q$, the latitude, c = the centrifugal force at the equator, c' = the centrifugal force at A, v = velocity of Q, and v' = velocity of A; then

Fig. 120.



$$c = \frac{v^2}{2\,r}, \text{ and } c' = \frac{v'^2}{2r'}.$$

But $v : v' :: r : r'$; whence $v' = \frac{v\,r'}{r}$. Again, from the triangle $A\,C\,O$ we have $r' = r \cos l$; hence $v' = v \cos l$, and $c' = \frac{v^2 \cos^2 l}{2\,r \cos l} = \frac{v^2 \cos l}{2\,r}$. Comparing the value of c' with that of c, we have

$$c' = c \cos l.$$

That is, *the centrifugal force at any point on the earth's surface is equal to the centrifugal force at the equator, multiplied by the cosine of the latitude of the place.*

A body at the equator loses by centrifugal force $\frac{1}{289}$ part of the weight which it would have if the earth did not revolve on its axis.

Let $A\,B$ represent the centrifugal force at A, and resolve it

into $A\ D$ on $C\ A$ produced, and $A\ F$, tangent to the meridian $N\ Q\ S$; then, since the angle $D\ A\ B = A\ C\ Q = l$, we have

$$A\ D = A\ B \cos l = c \cos l \,.\, \cos l = c \cos^2 l.$$

That is, *that component of the centrifugal force at any point, which opposes the force of gravity, is equal to the centrifugal force at the equator, multiplied by the square of the cosine of the latitude of the place.*

In like manner we find $A\ F = A\ B \sin l = c \cos l \sin l = \frac{c \sin 2\ l}{2}$. From this equation we see that the tangential component is 0 at the equator, increases till $l = 45°$; where it is a maximum; then goes on diminishing till $l = 90°$, when it again becomes 0.

The effect of $A\ D$ is to diminish the weight of the particle, while the effect of $A\ F$ is to urge it toward the equator.

169. Examples on Central Forces.—

1. A ball weighing 10 lbs. is whirled around in a circumference of 10 feet radius, with a velocity of 30 feet per second. What is the tension upon the cord which restrains the ball?

Ans. 28 lbs. nearly.

2. With what velocity must a body revolve in a circumference of 5 feet radius, in order that the centrifugal force may equal the weight of the body? *Ans.* $v = 12.7$ ft.

3. A ball weighing 2 lbs. is whirled round by a sling 3 feet long, making 4 revolutions per second. What is its centrifugal force? *Ans.* 117.84 lbs.

4. A weight of 5 lbs. is attached to the end of a cord 3 feet long just capable of sustaining a weight of 100 lbs. How many revolutions per second must the body make in order that the cord may be upon the point of breaking? *Ans.* $n = 2.3$ nearly

5. A railway carriage, weighing 7 tons, moving at the rate of 30 miles per hour, describes an arc whose radius is 400 yards. What is the outward pressure upon the track? *Ans.* 786 + lbs.

170. Composition of two Rotary Motions.—

When a body is rotating on an axis, and a force is applied which alone would cause it to rotate on some other axis, the body will commence rotation on an axis lying between them, and the velocities of rotation on the three axes are such, that each may be represented by the sine of the angle between the other two.

Suppose a body is rotating on an axis $A\ B$ in the plane of $H\ K$, and that a force is applied to make it rotate on the axis $C\ D$ in the same plane $H\ K$, these two axes intersecting within the body at some point called G.

Imagine a perpendicular to the plane of the axes to be drawn through G, and let P be a particle of the body in this perpendicular. Suppose the particle P, in an infinitely small time t, to pass over $P\,a$ perpendicular to $A\,B$, by the first rotation, and over $P\,c$, perpendicular to $C\,D$, by the second. Then, since the particle will describe the diagonal $P\,e$ in the time t, this line must indicate the direction and velocity of the resultant rotation. Therefore, if $E\,F$ be drawn through G, perpendicular to the plane $G\,P\,e$, $E\,F$ is the axis on which the body revolves in consequence of the two rotations given to it. Since $P\,G$ is perpendicular to the plane $A\,G\,C$, and also to the line $E\,F$, therefore $E\,F$ is in that plane; that is, the new axis of rotation is in the plane of the other two axes. The angles $A\,G\,E$ and $E\,G\,C$, are respectively equal to the angles $a\,P\,e$ and $e\,P\,c$, the inclinations of the planes of rotation. But the lines, $P\,a$, $P\,c$, $P\,e$, represent the velocities in those directions respectively; and (Art. 44) $P\,a : P\,c : P\,e :: \sin c\,P\,e : \sin a\,P\,e : \sin a\,P\,c$; therefore $P\,a : P\,c : P\,e :: \sin C\,G\,E : \sin A\,G\,E : \sin A\,G\,C$; or, the velocities on the three axes, (namely, the axes of the component rotations, and of the resultant rotation,) are such, that each may be represented by the sine of the angle between the other two axes.

Fig. 121.

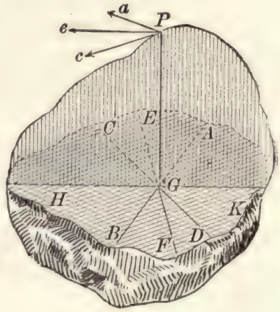
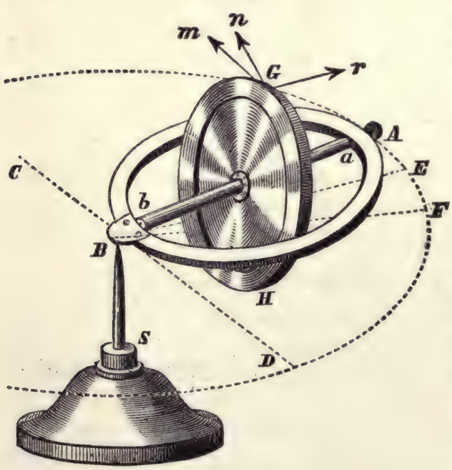


Fig. 122.



171. The Gyroscope.—The *gyroscope* affords an illustration of the composition of two rotations imparted to a body. As usually constructed, it consists of a heavy wheel $G\,H$ (Fig. 122), accurately balanced on the axis $a\,b$, which runs with as little friction as possible upon pivots in a metallic ring. In the direction of the axis, there is a projection B from the ring, having a socket

sunk into it on the under side, so that it may rest on the pointed standard *S*, without danger of slipping off.

The wheel is made to rotate swiftly by drawing off a cord wound upon *a b*, and then the socket in *B* is placed on the standard, and the whole left to itself. Immediately, instead of falling, the ring and wheel commence a slow revolution in a horizontal plane around the standard, the point *A* following the circumference *A E F*, in a direction contrary to the motion of the top of the wheel.

This revolution is explained by applying the principle of composition of rotations given in the preceding article. The particles of the wheel are rotating about the horizontal axis *a b* by the force imparted by the string. The force of gravity tends to make it fall, that is, to revolve in a vertical circle around the axis *C D* at right angles to *a b*. Hence, in a moment after dropping the ring, the system will be found revolving on an axis which lies in the direction *E B*, between *A B* and *C D*, the other two axes. *Now*, gravity bears it down around a new axis perpendicular to *E B*. Therefore, as before, it changes to still another axis *F B*, and thus continues to go round in a horizontal circle.

The only way possible for it to rotate on an axis in a new position, is to turn its present axis of rotation into that position. Hence, the whole instrument turns about, in order that its axis may take these successive positions.

The change of axis is seen also by observing the resultant of the motions of the particles at the top and bottom of the wheel. For example, *G* is moving swiftly in the direction *m* by the rotation around *a b*; by gravity it tends to move slowly in the line *r*, tangent to a vertical circle about the centre *B*. The resultant is in the line *n*, tangent to the wheel when its axis *a b* has taken the new position *E B*.

The centre of gravity of the ring and wheel tends to remain at rest, while the resultant of the two rotations carries around it all other parts, standard included, in horizontal circles. But the standard by its inertia and friction resists this effort, and the reaction causes the ring and wheel to go around the standard.

PART II.

HYDROSTATICS.

CHAPTER I.

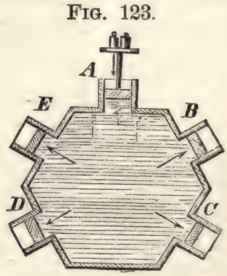
LIQUIDS AT REST.

172. Liquids Distinguished from Solids and Gases.—A *fluid* is a substance whose particles are moved among each other by a very slight force. In solid bodies the particles are held by the force of cohesion in fixed relations to each other; hence such bodies retain their form in spite of gravity or other small forces exerted upon them. If a solid be reduced to the finest powder, still each grain of the powder is a solid body, and its atoms are held together in a determinate shape. A pulverized solid, if piled up, will settle by the force of gravity to a certain inclination, according to the smallness and smoothness of its particles, while a liquid will not rest till its surface is horizontal.

Fluids are of two kinds, liquids and gases. In a *liquid*, there is a perceptible cohesion among its particles; but in a *gas*, the particles mutually repel each other. These fluids are also distinguished by the fact that liquids cannot be compressed except in a very slight degree, while the gases are very compressible. A force of 15 pounds on a square inch, applied to a mass of water, will compress it only about .000046 of its volume, as is shown by an instrument devised by Oersted. But the same force applied to a quantity of air of the usual density at the earth's surface will reduce it to one-half of its former volume.

173. Transmitted Pressure.—It is an observed property of fluids that a force which is applied to one part is transmitted undivided to all parts. For instance, if a piston *A* (Fig. 123) is pressed upon the water in the vessel *A D C* with a force of *one pound*, every other piston of the same size, as *B, C, D,* or *E,* receives a pressure of *one pound* in addition to the previous pressure of the water itself. Hence the whole amount of bursting pressure exerted within the vessel by the weight

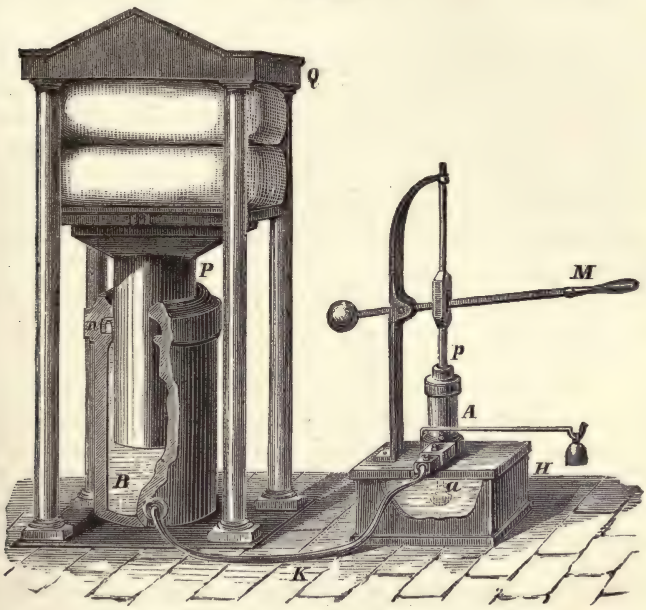
FIG. 123.



upon A equals as many pounds as there are portions of surface equal to the area of A. And if the pressure is increased till the vessel bursts, the fracture is as likely to occur in some other part as in that toward which the force is directed.

174. The Hydraulic Press.—An important application of the principle of transmitted pressure occurs in Bramah's hydraulic press, represented in Fig. 124. The walls of the cylinder and

FIG. 124.



reservoir are partly removed, to show the interior. A is a small forcing pump, worked by the lever M, by which water is raised in the pipe a from the reservoir H, and driven through the tube K into the cylinder B, where it presses up the piston P, and the iron plate on the top of it, against the substance above. At each downward stroke of the small piston p, a quantity of water is transferred to the cylinder B, and presses up the large piston with a force as many times greater than that exerted on the small one as the under surface of P is greater than that of p (Art 173). If the diameter of p is *one* inch, and that of P is *ten* inches, then any pressure on p exerts a pressure 100 times as great on P. The lever M gives an additional advantage. If the distances from the fulcrum to the rod p and to the hand are as 1 : 5, this ratio compounded with the other, 1 : 100, gives the ratio of power at M to

the pressure at Q as 1 : 500; so that a power of 100 lbs. exerts a pressure of 50000 lbs.

This machine has the special advantage of working with a small amount of friction. It is used for pressing paper and books, packing cotton, hay, &c.; also for testing the strength of cables and steam-boilers. It has been sometimes employed to raise great weights, as, for instance, the tubular bridge over the Menai straits; the two portions, after being constructed at the water level, were raised more than 100 feet to the top of the piers, by two hydraulic presses. The weight of each length lifted at once was more than 1800 tons.

The relation of power to weight in the hydraulic press is in accordance with the principle of virtual velocities (Art. 141). For, while a given quantity of water is transferred from the smaller to the larger cylinder, the velocity of the large piston is as much less than that of the small one as its area is greater. But we have seen that the pressures are directly as the areas. Therefore, in this as in other machines, the intensities of the forces are inversely as their virtual velocities.

Ex. 1. A press of the same form as in Fig. 124 has a piston whose cross section is one sq. ft.; the feed-pump piston is 2 sq. in. cross section, and stroke 6 inches. The lever has a short arm of 1 ft. and long arm of 4 ft. (measured from fulcrum in each case), Find the greatest pressure that can be produced by a man who exerts a force of 174 lbs., friction and difference of level of the liquid in the cylinders being disregarded. *Ans.* 50112 lbs.

Ex. 2. How many strokes of the pump will it take to raise the press piston one foot in the last example? *Ans.* 144.

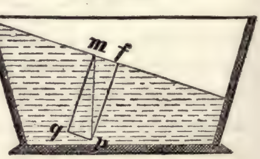
175. Equilibrium of a Fluid.—In order that a fluid may be at rest,

1. *The pressures at any one point must be equal in all directions.*

2. *The surface must be perpendicular to the resultant of the forces which act upon it.*

Both of these conditions result from the mobility of the particles. It is obvious that the first must be true, since, if any particle were pressed more in one direction than another, it would move in the direction of the greater force, and therefore not be at rest, as supposed.

FIG. 125.



In order to show the truth of the second condition, let $m\,p$ (Fig. 125) represent the resultant of the forces which act on the fluid. Then, if the

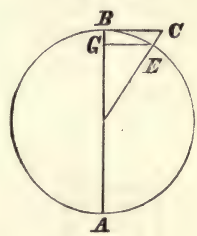
surface is not perpendicular to $m\ p$, that force may be resolved into $m\ q$ perpendicular to the surface, and $m\ f$ parallel to it. The latter, $m\ f$, not being opposed, the particles move in that direction.

As gravity is the principal force which acts on all the particles, the surface of a fluid at rest is ordinarily *level*, that is, perpendicular to a vertical or plumb line. If the surface is of small extent, it is sensibly a plane, though it is really curved, because the vertical lines, to which it is perpendicular, converge toward the centre of the earth.

176. The Curvature of a Liquid Surface.—The earth being 7912 miles in diameter, a distance of 100 feet on its surface subtends an angle of about one second at the centre, and therefore the levels of two places 100 feet apart are inclined one second to each other.

The amount of depression for moderate distances is found by the formula, $d = \frac{2}{3} L^2$, in which d is the depression in feet, and L the length of arc in miles. Let $B\ E$ (Fig. 126) be a small arc of a great circle on the earth; then $C\ E$ is the depression. As $B\ E$ is small, its chord may be considered equal to the arc, and $B\ G$ equal to the depression. But $B\ G : B\ E :: B\ E : B\ A$; that is, $d : L :: L : 7912$; or $d = \frac{L^2}{7912}$. In order to express d in feet, while the other lines are in miles, we have

FIG. 126

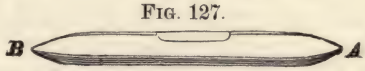


$$d = \frac{L^2 \times 5280^2}{7912 \times 5280} = \frac{L^2 \times 5280}{7912} = \frac{2}{3} L^2, \text{ very nearly.}$$

This gives, for one mile, $d = 8$ inches; for two miles, $d = 2$ ft. 8 in.; and for 100 miles, $d = 6667$ ft., &c. If a canal is 100 miles long, each end is more than a mile below the tangent to the surface of the water at the other end.

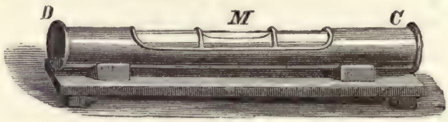
177. The Spirit Level.—Since the surface of a liquid at rest is level, any straight line which is placed parallel to such a surface is also level. Leveling instruments are constructed on this principle. The most accurate kind is the one called the *spirit* level. Its most essential part is a glass tube, $A\ B$ (Fig. 127), nearly filled with alcohol (because water would be liable to freeze), and hermetically sealed. The tube having a little convexity upward from end to end, though so slight as not to be

FIG. 127.



visible, the bubble of air moves to the highest part, and changes its place by the least inclination of the tube. The tube is so connected with a straight bar of wood or metal, as *D C* (Fig. 128), or for nicer purposes, with a telescope, that the bubble is at the middle *M* when the bar or the axis of the telescope is exactly level. The tube usually has graduation lines upon it for adjusting the bubble accurately to the middle.

FIG. 128.



178. Pressure as Depth.—From the principle of equal transmission of force in a fluid, it follows that, if a liquid is uniformly dense, its pressure on a given area varies as the perpendicular depth, whatever the form or size of the reservoir. Let the vessel *A B C D* (Fig. 129), having the form of a right prism, be filled with water, and imagine the water to be divided by horizontal planes into strata of equal thickness. If the density is everywhere the same, the weights of these strata are equal. But the pressure on each stratum is the sum of the weights of all the strata above it. Therefore, in this case, the pressure varies as the depth.

FIG. 129.

FIG. 130.

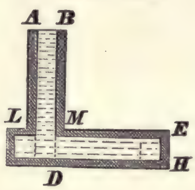
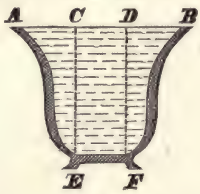


FIG. 131.



But let the reservoir (Fig. 130) contain water. The pressure of the column *A B L M* is transmitted equally in every direction (Art. 173). If the area of the section *L M* is one sq. inch and the weight of the column *A L* is one pound, then every square inch of the side *E H* will receive a pressure of one pound, on account of the column *A L*, in addition to the pressure it sustains from the contained water. So also every square inch of the bottom *D H* will sustain an added pressure of 1 lb., and also every square inch of the top *M E* will sustain an *upward* pressure of 1 lb. That is to say, the added pressure upon every part of the containing vessel *L E H D*, whose area equals the area of the base of the column *A L M B*, is equal to the weight of that column.

Again, if the base is smaller than the top, as in the vessel

$A\ B\ E\ F$ (Fig. 131), then the pressure on $E\ F$ equals only the weight of the column $C\ D\ E\ F$. The water in the surrounding space $A\ C\ E$, $B\ D\ F$, simply serves as a vertical wall to balance the lateral pressures of the central column.

If the surface pressed upon is oblique or vertical, then the points of it are at unequal depths ; in this case, the depth of the area is understood to be the *average* depth of all its parts ; that is, the depth of its centre of gravity.

If the fluid were compressible, the lower strata would be more dense than the upper ones, and therefore the pressure would increase at a faster rate than the depth.

The following experiment will show that the pressure of a liquid upon a given base is due to the depth of the liquid and is independent of the volume. Bend a glass tube $A\ B\ C$, as shown in Fig. 132 (*a*), and attach a cup $D\ E$, into which may be screwed

FIG. 132 (*a*).

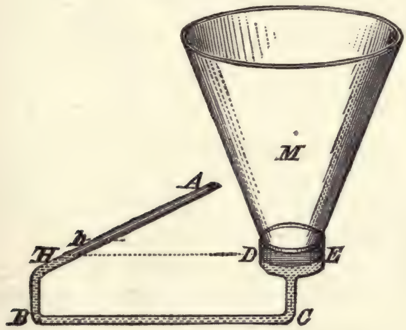
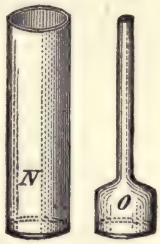


FIG. 132 (*b*).



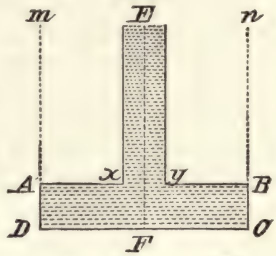
various shaped receivers M, N, O, &c. Into the cup $D\ E$ pour mercury, which will stand at a level, say $H\ D\ E$. Now screw into the cup any one of the receivers, as M, and pour in water to any desired height. The mercury will be depressed in the cup $D\ E$ by the pressure of the water, and will rise in $A\ B$ to some point h. Now remove the vessel M and substitute in succession each of the others, filling to the same height as before. The mercury in each case will rise to the point h, showing that the pressure upon the area of mercury in the cup $D\ E$ is the same in all cases, for a given height of liquid.

179. Hydrostatic Paradox.—To guard against a possible misapprehension in this connection, the student must be cautioned to distinguish between the *pressure* upon the bottom and the *weight* of the contained liquid.

In the vessel $A\ B\ C\ D$ (Fig. 133), the *pressure* upon the bot-

tom is equal to the area of the base $D\,C$ in square inches multiplied by the weight of a column of water of one sq. inch cross-section and height $E\,F$, which product is equal to the weight of a column of cross-section $D\,C$ and height $E\,F$, or the whole volume $m\,D\,C\,n$. But the *weight* of the contained water is less than this, as shown by the figure.

FIG. 133.



To illustrate, suppose area of base $D\,C = 12$ sq. inches, $A\,D = 1$ inch, $E\,F = 11$ inches, and $x\,y = 1$ sq. inch, and call the weight of one cubic inch of water w. Then the *pressure* upon the base $D\,C = 12 \times 11 \times w = 132\,w$. The *weight* of the liquid $= 12 \times 1 \times w + 10 \times 1 \times w = 22\,w$.

The *downward* pressure upon the base is, as above, $132\,w$. The *upward* pressure upon the upper base $A\,B$ is equal to the area of the *ring* $A\,B$ multiplied by the height $m\,A$, or equal to $11 \times 10 \times w = 110\,w$. Downward pressure minus upward pressure = weight.

$$132\,w - 110\,w = 22\,w, \text{ as before.}$$

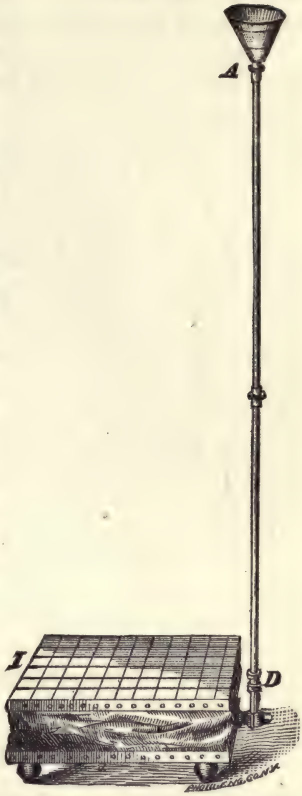
180. Amount of Pressure in Water.—One cubic foot of water weighs 1000 ounces, or 62.5 pounds avoirdupois. Therefore, the pressure on *one square foot*, at the depth of *one foot*, is 62.5 pounds. From this, as the *unit* of hydrostatic pressure, it is easy to determine the pressures on all surfaces, at all depths; for it is obvious that, when the depth is the same, the pressure varies as the surface pressed upon; and it has been shown that, on a given surface, the pressure varies as the depth of its centre of gravity; it therefore varies as the product of the two. Let p = pressure; a = area pressed upon; and d = the depth of its centre of gravity; then $p = a\,d \times 62.5$.

Depth.	*Lbs. per sq. ft.*	*Depth.*	*Lbs. per sq. ft.*
1 ft.	62.5	100 ft.	6,250
10	625	1 mile	330,000
16	1000	5 miles	1,650,000

From the above table it may be inferred that the pressure on a square foot in the deepest parts of the ocean must be not far from two millions of pounds; for the depth in some places is more than five miles, and sea-water weighs 64.37 pounds, instead of 62.5 pounds. A brass vessel full of air, containing only a pint, and whose walls were one inch thick, has been known to be crushed in by this great pressure, when sunk to the bottom of the ocean.

Owing to the increase of pressure with depth, there is great difficulty in confining a high column of water by artificial structures. The strength of banks, dams, flood-gates, and aqueduct pipes, must increase in the same ratio as the perpendicular depth from the surface of the water, without regard to its horizontal extent.

FIG. 134.



181. Column of Water whose Weight Equals the Pressure.—A convenient mode of conceiving readily of the amount of pressure on an area, in any given circumstances, is this: consider the area pressed upon to form the horizontal base of a hollow prism; let the height of the prism equal the average depth of the area; and then suppose it filled with water. The weight of this column of water is equal to the pressure. For the contents of the prism (whose base $= a$, and its height $= d$), $= a\,d$; and the weight of the same $= a\,d \times 62.5$ lbs.; which is the same expression as was obtained above for the pressure.

On the bottom of a cubical vessel full of water, the pressure equals the *weight* of the water; on each side of the same the pressure is *one-half* the weight of the water; hence, on all the five sides the pressure is *three times* the weight of the water; and if the top were closed, on which the pressure is zero, the pressure on the six sides is the same, three times the weight of the water.

182. Illustrations of Hydrostatic Pressure.—A vessel may be formed so that both its base and height shall be great, but its cubical contents small; in which case, a great pressure is produced by a small quantity of water. The hydrostatic bellows is an example. In Fig. 134 the weight which can be sustained on the lid $D\,I$ by the column $A\,D$ is equal to that of a prism or cylinder of water, whose base is $D\,I$, and its height $D\,A$. It is immaterial how shallow is the stratum of water on the base, or how slender the tube $A\,D$, if greater than a capillary size.

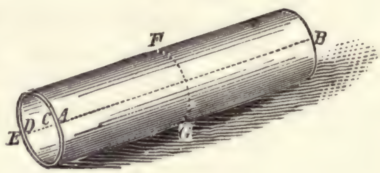
In like manner, a cask, after being filled, may be burst by an additional pint of water; for, by screwing a long and slender pipe

into the top of the cask, and filling it with water, the pressure is easily made greater than the strength of the cask can bear.

183. Determination of Thickness of Cylinder.—To determine the thickness of plate required in a cylindrical vessel that it may sustain a given pressure, we assume that the bursting results from tearing asunder the material of the plate.

Let $A\ B\ E$ (Fig. 135) represent the cylindrical vessel; $E\ D\ C\ A\ B$ a longitudinal section through the axis; put $a = A\ B =$ length in inches, $2\ r = C\ D =$ internal diameter in inches, $e = A\ C = D\ E =$ thickness of plate in inches, $T =$ tenacity of the material in lbs. per sq. inch; then

Fig. 135.



$a \times e \times T =$ strength, or resistance to tearing apart, of section $B\ A\ C$. As there are two such sections which resist the internal pressure the total strength through the section $E\ D\ C\ A\ B$, is $2\ a \times e \times T$.

Call the internal pressure in lbs. per sq. inch P. The total bursting pressure through the section $C\ D$, acting upward and downward to cause separation in that plane, is equal to the area multiplied by the pressure per sq. inch, or = rectangle $2\ r \times a \times P$. But at the moment of rupture these two must be equal, therefore

$2\ a\ e\ T = 2\ r \times a \times P$, whence $e = \frac{r \times P}{T}$ which gives the thickness when the internal diameter, the tenacity and the pressure are known. The longitudinal section through the axis is the weakest longitudinal section that can be taken, hence we need consider no other.

To determine the thickness to withstand rupture through the transverse section $G\ F$, we have, area of section of material through $G\ F = \pi\ (r + e)^2 - \pi\ r^2 = \pi\ e\ (e + 2\ r)$, and the tenacity of section $= \pi\ e\ (e + 2\ r)\ T$.

The bursting pressure upon the plane through $G\ F$, exerted upon the heads of the cylinder, $= \pi \times r^2 \times P$. These being equal we have,

$$\pi\ e\ (e + 2\ r)\ T = \pi\ r^2\ P,$$

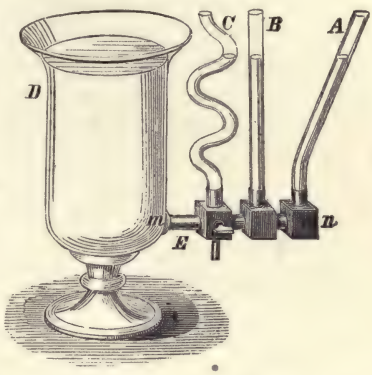
$$2\ e \left(\frac{e}{2\ r} + 1\right) T = r\ P;$$

neglecting $\frac{e}{2\ r}$, which will usually be a small fraction, we get $e = \frac{r\ P}{2\ T}$.

Comparing this with the previous result we find that the transverse section requires only half the thickness of material, for a given pressure, which is required by the longitudinal section, hence this section need not be considered in determining the thickness.

184. The Same Level in Connected Vessels.—In tubes or reservoirs which communicate with each other, water will rest only when its surface is at the same level in them all. If water is poured into *D* (Fig. 136), it will rise in the vertical tube *B*, so as to stand at the same level as in *D*. For, the pressure toward the right on any cross-section *E* of the horizontal pipe *m n* equals the product of its area by its depth below *D*. So the pressure on the same section towards the left equals the product of its area by its depth below *B*. But these pressures are equal, since the liquid is at rest. Therefore *E* is at equal depths below *B* and *D*; in other words, *B* and *D* are on the same level. The same reasoning applies to the irregular tubes *A* and *C*, and to any others, of whatever form or size.

Fig. 136.



Water conveyed in aqueducts, or running in natural channels in the earth, will rise just as high as the source, but no higher.

Artesian wells illustrate the same tendency of water to rise to its level in the different branches of a tube. When a deep boring is made in the earth, it may strike a layer or channel of water which descends from elevated land, sometimes very distant. The pressure causes it to rise in the tube, and often throws it many feet above the surface. Fig. 137 shows an artesian well, through

Fig. 137.

which is discharged the water that descends in the porous stratum $K\ K$, confined between the strata of clay $A\ B$ and $C\ D$.

A tube driven to the water bed anywhere between A or B and the lowest point in the diagram, might also bring water to the surface if the flow *below* the end of the tube were sufficiently obstructed by friction ; hence an artesian well might be successfully driven when the inclination of the water bed is wholly in one direction.

185. Centre of Pressure.—The centre of pressure of any surface immersed in water is that point through which passes the resultant of all the pressures on the surface. It is the point, therefore, at which a single force must be applied in order to counterbalance all the pressures exerted on the surface. If the surface be a plane, and horizontal, the centre of pressure coincides with the centre of gravity, because the pressures are equal on every part of it, just as the force of gravity is. But if the plane surface makes an angle with the horizon, the centre of pressure is lower than the centre of gravity, since the pressure increases with the depth. For example, if the vertical side of a vessel full of water is rectangular, the centre is *one-third* of the distance from the middle of the base to the middle of the upper side. If triangular, with its lower side horizontal, the centre of pressure is *one-fourth* of the distance from the middle of the base to the vertex. If triangular, with the top horizontal, the centre of pressure is *half* way up on the bisecting line.

[See Appendix for calculations of the place of the centre of pressure.]

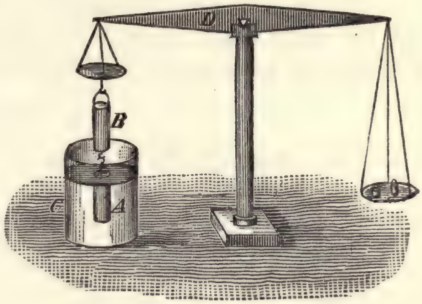
186. The Loss of Weight in Water.—When a body is immersed in water, it suffers a pressure on every side, which is proportional to the depth. Opposite components of lateral pressures, being exerted on surfaces at the same depth, balance each other ; but this cannot be true of the vertical pressures, since the top and bottom of the body are at unequal depths. The upward pressure on the bottom exceeds the downward pressure on the top ; and this excess constitutes the *buoyant power* of a fluid, which causes a loss of weight.

A body immersed in water loses weight equal to the weight of water displaced.

For before the body was immersed, the water occupying the same space was exactly supported, being pressed upward more than downward by a force equal to its own weight. The weight of the *body*, therefore, is diminished by this same difference of pressures, that is, by the weight of the displaced water.

To show this experimentally, suspend a solid cylinder A (Fig. 138) below a hollow cylinder B, into which it will fit with great nicety; attach both to the arm of a balance and carefully counterpoise them; now pour water, or any other liquid, into the beaker C until it is full, and the equilibrium will be destroyed, the end of the beam D rising. Fill the cylinder B with the same liquid, and when it is exactly full, the cylinder A will be found to be submerged exactly to its upper edge, thus showing that the buoyancy of the liquid in this case is counteracted by a volume of the same liquid equal to the volume of the submerged body.

FIG. 138.



On the supposition of the complete incompressibility of water, this loss is the same at all depths, because the weight of displaced water is the same. As water, however, is slightly compressible, its buoyant power must increase a little at great depths. Calling the compression .000046 for one atmosphere ($=$ 34 feet of water), the bulk of water at the depth of a mile is reduced by about $\frac{1}{140}$, and its specific gravity increased in the same ratio; so that, *possibly*, a body might sink near the surface, and float at great depths in the ocean. But this is not *probable* in any case, since the same compressing force may reduce the volume of the solid as much as that of the water. And, furthermore, the increase of density by increased depth is so slow, that even if solids were incompressible, most of those which sink at all would not find their floating place within the greatest depths of the ocean. For example, a stone twice as heavy as water must sink 100 miles before it could float.

187. Equilibrium of Floating Bodies.—If the body which is immersed has the same density as water, it simply loses its whole weight, and remains wherever it is placed. But if it is less dense than water, the excess of upward pressure is more than sufficient to support it; it is, therefore, raised to the surface, and comes to a state of equilibrium after partly emerging. In order that a floating body may have a stable equilibrium, the three following conditions must be fulfilled:

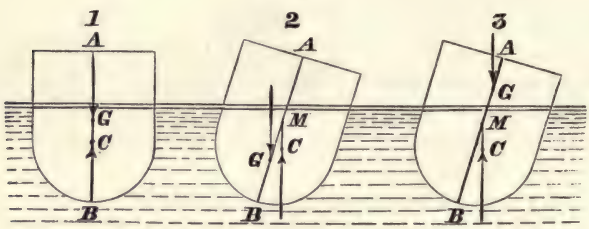
1. *It displaces an amount of water whose weight is equal to its own.*

2. *The centre of gravity of the body is in the same vertical line with that of the displaced water.*

3. *The metacenter is higher than the centre of gravity of the body.*

The reason for the *first* condition is obvious; for both the body and the water displaced by it are sustained by the same upward pressures, and therefore must be of equal weight.

Fig. 139.



That the *second* is true, is proved as follows: Let C (Fig. 139, 1) be the centre of gravity of the displaced water, while that of the body is at G. Now the fluid, previous to its removal, was sustained by an upward force equal to its own weight, acting through its centre of gravity C; and the same upward force now acts upon the floating body through the same point. But the body is urged downward by gravity in the direction of the vertical line $A\ G\ B$. Were those two forces exactly opposite and equal, they would keep the body at rest; but this is the case only when the points C and G are in the same vertical line; in every other position of these points, the two parallel forces tend to turn the body round on a point between them.

188. The Metacenter.—To understand the *third* condition, the metacenter must be defined. A floating body assumes a position such that the line through the centres of gravity of the body and of the displaced water shall be vertical; now, regard this line so determined as fixed with respect to the body, moving with it to any degree of inclination; then move the body so that this line shall make an indefinitely small angle with its vertical position; the intersection of the line as now placed with the vertical through the new centre of gravity of the displaced water is called the metacentre. When the centre of gravity of the body G is *lower* than the metacenter, as in Fig. 139, 2, the parallel forces, downward through G and upward through C, revolve the body back to its position of equilibrium, which is then called a stable equilibrium. But if the centre of gravity of the body is

higher than the metacenter, as in Fig. 139, 3, the rotation is in the opposite direction, and the body is upset, the equilibrium being unstable. Once more, if the centre of gravity of the body is *at* the metacenter, the body rests indifferently in any position, as, for example, a sphere of uniform density. The equilibrium in this case is called neutral.

If only the first condition is fulfilled, there is *no* equilibrium; if only the first and second, the equilibrium is *unstable;* if all the three, the equilibrium is *stable.*

In accordance with the third condition, it is necessary to place the heaviest parts of a ship's cargo in the bottom of the vessel, and sometimes, if the cargo consists of light materials, to fill the bottom with stone or iron, called *ballast,* lest the masts and rigging should raise the centre of gravity too high for stability. On the same principle, those articles which are prepared for life-preservers, in case of shipwreck, should be attached to the upper part of the body, that the head may be kept above water. The danger arising from several persons standing up in a small boat is quite apparent; for the centre of gravity is elevated, and liable to become higher than the metacenter, thus producing an unstable equilibrium.

189. Floating in a Small Quantity of Water.—As pressure on a given surface depends solely on the depth, and not at all on the extent or quantity of water, it follows that a body will float as freely in a space slightly larger than itself as on the open water of a lake. For instance, a ship may be floated by a few hogsheads of water in a dock whose form is adapted to it. In such a case, it cannot be literally true that the displaced water weighs as much as the vessel, when *all* the water in the dock may not weigh a hundredth part as much. The expression "displaced water" means the amount which would fill the place occupied by the immersed portion of the body. An experiment illustrative of the above is, to float a tumbler within another by means of a spoonful of water between.

190. Floating of Heavy Substances.—A body of the most dense material may float, if it has such a form given it as to exclude the water from the upper side, till the required amount is displaced. Ships are built of iron, and laden with substances of greater specific gravity than water, and yet ride safely on the ocean. A block of any heavy material, as lead, may be sustained by the upward pressure beneath it, provided the water is excluded from the upper side by a tube fitted to it by a water-tight joint.

191. Specific Gravity.—The weight of a body compared

with the weight of the same volume of the standard, is called its *specific gravity.*

Distilled water at about 39° F., the temperature of its greatest density, is the standard for all solids and liquids, and common air, at 32°, for gases. Therefore the specific gravity of a solid or a liquid body, is the ratio of its weight to the weight of an equal volume of water; and the specific gravity of an aeriform body is the ratio of its weight to the weight of an equal volume of air. Hence, to find the specific gravity of a solid or liquid, divide its weight by the weight of the same volume of water; but in the case of a gas, divide by the weight of the same volume of air.

192. Methods of Finding Specific Gravity.—

1. For a solid heavier than water, *divide its weight by its loss of weight in water.*

The reason for this rule is obvious. The weight which a submerged body loses (Art. 186) is equal to the weight of the displaced water, which has, of course, the same volume as the body; therefore, dividing by the loss is the same as dividing by the weight of the same volume of water.

2. For a solid lighter than water, *divide its weight by its weight added to the loss it occasions to a heavier body previously balanced in water.*

For, if the light body be attached to a body heavy enough to sink it, it loses all its own weight, and causes loss to the other which was previously balanced. And the whole loss equals the weight of water displaced by the light body. Hence, as before, we in fact divide the weight of the body by the weight of the same volume of water.

If the body whose specific gravity is required be soluble in water, its specific gravity must be determined with reference to some liquid which will not dissolve it, such as alcohol, turpentine, a saturated solution of the substance itself, &c., and then the specific gravity so obtained must be multiplied by the specific gravity of the liquid used, as compared with water.

In all the above cases any air adhering to the bodies must be removed after immersion.

3. For a liquid, *find the loss which a body sustains weighed in the liquid and then in water, and divide the first loss by the second.*

For the first loss equals the weight of the displaced liquid, and the second that of the displaced water; and the volume in each case is the same, namely, that of the body weighed in them.

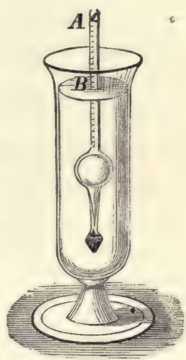
But the specific gravity of a liquid may be more directly obtained by measuring equal volumes of it and of water in a flask,

and finding the weight of each. Then the weight of the liquid divided by that of the water is the specific gravity required.

Flasks for the purpose are made with carefully ground stoppers through which is pierced a fine hole so that in inserting the stopper there may be an overflow through the hole, after which the flask having been carefully wiped off, it is ready for weighing.

193. The Hydrometer, or Areometer.—In commerce and the arts, the specific gravities of substances are obtained in a more direct and sufficiently accurate way, by instruments constructed for the purpose. The general name for such instruments is the *hydrometer*, or *areometer*. But other names are given to such as are limited to particular uses; as, for example, the *alcoömeter* for alcohol, and the *lactometer* for milk. The hydrometer, represented in Fig. 140, consists of a hollow ball, with a graduated stem. Below the ball is a bulb containing mercury, which gives the instrument a stable equilibrium when in an upright position. Since it will descend until it has displaced a quantity of the fluid equal in weight to itself, it will of course sink to a greater depth if the fluid is lighter. From the depths to which it sinks, therefore, as indicated by the graduated stem, the corresponding specific gravities are estimated.

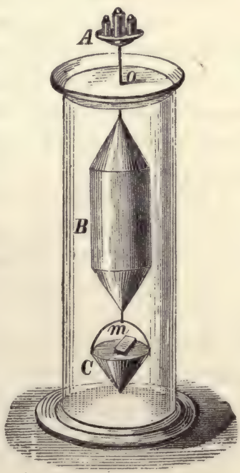
Fig. 140.



The sensibility of instruments of this class is increased by diminishing the diameter of the stem.

Nicholson's hydrometer (Fig. 141) is the most useful of this class of instruments, since it may be applied to finding the specific gravities of solid as well as liquid bodies. In addition to the hollow ball of the common hydrometer, it is furnished at the top with a pan *A* for receiving weights, and a cavity beneath for holding the substance under trial. The instrument is so adjusted that when 1000 grains are placed in the pan, the instrument sinks in distilled water at the temperature of $39\frac{1}{2}°$ F. to a fixed mark, 0, on the stem. Calling the weight of the instrument W, the weight of displaced water is $W+1000$.

Fig. 141.

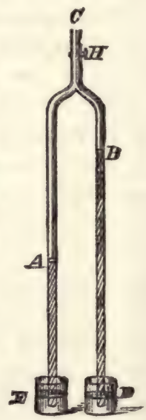


To find the specific gravity of a *liquid*, place in the pan such a weight w as will just bring the mark to the surface. Then the weight of the liquid displaced is $W+w$. But its volume is equal to that of the displaced water. Therefore its specific gravity is $\frac{W+w}{W+1000}$.

To find the specific gravity of a *solid*, place in the pan a fragment of it weighing less than 1000 grains, and add the weight w required to sink the mark to the water-level. Then the weight of the substance in air is $1000 - w$. Remove the substance to the cavity at the bottom of the instrument, and add to the weight in the pan a sufficient number of grains w' to sink the mark to the surface. Then w' is the *loss* of weight in water; therefore, $\frac{1000 - w}{w'}$ is the specific gravity of the substance.

194. Specific Gravity of Liquids by Means of Heights.—This method depends upon the fact that the heights at which columns of liquids will be sustained by any given atmospheric pressure are inversely as their specific gravities.

FIG. 142.



Arrange two glass tubes A and B (Fig. 142) connected at the top with a common outlet C, their lower ends being immersed in the liquids contained in the beakers E and D. Exhaust the air by the outlet C till the liquids rise to any desired height, say to A and B, and suppose the height of the column A, measured from the surface of the liquid in beaker E to be one-half that of the column B measured from the surface in beaker D; then the specific gravity of liquid E is twice that of liquid D.

This method gives only approximate results, depending upon the fineness of division of the scales used, corrected for capillarity of the tubes.

195. Table of Specific Gravities.—An accurate knowledge of the specific gravities of bodies is important for many purposes of science and art, and they have therefore been determined with the greatest possible precision. The heaviest of all known substances is *platinum*, whose specific gravity, when compressed by rolling, is 22, water being 1 ; and the lightest is *hydrogen*, whose specific gravity is = .073, common air being 1. Now, as water is about 800 times as heavy as air, it is ($800 \div .073 =$) 10.959 times as heavy as hydrogen. Therefore platinum is about ($10.959 \times 22 =$) 241,000 times as heavy as hydrogen. Between

these limits, 1 and 241,000, there is a wide range for the specific gravities of all other substances. As a class, the common metals are the heaviest bodies ; next to these come the metallic ores ; then the precious gems ; minerals in general, animal and vegetable substances, as shown in the following table :

Metals (pure), not including the bases of the alkalies and earths, from .... 5—22

Platinum	22.0	Copper	8.90
Gold	19.25	Steel	7.84
Mercury	13.58	Iron	7.78
Lead	11.35	Tin	7.29
Silver	10.47	Zinc	7.00

Metallic ores, lighter than the pure metals, but usually above	4.00
Precious gems, as the ruby, sapphire, and diamond	3—4
Minerals, comprehending most stony bodies	2—3
Liquids, from ether highly rectified to sulphuric acid highly concentrated	$\frac{3}{4}$—2
Acids in general, heavier than water.	
Oils in general, lighter ; but the oils of cloves and cinnamon are heavier than water ; the greater part lie between .9 and 1	.9—1
Milk	1.032
Alcohol (perfectly pure)	.797
" of commerce	.835
Proof spirit	.923
Wines ; the specific gravity of the lighter wines, as Champagne and Burgundy, is a little less, and of the heavier wines, as Malaga, a little greater than that of water.	
Woods, cork being the lightest, and lignum vitæ the heaviest	.24—1.34

196. Floating.—The human body, when the lungs are filled with air, is lighter than water, and but for the difficulty of keeping the lungs constantly inflated, it would naturally float. With a moderate degree of skill, therefore, swimming becomes a very easy process, especially in salt water. When, however, a man plunges, as divers sometimes do, to a great depth, the air in the lungs becomes compressed, and the body does not rise except by muscular effort. The bodies of drowned persons rise and float after a few days, in consequence of the inflation occasioned by putrefaction.

As rocks are generally not much more than twice as heavy as water, nearly half their weight is sustained while they are under water ; hence, their weight seems to be greatly increased as soon as they are raised above the surface. It is in part owing to their diminished weight that large masses of rock are transported with

great facility by a torrent. While bathing, a person's limbs feel as if they had nearly lost their weight, and when he leaves the water, they seem unusually heavy.

197. To find the Magnitude of an Irregular Body.—It would be a long and difficult operation to find the exact contents of an irregular mineral by direct measurement. But it might be found with facility and accuracy by weighing it in air, and then finding its loss of weight in water. The loss is the weight of a mass of water having the same volume. Now, as 1000 ounces avoirdupois of water measure 1728 cubic inches, a direct proportion will show what is the volume of the displaced water; that is, of the mineral itself.

198. Cohesion and Adhesion.—What distinguishes a liquid from a solid is not its want of cohesion so much as the mobility of its particles. It is proved in many ways that the particles of a liquid strongly attract each other. It is owing to this that water so readily forms itself into drops. The same property is still more observable in mercury, which, when minutely divided, will roll over surfaces in spherical forms. When a disk of almost any substance is laid upon water, and then raised gently, it lifts a column of water after it by adhesion, till at length the edge of the fluid begins to divide, and the column is detached, not in all parts at once, but by a successive rupturing of the lateral surface. It is proved that the whole attraction of the liquid would be far too great to be overcome by the force applied to pull off the disk, were it not that it is encountered by little and little, at the edges of the column. But it is the cohesion of the water which is overcome in this experiment; for the upper lamina still adheres to the disk. By a pair of scales we find that it requires the same force to draw off disks of a given size, whatever the materials may be, provided they are *wet* when detached. This is what might be expected, since in each case we break the attraction between two laminæ of water. But if we use disks which are not wet by the liquid, it is not generally true that those of different material will be removed by the same force; indicating that some substances adhere to a given liquid more strongly than others.

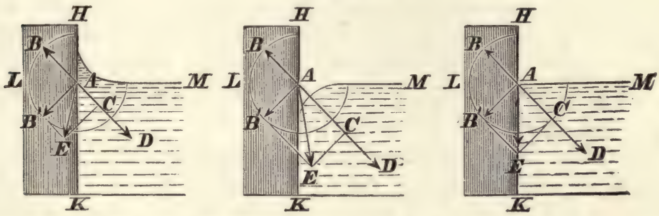
These molecular attractions extend to an exceedingly small distance, as is proved by many facts. A lamina of water adheres as strongly to the thinnest disk that can be used as to a thick one; so, also, the upper lamina coheres with equal force to the next below it, whether the layer be deep or shallow.

199. Capillary Action.—This name is given to the molecular forces, adhesion and cohesion, when they produce disturbing

effects on the surface of a liquid, elevating it above or depressing it below the general level. These effects are called *capillary*, because most strikingly exhibited in very fine (*hair-sized*) tubes.

The liquid will be elevated in a concave curve, or depressed in a convex curve, by the side of the solid, according as the attraction of the liquid molecules for each other is less or greater than twice the attraction between the liquid and the solid.

FIG. 143.



Case 1st. Let $H\ K$ (Fig. 143, 1) and $L\ M$ be a section of the vertical side of a solid, and of the general level of the liquid. The particle A, where these lines meet, is attracted (so far as this section is concerned) by all the particles of an insensibly small quadrant of the liquid, the resultant of which attractions is in the line $A\ D$, 45° below $A\ M$. It is also attracted by all the particles in two quadrants of the solid, and the resultants are in the directions $A\ B$, 45° above, and $A\ B'$, 45° below $L\ M$.

Now suppose the force $A\ D$ to be *less than twice* $A\ B$ or $A\ B'$. Cut off $C\ D = A\ B$; then $A\ B$, being opposite and equal to $C\ D$, is in equilibrium with it. The remainder $A\ C$, being less than $A\ B'$, their resultant $A\ E$ will be directed toward the solid; and therefore the surface of the liquid, since it must be perpendicular to the resultant of forces acting on it (Art. 175), takes the direction represented; that is, concave upward.

Case 2d. Let $A\ D$ (Fig. 143, 2), the attraction of A toward the liquid particles, be *more than twice* $A\ B$, the attraction toward a quadrant of the solid. Then, making $C\ D$ equal to $A\ B$, these two resultants balance as before; and as $A\ C$ is greater than $A\ B'$, the angle between $A\ C$ and the resultant $A\ E$ is less than 45°, and A is drawn away from the solid. Therefore the surface, being perpendicular to the resultant of the molecular forces acting on it, is convex upward.

Case 3d. If $A\ D$ (Fig. 143, 3) be exactly twice $A\ B$, then $C\ D$ balances $A\ B$, and the resultant of $A\ C$ and $A\ B'$ is $A\ E$ in a vertical direction; therefore the surface at A is level, being neither elevated nor depressed.

Case 1st occurs whenever a liquid readily *wets* a solid, if brought in contact with it, as, for example, water and clean glass.

Case 2d occurs when a solid *cannot be wet* by a liquid, as glass and mercury. Case 3d is rare, and occurs at the limit between the other two; water and steel afford as good an example as any.

200. Capillary Tubes.—In fine tubes these molecular forces affect the entire columns as well as their edges. If the material of the tube can be wet by a liquid, it will raise a column of that liquid above the level, at the same time making the top of the column concave. If it is not capable of being wet, the liquid is depressed, and the top of the column is convex. The first case is illustrated by glass and water; the second by glass and mercury.

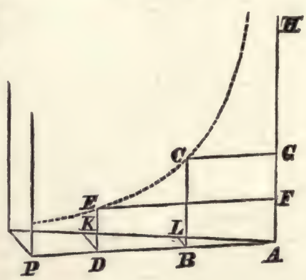
The materials being given, the distance by which the liquid is elevated or depressed varies inversely as the diameter. Therefore the product of the two is constant.

The amount of elevation and depression depends on the strength of the molecular forces, rather than on the specific gravity of the liquids. Alcohol, though lighter than water, is raised only half as high in a glass tube.

201. Parallel and Inclined Plates.—Between parallel plates a liquid rises or falls half as far as in a tube of the same diameter. This is because the sustaining force acts only on two sides of each filament, while in a tube it acts on all sides. Therefore, as in tubes the height varies inversely as the diameter, so in plates the height varies inversely as the distance between them.

If the plates are inclined to each other, having their edge of meeting perpendicular to the horizon, the surface of a liquid rising between them assumes the form of a *hyperbola*, whose branches approach the vertical edge, and the water-level, as the asymptotes of the curve. This results from the law already stated, that the height varies inversely as the distance between the plates. Let the edge of meeting, $A\ H$ (Fig. 144), be the axis of ordinates, and the line in which the level surface of the water intersects the glass, $A\ P$, the axis of abscissas. Let $B\ C$, $D\ E$, be any ordinates, and $A\ B$, $A\ D$, their abscissas, and $B\ L$, $D\ K$, the distances between the plates. By the law of capillarity, the heights $B\ C$, $D\ E$, are inversely as $B\ L$, $D\ K$. But, by the similar triangles, $A\ B\ L$, $A\ D\ K$, $B\ L$, $D\ K$, are as $A\ B$, $A\ D$; therefore, $B\ C$, $D\ E$, are inversely as $A\ B$, $A\ D$; and this is a property of the hyperbola with reference to the centre and asymptotes, that the ordinates are inversely as the abscissas.

Fig. 144.



202. Effects of Capillarity on Floating Bodies.—Some cases of apparent attractions and repulsions between floating bodies are caused by the forms which the liquid assumes on the sides of the bodies. If two balls raise the water about them, and are so near to each other that the concave surfaces between them meet in one, they immediately approach each other till they touch; and then, if either be moved, the other will follow it. The water, which is raised and hangs suspended between them, draws them together.

Again, if each ball depresses the water around it, they will also move to each other, and be held together, so soon as they are near enough for the convex surfaces to meet. In this case, they are not pulled, but pushed together by the hydrostatic pressure of the higher water on the outside.

Once more, if one ball raises the water, and the other depresses it, and they are brought so near each other that the curves meet, they immediately move apart, as if repelled. For now the equilibrium is destroyed in a way just the reverse of the preceding cases. The water between the balls is too high for that which depresses, and too low for that which raises the water, so that the former is pushed away, and the latter is drawn away.

The first case, which is by far the most common, explains the fact often observed, that floating fragments are liable to be gathered into clusters; for most substances are capable of being wet, and therefore they raise the water about them.

203. Illustrations of Capillary Action.—It is by capillary action that a part of the water which falls on the earth is kept near its surface, instead of sinking to the lowest depths of the soil. This force aids the ascent of sap in the pores of plants. It lifts the oil between the fibres of the lamp-wick to the place of combustion. Cloth rapidly imbibes moisture by its numerous capillary spaces, so that it can be used for wiping things dry. If paper is not *sized*, it also imbibes moisture quickly, and can be used as *blotting-paper ;* but when its pores are filled with sizing, to fit it for writing, it absorbs moisture only in a slight degree and the ink which is applied to it must dry by evaporation.

The great strength of the capillary force is shown in the effects produced by the swelling of wood and other substances when kept wet. Dry wooden wedges, driven into a groove cut around a cylinder of stone, and then occasionally wet, will at length cause it to break asunder. As the pores between the fibres of a rope run around it in spiral lines, the swelling of a rope caused by keeping it wet will contract its length with immense force.

204. Questions in Hydrostatics.—

1. The diameters of the two cylinders of a hydraulic press are *one inch* and *one foot*, respectively; before the piston descends, the column of water in the small cylinder is *two feet* higher than the bottom of the large piston. Suppose that by a screw a force of 500 lbs. is applied to the small piston; what is the whole force exerted on the large piston at the beginning of the stroke?

Ans. 72098.17 lbs.

2. A junk bottle, whose lateral surface contained 50 square inches, being let down into the sea 3000 feet, what pressure do the sides of the bottle sustain, a cubic foot of sea water weighing 64.37 lbs.? *Ans.* 67052.08 + lbs.

3. A Greenland whale sometimes has a surface of 3600 square feet; what pressure would he bear at the depth of 800 fathoms?

Ans. 1,112,313,600 lbs.

4. A mill-dam, running perpendicularly across a river, slopes at an angle of 30 degrees with the horizon. The average depth of the stream is 12 feet, and its breadth 500 yards; required the amount of pressure on the dam? *Ans.* 13,500,000 lbs.

5. A mineral weighs 960 grains in air, and 739 grains in water; what is its specific gravity? *Ans.* 4.344.

6. What are the respective weights of two equal cubical masses of gold and cork, each measuring 2 feet on its linear edge?

Ans. The gold weighs 9625 lbs. = 4.812 tons; the cork weighs 120 lbs.

7. A mass of granite contains 5949 cubic feet. The specific gravity of a fragment of it is found to be 2.6; what does the mass weigh? *Ans.* 483.356 tons.

8. An island of ice rises 30 feet out of water, and its upper surface is a circular plane, containing $\frac{3}{4}$ths of an acre. On the supposition that the mass is cylindrical, required its weight, and depth below the water, the specific gravity of sea-water being 1.0263, and that of ice .92. *Ans.* Weight, 272048 tons; depth, 259.64 feet.

9. Wishing to ascertain the exact number of cubic inches in a very irregular fragment of stone, I ascertained its loss of weight in water to be 5.346 ounces; required its volume.

Ans. 9.238 cubic inches.

10. Hiero, king of Syracuse, ordered his jeweller to make him a crown of gold containing 63 ounces. The artist attempted a fraud by substituting a certain portion of silver; which being suspected, the king appointed Archimedes to examine it. Archimedes, putting it into water, found it displaced 8.2245 cubic inches of the fluid; and having found that the inch of gold weighs 10.36 ounces, and that of silver 5.85 ounces, he discovered what

part of the king's gold had been purloined; it is required to repeat the process. *Ans.* 28.8 ounces.

11. The specific gravity of lead being 11.35; of cork, .24; of fir, .45; how much cork must be added to 60 lbs. of lead, that the united mass may weigh as much as an equal bulk of fir?

Ans. 65.8527 lbs.

CHAPTER II.

LIQUIDS IN MOTION.

205. Depth and Velocity of Discharge.—From an aperture which is small, compared with the breadth of the reservoir, *the velocity of discharge varies as the square root of the depth.* For the pressure on a given area varies as the depth (Art. 178). If the area is removed, this pressure is a force which is measured by the momentum of the water; therefore the *momentum* varies as the depth (d). But momentum varies as the mass (q) multiplied by the velocity (v); hence $q\,v \propto d$. But it is obvious that q and v vary alike, since the greater the velocity, the greater in the same ratio is the quantity discharged. Therefore, $q^2 \propto d$, or $q \propto d^{\frac{1}{2}}$; also $v^2 \propto d$, or $v \propto d^{\frac{1}{2}}$.

Not only does the velocity vary as the square root of the depth of the orifice, but *it is equal to that acquired by a body falling through the depth.*

Let $h =$ the height of the liquid above the orifice, and $h' =$ the height of an infinitely thin layer at the orifice.

If this thin layer were to fall through the height h', under the action of its own weight or pressure, the velocity acquired would be $v' = \sqrt{2gh'}$ (Art. 27).

Denoting the velocity generated by the pressure of the entire column by v, we have, since velocity $\propto \sqrt{\text{depth}}$,

$$v : v' :: \sqrt{h} : \sqrt{h'}, \text{ or}$$
$$v : \sqrt{2gh'} :: \sqrt{h} : \sqrt{h'};$$
$$\therefore v = \sqrt{2gh}.$$

But $\sqrt{2gh}$ is also the velocity acquired in falling through the distance h (Art. 27).

From an orifice 16.1 feet below the surface of water, the velocity of discharge is 32.2 feet per second, because this is the velocity acquired in falling 16.1 feet; and at a depth *four* times as great,

that is, 64.4 feet, the velocity will only be doubled, that is, 64.4 feet per second.

As the velocity of discharge at any depth is equal to that of a body which has fallen a distance equal to the depth, it is theoretically immaterial whether water is taken upon a wheel from a gate at the same level, or allowed to fall on the wheel from the top of the reservoir. In practice, however, the former is best, on account of the resistance which water meets with in falling through the air.

206. Descent of Surface.—When water is discharged from the bottom of a cylindric or prismatic vessel, the surface descends with a *uniformly retarded* motion. For the velocity with which the surface descends varies as the velocity of the stream, and therefore as the square root of the depth (Art. 205). But this is a characteristic of uniformly retarded motion, that the velocity varies as the square root of the distance from the point where the motion terminates, as in the case of a body ascending perpendicularly from the earth.

The descent of the surface of water in a prismatic vessel has been used for measuring time. The *clepsydra*, or water-clock of the Romans, was a time-keeper of this description. The graduation must increase upward, as the odd numbers 1, 3, 5, 7, &c.; since, by the law of this kind of motion, the spaces passed over in equal times are as those numbers.

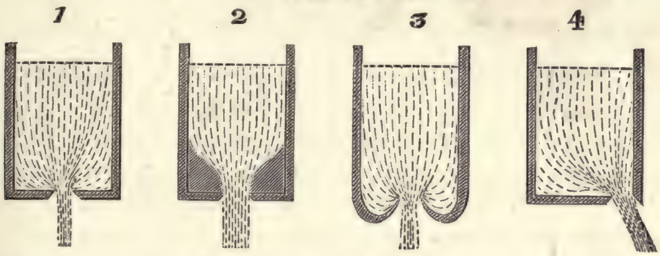
If a prismatic vessel is kept full, it discharges *twice* as much water in the same time as if it is allowed to empty itself. For the velocity in the first instance, is *uniform;* and in the second it is *uniformly retarded*, till it becomes zero. We reason in this case, therefore, as in regard to bodies moving uniformly, and with motion uniformly accelerated from rest, or uniformly retarded till it ceases (Art. 21), that the former motion is *twice* as great as the latter.

207. Discharge from Orifices in Different Situations.—Other circumstances besides *area* and *depth* of the aperture are found to have considerable influence on the velocity of discharge. Observations on the directions of the filaments are made by introducing into the water particles of some opaque substance, having the same density as water, whose movements are visible. From such observations it appears that the particles of water descend in vertical lines, until they arrive within three or four inches of the aperture, when they gradually turn in a direction more or less oblique toward the place of discharge. This convergence of the filaments extends outside of the vessel, and causes the stream to

diminish for a short distance, and then increase. The smallest section of the stream, called the *vena contracta,* is at a distance from the aperture varying from *one-half* of its diameter to the *whole.*

If water is discharged through a circular aperture in a thin plate in the bottom of the reservoir, and at a distance from the sides, as in Fig. 145, 1, the filaments form the vena contracta at a distance beyond the aperture equal to *one-half* of its diameter; the area of the section at the vena contracta is less than *two-thirds* (0.64) of the area of the aperture; this contraction also lessens the theoretical velocity by about four per cent., leaving .96 v for the final velocity; combining these two causes, it is found that for circular orifices of $\frac{1}{2}$ to 6 inches in diameter, with from four to 20 feet head of water, the actual discharge is only .615 of the theoretical discharge.

FIG. 145.



If the reservoir terminates in a short pipe or ajutage, whose interior is adapted to the curvature of the filaments, as far as to the vena contracta, or a little beyond, as in Fig. 145, 2, it is found the most favorable for free discharge, which in some cases reaches 0.98 of the theoretical discharge. The stream is smooth and pellucid like a rod of glass. The most unfavorable form is that in which the ajutage, instead of being external, as in the case just described, projects inward, as in Fig. 145, 3; the filaments in this case reach the aperture, some ascending, others descending, and therefore interfere with each other. Hence the stream is much roughened in its appearance, and the flow is only 0.53 of what is due to the size of the aperture and its depth.

When the aperture is through a thin plate, the contraction of the stream and the amount of discharge are both modified by the circumstance of being near one or more sides of the reservoir. There is little or no contraction on the side next the wall of the vessel, since the filaments have no obliquity on that side; and the quantity is on that account increased. The filaments from the opposite side also divert the stream a few degrees from the perpendicular (Fig. 145, 4).

208. Friction in Pipes.—As has just been stated, an ajutage extending to or slightly beyond the vena contracta, and adapted to the form of the stream, very much increases the quantity discharged; but beyond that, the longer the pipe, the more does it impede the discharge by friction. For a given quantity of water flowing through a pipe the resistance of friction increases with the number of points with which the water comes in contact; that is, the resistance is in proportion to the wetted surface; for every particle of water in contact with the interior surface of the pipe, acts as a retarding force. Now let f be the resistance of friction in a pipe of *unit* diameter, length and velocity; then the resistance in a pipe l feet long and d feet in diameter with a unit of velocity will be $f\,d\,l$; but the quantity of water delivered by this pipe will be d^2 times that delivered by the former, in unit of time with same velocity, since areas of cross-sections are to each other as squares of their diameters; therefore for the same quantity of water delivered, the resistance of friction in the latter pipe will be $\frac{f\,d\,l}{d^2}$ or $\frac{f\,l}{d}$, that is to say, *the resistance of friction in pipes is directly as their lengths and inversely as their diameters, the velocity being constant.* In order, therefore, to convey water at a given rate through a long pipe, it is necessary either to increase the head of water or to enlarge the pipe, so as to compensate for friction.

An aqueduct should be as *straight* as possible, not only to avoid unnecessary increase of length, but because the force of the stream is diminished by all changes of direction. If there must be change, it should be a gradual curve, and not an abrupt turn. When a pipe changes its direction by an *angle*, instead of a curve, there is a useless expenditure of force; a change of 90° requires that the head of water should be increased by nearly the height due to the velocity of discharge. For instance, if the discharge is *eight* feet per second (which is the velocity due to one foot of fall), then a right angle in the pipe requires that the head of water should be increased by nearly *one* foot, in order to maintain that velocity.

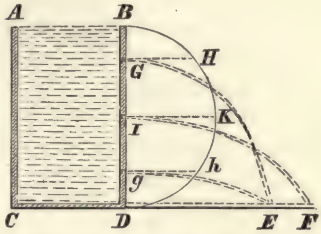
Empirical formulæ, based upon the results of experiments for the velocity of flow in pipes, and for the loss of head due to bends and angles in the pipe, are given in works treating of Practical Hydraulics. The derivation and development of such formulæ is beyond the scope of a work like this.

209. Jets.—Since a body, when projected upward with a certain velocity, will rise to the same height as that from which it must have fallen to acquire that velocity, therefore, if water issue from the side of a vessel through a pipe bent upward, it would,

were it not for the resistance of the air and friction at the orifice, rise to the level of the water in the reservoir. If water is discharged from an orifice in any other than a vertical direction, it describes a parabola, since each particle may be regarded as a projectile (Art. 47).

If a semicircle be described on the perpendicular side of a vessel as a diameter, and water issue horizontally from any point, its *range*, measured on the level of the base, equals *twice the ordinate* of that point. For, the velocity with which the fluid issues from the vessel, being that which is due to the height $B\,G$ (Fig. 146), is $\sqrt{2\,g\,.\,B\,G}$ (Art 27). But after leaving the orifice, it arrives at the horizontal plane in the time in which a body would fall freely through $G\,D$, which is $\sqrt{\frac{2\,G\,D}{g}}$. Since the horizontal motion is uniform, the space equals the product of the time by the velocity; that is, $D\,E = \sqrt{\frac{2\,G\,D}{g}} \times \sqrt{2\,g\,.\,B\,G} = 2\sqrt{B\,G\,.\,G\,D} = 2\,G\,H$, or twice the ordinate of the semicircle at the place of discharge.

Fig. 146.



The greatest range occurs when the fluid issues from the centre, for then the ordinate is greatest; and the range at equal distances above and below the centre is the same.

The remarks already made respecting pipes apply to those which convey water to the jets of fire-engines and fountains. If the pipe or hose is very long, or narrow, or crooked, or if the jet-pipe is not smoothly tapered from the full diameter of the hose to the aperture, much force is lost by friction and other resistances, especially in great velocities. If the length of hose is even *twenty* times as great as its diameter, 32 per cent. of height is lost in the jet, and more still when the ratio of length to diameter is greater than this.

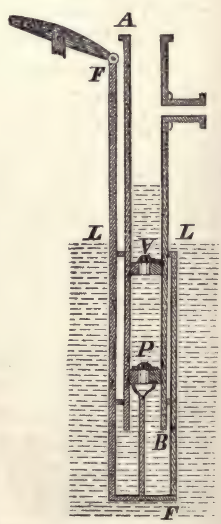
310. Rivers.—Friction and change of direction have great influence on the flow of rivers. A *dynamical equilibrium*, as it is called, exists between gravity, which causes the descent, and the resistances, which prevent acceleration at any given point, beyond a certain moderate limit; as the same quantity of water must pass every cross section of the stream in the same unit of time, under ordinary conditions, the velocity varies inversely as the area of

the cross section. The velocity in all parts of the same section, however, is not the same; it is greatest at that part of the surface where the depth is greatest, and least in contact with the bed of the stream.

To find the mean *velocity* through a given section, it is necessary to float bodies at various places on the surface, and also below it, to the bottom, and to divide the sum of all the velocities thus obtained, by the number of observations. To obtain the *quantity* of water which flows through a given section of a river, having determined the velocity as above, find next the area of the section, by taking the depth at various points of it, and multiplying the mean depth by the breadth. The quantity of water is then found by multiplying the area by the velocity.

The increased velocity of a stream during a freshet, while the stream is confined within its banks, exhibits something of the acceleration which belongs to bodies descending on an inclined plane. It presents the case of a river flowing upon the top of another river, and consequently meeting with much less resistance than when it runs upon the rough surface of the earth itself. The augmented force of a stream in a freshet arises from the simultaneous increase of the quantity of water and the velocity. In consequence of the friction of the banks and beds of rivers, and the numerous obstacles they meet with in their winding course, their velocity is usually very small, not more than three or four miles per hour; whereas, were it not for these impediments, it would become immensely great, and its effects would be exceedingly disastrous. A very slight declivity is sufficient for giving the running motion to water. The largest rivers in the world fall about five or six inches in a mile.

FIG. 147.



211. Hydraulic Pumps.—The most common pumps for raising water operate on a principle of pneumatics, and will be described under that subject.

In the *lifting pump* the water is pushed up in the pump tube by a piston placed below the water-level. In the tube $A\ B$ (Fig. 147) is a fixed valve V, a little below the water-level $L\ L$, while still lower is the piston P, in which there is a valve. Both of these valves open upward. The piston is attached to a rod, which extends downward to the frame $F\ F$. This frame can be moved

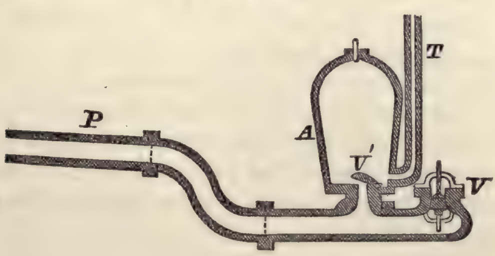
up and down on the outside of the tube by a lever. When the piston descends, the water passes through its valve by hydrostatic pressure; and when raised, it pushes the water before it through the fixed valve, which then prevents its return. In this manner, by repeated strokes, the water can be driven to any height which the instrument can bear.

The *chain pump* consists of an endless chain with circular disks attached to it at intervals of a few inches, which raise the water before them in a tube, by means of a wheel over which the chain passes; the wheel may be turned by a crank. The disks cannot fit closely in the tube without causing too great resistance; hence, a certain velocity is requisite in order to raise water to the place of discharge; and after the working of the pump ceases, the water soon descends to the level in the well.

212. Centrifugal Pumps.—Water may also be raised through small heights and in great volume by the centrifugal pump. This consists of revolving curved, hollow arms, connected with a hollow axis through which the water enters. As this axis is made to rotate in a direction contrary to the curvature of the arms the centrifugal force causes the water to leave the arms and move off in tangents; a casing drum inclosing the revolving portion forces the water to move around in a vortex till it reaches a delivery pipe entering the drum as a tangent, through which it is discharged. A high delivery requires so great velocity that the pump becomes inferior in efficiency to other forms.

213. The Hydraulic Ram.—When a large quantity of water is descending through an inclined pipe, if the lower extremity is suddenly closed, since water is nearly incompressible, the shock of the whole column is received in a single instant, and if no escape is provided, is very likely to burst the pipe. The intensity of the shock of water when stopped is made the means of raising a portion of it above the level of the head. The instrument for effecting this is called the *hydraulic ram*. At the lower end of a long pipe, *P* (Fig. 148), is a valve, *V*, opening downward;

FIG. 148.



near it, another valve, V', opens into the air-vessel, A; and from this ascends the pipe, T, in which the water is to be raised. As the valve V lies open by its weight, the water runs out, till its momentum at length shuts it, and the entire column is suddenly stopped; this impulse forces the water into the air-vessel, and thence, by the compressed air, up the tube T. As soon as the momentum is expended, the valve V drops, and the process is repeated.

214. Water-Wheels with a Horizontal Axis.—The *overshot wheel* (Fig. 149) is constructed with buckets on the circumference, which receive the water just after passing the highest point, and empty themselves before reaching the bottom. The weight of the water, as it is all on one side of a vertical diameter, causes the wheel to revolve. It is usually made as large as the fall will allow, and will carry machinery with a very small supply of water, if the fall is only considerable. The *moment* of each bucket-full constantly increases from a, where it is filled, to F, where its acting distance is radius, and therefore a maximum. From F downward the moment decreases, both by loss of water and diminution of acting distance, and becomes zero at L. These wheels deliver from 70 to 80 per cent. of the horse-power of the fall of water received upon them.

FIG. 149.

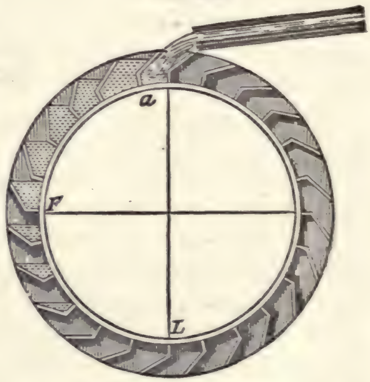
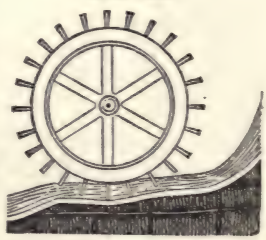


FIG. 150.

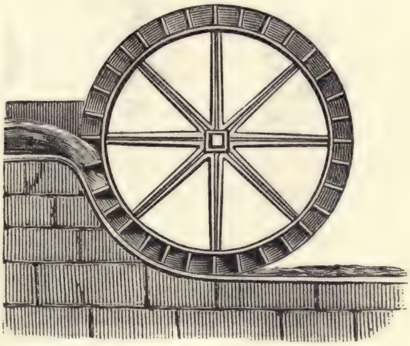


The *undershot wheel* (Fig. 150) is revolved by the momentum of running water, which strikes the float-boards on the lower side. When these are placed, as in the figure, perpendicular to the circumference, the wheel may turn either way; this is the construction adopted in tide-mills. When the wheel is required to turn only in one direction, an advantage is gained by placing the float-boards so as to present an acute angle toward the current, by which means the water acts partly by its weight, as in the overshot wheel. The undershot wheel is adapted to situations where the supply of water is always abundant.

The maximum efficiency of these wheels is obtained when the circumferential velocity is one half the velocity of the water, and is about 30 per cent. of the theoretical work of the water used. With curved float-boards the efficiency may reach about 60 per cent.

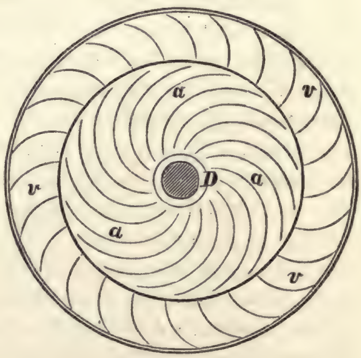
In the *breast wheel* (Fig. 151) the water is received upon the float-boards at about the height of the axis, and acts partly by its weight, and partly by its momentum. The planes of the float-boards are set at right angles to the circumference of the wheel, and are brought so near the mill-course that the water is held and acts by its weight, as in buckets. The efficiency is about 40 to 50 per cent.

FIG. 151.



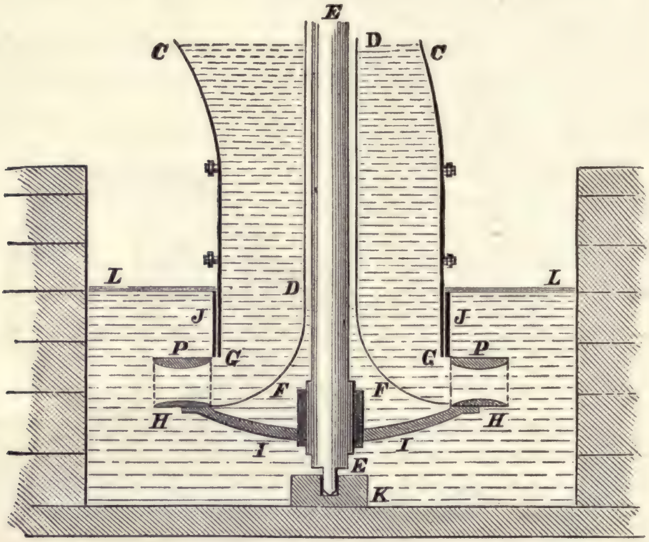
215. The Turbine.—This very efficient water-wheel, frequently called the French turbine, is of modern invention, and has received its chief improvements in this country. It revolves on a vertical axis, and surrounds the bottom of the reservoir from which it receives the water. The lower part of the reservoir is divided into a large number of sluices by curved partitions, which direct the water nearly into the line of a tangent, as it issues upon the wheel. The vanes of the wheel are curved in the opposite direction, so as to receive the force of the issuing streams at right angles. The horizontal section (Fig. 152) shows the lower part of the reservoir with its curved guides, *a*, *a*, *a*, and the wheel with its curved vanes, *v*, *v*, *v*, surrounding the reservoir; *D* is the central tube, through which the axis of the wheel passes. Fig. 153 is a vertical section of the turbine; but it does not present the guides of the reservoir, nor the vanes of the wheel. *C G*, *C G*, is the outer wall of the reservoir; *D*, *D*, its inner wall or tube; *F*, *F*, the base, curved so as to turn the

FIG. 152.



descending water gradually into a horizontal direction. The outer wall, which terminates at *G,G,* is connected with the base

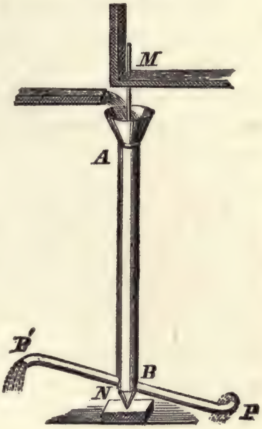
FIG. 153.



and tube by the guides which are shown at *a, a,* in Fig. 152. The lower rim of the wheel, *H, H,* is connected with the upper rim, *P, P,* by the vanes between them, *v, v* (Fig. 152), and to the axis, *E, E,* by the spokes *I, I.* The gate, *J, J,* is a thin cylinder which is raised or lowered between the wheel and the sluices of the reservoir. The bottom of the axis revolves in the socket *K,* and the top connects with the machinery. As the reservoir cannot be supported from below, it is suspended by flanges on the masonry of the wheel-pit, or on pillars outside of the wheel. To prevent confusion in the figure, the supports of the reservoir and the machinery for raising the gate are omitted. By the curved base and guides of the reservoir, the water is conducted in a spiral course to the wheel, with no sudden change of direction, and thus loses very little of its force. The wheel usually runs below the level of the water in the wheel-pit, as represented in the figure, *L L* being the surface of the water. The reservoir is sometimes merely the extremity of a large tapering tube or supply pipe, bent from a horizontal to a vertical direction. In such a case, the tube *D D,* in which the axis runs, passes through the upper side of the supply pipe. The figure represents only the lower part. The efficiency is about 80 per cent., though many claim a much higher efficiency than this.

216. Barker's Mill.—This machine operates on the principle of *unbalanced hydrostatic pressure.* It consists of a vertical hollow cylinder, $A\ B$ (Fig. 154), free to revolve on its axis $M\ N$, and having a horizontal tube connected with it at the bottom. Near each end of the horizontal tube, at P and P', is an orifice, one on one side, and one on the opposite. The cylinder, being kept full of water, whirls in a direction opposite to that of the discharging streams from P and P'. This is owing to the fact that hydrostatic pressure is removed from the apertures, while on the interior of the tube, at points exactly opposite to them, are pressures which are now unbalanced, but which would be counteracted by the pressures at the apertures, if they were closed. The tube $P\ P'$ may revolve either in the air, or beneath the surface of the water. The speed of rotation is increased by lengthening the tube $A\ B$.

FIG. 154.



217. Resistance to Motion in a Liquid.—The resistance which a body encounters in moving through any fluid arises from the inertia of the particles of the fluid, their want of perfect mobility among each other, and friction. Only the first of these admits of theoretical determination. So far as the inertia of the fluid is concerned, the *resistance* which a surface meets with in moving perpendicularly through it *varies as the square of the velocity.* For the resistance is measured by the *momentum* imparted by the moving body to the fluid. And this momentum (m) varies as the product of the quantity of fluid set in motion (q), and its velocity (v); or $m \propto q\ v$. But it is obvious that the quantity displaced varies as the velocity of the body, or $q \propto v$; hence $m \propto v^2$. Therefore the resistance varies as the square of the velocity.

This proposition is found to hold good in practice, where the velocity is small, as the motions of boats or ships in water; but when the velocity becomes very great, as that of a cannon ball, the resistance increases in a much higher ratio than as the square of the velocity. Since action and reaction are equal, it makes no difference, in the foregoing proposition, whether we consider the body in motion and the fluid at rest, or the fluid in motion and striking against the body at rest.

Since the resistance increases so rapidly, there is a wasteful

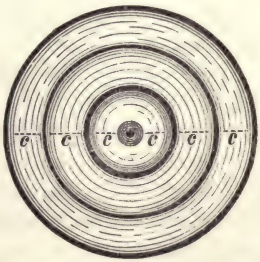
expenditure of force in trying to attain great velocities in navigation. For, in order to double the velocity of a steamboat, the force of the steam must be increased four fold; and, in order to triple its velocity, the force must become nine times as great.

When the resistance becomes equal to the moving force, the body moves uniformly, and is said to be in a state of *dynamical equilibrium.* Thus, a body falling freely through the air by gravity does not continue to be accelerated beyond a certain limit, but is finally brought, by the resistance of the air, to a uniform motion.

218. Water Waves.—These are moving elevations of water, caused by some force which acts unequally on its surface. There are two very different kinds of waves, called, respectively, *waves of oscillation* and *waves of translation.* In the first kind the particles of water have a vibratory or reciprocating motion, by which the vertical columns are alternately lengthened and shortened. A familiar example of this kind is the *sea-wave.* In the waves of translation the particles are raised, transferred forward, and then deposited in a new place, without any vibratory movement.

219. Waves of Oscillation.—If a pebble be tossed upon still water, it crowds aside the particles beneath it, and raises them above the level, forming a wave around it in the shape of a ring. As soon as this ring begins to descend, it elevates above the level another portion around itself, and thus the ring-wave continues to spread outward every way from the centre. But in the meantime the water at the centre, as it rises toward the level, acquires a momentum which lifts it above that level. From that position it descends, and once more passes below the level, thus starting a new wave around it, as at first, only of less height. Hence, we see as the result of the first disturbance, a series of concentric waves continually spreading outward and diminishing in height at greater distances, until they cease to be visible. In Fig. 155 are represented three circular waves at one of the moments of time when the center is lowest. The shaded parts are the basins or *troughs,* and the light parts, *c, c, c,* are the ridges or *crests.* Fig. 156 is a vertical section along the line, *c, c,* through the centre of the system, corresponding to the momentary arrangement of Fig. 155. The central basin is at *b,* and

Fig. 155.



the crests at *c*, *c*, *c*. A little later, when either crest has moved half way to the place of the next one, both figures will have become reversed; the centre will be a hillock, the troughs will be at *c*, *c*, and the crests at the middle points between them.

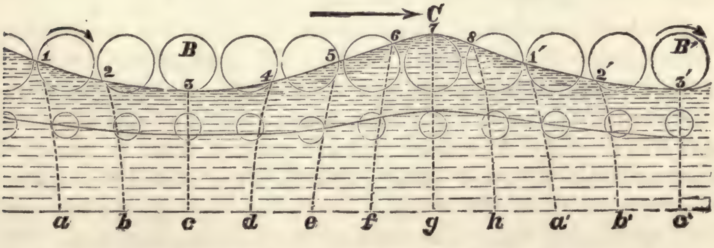
FIG. 156.

Except in the circular arrangement of the crests and troughs around a centre, the waves of the foregoing experiment illustrate common sea-waves. They constitute a system of elevations and depressions moving along the surface at right angles to the line of the wave-crest.

220. Phases.—In the cross-section (Fig. 156), where the waves are shown in profile, any particular part of the curve is called a *phase*. Different phases are generally unlike, both in elevation and in movement. The corresponding parts of different waves are called *like phases;* and those points in which the molecular motions are reversed are called *opposite phases*. The highest points of the crests of two waves are like phases; the highest point of the crest and the lowest point of the trough are opposite phases. Two points half way from crest to trough, one on the front of the wave, and the other on the rear of it, are also opposite phases, although they are at the same elevation; for they are moving in opposite directions. The *length* of a wave is the horizontal distance between two successive like phases.

221. Molecular Movements.—The water which constitutes a system of waves does not advance along the surface, as the waves themselves do; for a floating body is not borne along by them, but alternately rises and falls as the waves pass under it. Each particle of water, instead of advancing with the wave, oscillates about its mean place, alternately rising as high as the crest, and falling as low as the trough. Its path is the circumference of a vertical circle. Let $B\,B'$ (Fig. 157) represent two successive troughs, and

FIG. 157.



C the intervening crest; and for convenience suppose $a\,a'$, the wave length, to be divided into eight equal parts. The waves

move in the direction of the straight arrow, while the particles of water revolve in the direction of the bent arrows. The points 1, 2, 3, &c., represent particles which, if the water were at rest, would be directly above the points *a, b, c,* &c. At the moment represented, 1 is at the extreme left of its revolution, 2 is at 45° below, 3 at the lowest point, &c. When the wave has advanced one-eighth of its length, 1 will have ascended 45°, 2 will have ascended to the extreme left, and each of the eight particles will have revolved one-eighth of the circumference shown in the figure. Then 4 will be at the bottom, and 8 at the top. Each particle of water on the front of the wave, from 1 to 3, and from 7 to 3′, is ascending; each one on the rear, from 3 to 7, is descending. It is plain that while the wave advances its whole length, that is, while the phase *B* is moving to *B′*, each particle makes a complete revolution; 3′, which is now lowest, will be lowest again, having in the meantime occupied all other points of the circumference.

Particles *below* the surface, as far as the wave disturbance reaches, perform synchronous revolutions, but in smaller circles, as represented in the figure.

222. Form of Waves of Oscillation.—The sectional form of these waves is that of the inverted *trochoid*, a curve described by a point in a circle as it rolls on a straight line. The curvature of the crest is always greater than that of the trough, and the summit may possibly be a sharp ridge, in which case the section of the trough is a *cycloid*, the describing point of the rolling circle being on the circumference; the height of such waves is to their length as the diameter of a circle to the circumference. If waves are ever higher than about one-third of their length, the summits are broken into spray.

223. Distortion of the Vertical Columns.—Where the surface is depressed below its level, some of the water must be crowded laterally out of its place, and the vertical columns, being shorter, must necessarily be wider, at least in the upper part. So, too, where the surface is raised above its level, the lengthened columns must be narrower. In Fig. 157 these effects are made apparent as the necessary result of the revolutions of the particles. The dotted lines, 1 *a*, 2 *b*, 3 *c*, &c., were all vertical lines when the water was at rest. But now they are swayed, some to the right and some to the left, none being vertical, except under the highest and lowest points of the waves. Under the trough the lines are spread apart, and under the crest they are drawn together. The sectional figures 1 *a b* 2, 2 *b c* 3, &c., which would all be rectangular if the water were at rest, are now distorted in form, the upper

parts being alternately expanded and contracted in breadth as the successive phases pass them.

224. Sea-Waves.—The waves raised by the wind rarely exhibit the precise forms above described, and the particles rarely revolve in exact circles, partly because there is scarcely ever a system of waves undisturbed by other systems, which are passing over the water at the same time, and partly because the wind, which was the original cause of the waves, acts continually upon their surfaces to distort and confuse them.

The *interference* of waves denotes, in general, the resultant system, which is produced by the combination of two or more separate systems. The joint effect of two systems is various, according as they are more or less unlike as to length of waves. But even if two systems are just alike, still the effect of interference will vary, according to the coincidence or the degree of discrepancy of their like phases. For instance, if two similar systems exactly coincide, phase for phase, the waves simply have double height; or, in general terms, there is double intensity in the wave motion. But if the phases of one system exactly coincide with the *opposite* phases of the other, then the water is nearly level, the crests of each system filling the troughs of the other. These two effects may be plainly seen in the intersections of ring-waves formed by dropping two pebbles on still water.

225. Waves of Translation.—The principal characteristics of the wave of translation are, that it is solitary—i. e., it does not belong to a system, like the other kind; and that its length and velocity both depend on the depth of the water. Where the water is deeper, the wave travels faster, and its length (measured in the direction of its progress) is longer. A wave of this character is started in a canal by a moving boat; and when the boat stops, it moves on alone. A grand example of this species is found in the tide-wave of the ocean. It is called the wave of translation because the particles of water are borne forward a certain distance while the wave is passing, and then remain at rest.

PART III.

PNEUMATICS.

CHAPTER I.

PROPERTIES OF GASES.—INSTRUMENTS FOR INVESTIGATION.

226. Gases Distinguished from Liquids.—The property of *mobility* of particles, which belongs to all fluids, is more remarkable in gases than in liquids.

While gaseous substances are compressed with ease, they are always ready to expand and occupy more space. This property, called *dilatability,* scarcely belongs to liquids at all.

This property may be experimentally illustrated by placing a bag only partly full of air under the receiver of an air pump and exhausting the air; the external pressure having been removed, the bag will seem full almost to bursting, the contained air having dilated to many times its former volume.

Invert a flask containing air into a beaker of colored water, and place the whole under the receiver of an air pump. As the air is exhausted the contained air in the flask will expand and, driving the water out of the neck of the flask, will rise in bubbles to the surface. Upon admitting air again to the receiver, the water will be forced into the flask to take the place of the escaped air, and will rise until the tension of the contained air, together with the weight of the water column, is equal to that of the air in the receiver.

227. Tension of Gases.—By the term *tension* just used, is meant the force exerted by the gas at each instant in opposition to any compressing or restraining force; or, in other words, the force of expansion. The molecules of the gas are supposed to be flying through space with great velocity in straight lines. The combined effect of the impact of these molecules upon the walls of the containing vessel is an outward pressure which is opposed by the strength of the material of the vessel. In the first experi-

ment given, the impact of the molecules of the air in the room upon the *outside* of the bag counterbalanced the impact of the molecules of the contained air upon the *inside;* but when the external air was removed, there was no counterbalancing force, until the bag expanded so much that the strength of elasticity of the rubber itself equaled the resultant of the impacts within. This theory of tension will be of great help in discussing the subject of expansion by heat.

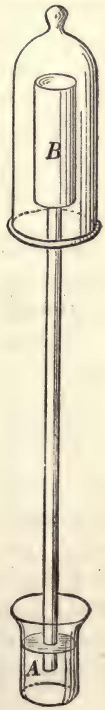
228. Change of Condition.—Liquids, and even solids, may be changed into the gaseous or aeriform condition by heating them sufficiently. By being cooled, they return again to their former state. In the gaseous form they are called *vapors.* All substances which are ordinarily gases can be so far cooled, especially under great pressure, as to be reduced to the liquid or solid form.

Those which can only be thus reduced under very great pressures, and at very low temperatures, are regarded as types of a theoretically perfect gas.

229. Diffusion of Gases.—If two flasks, *A* and *B*, be connected by a tube, as in Fig. 158, and the upper, *A*, be filled with hydrogen, or illuminating gas, and the lower *B* with carbonic dioxide, after a time some of the lighter gas will be found in *B*, having passed down through the tube, while a part of the heavy gas in *B* will have passed upwards to *A*. This result must follow from the theory of molecular motion given before. The action is called *diffusion.*

Fig. 158.

Fig. 159.



230. Osmose of Gases.—Cement a glass tube, about twenty-four inches long, to a porous cell *B* (Fig. 159), and dip the lower end of the tube into colored liquid *A*. Now fill an inverted bell jar with hydrogen, or illuminating gas, and place it over *B*. The gas will make its way into the porous cell more rapidly than the air makes its way out, diffusion inwards being more rapid than diffusion outwards, and, in consequence, some air will be driven out through the glass tube, escaping in bubbles through the liquid in *A*. Upon removing the bell jar the gas within the porous cell will pass out again more rapidly than air can pass in, and a partial vacuum will be

formed, causing a rise of the colored liquid in the tube. This mixing of gases through a porous cell, or a thin moistened membrane, is denominated *Osmose of Gases* to distinguish it from a similar action of liquids under like conditions. The inward flow is termed *endosmose,* and the outward flow *exosmose,* distinctions which are of no great importance.

231. Weight of Gases.—Like all other forms of matter, gases have weight. Some are relatively light, some heavy. Take a copper globe, and hang it upon one scale pan of a delicate balance, and accurately counterpoise it. Next exhaust the air from the globe, and it will be found lighter than before; fill with carbonic dioxide, and it will weigh much more than at first. The heavier gases may be poured from one vessel to another like water; carbonic dioxide may be poured from a beaker upon a burning candle, which may thus be extinguished.

232.—Pressure of Gases.—As a consequence of the weight of gases we have to consider the pressure exerted by them. We shall use the atmosphere as a type of all gases. Across the open top of a cylindric receiver stretch a sheet of rubber; upon exhausting the air from the receiver, the rubber will be pressed inwards by the external air. Substitute for the sheet of rubber a sheet of wetted bladder, which allow to dry. Upon exhausting the air the bladder will burst, under the pressure inwards, with a loud report.

Exhaust the air from a receiver, into which projects a jet tube closed with a stop-cock; upon submerging the outer end of the jet and opening the stop-cock, a fountain in vacuo will be produced.

Exhaust the air from two closely fitted hemispheres, called Magdeburg hemispheres, of about four inches diameter; a force of over 175 lbs. will be required to separate them.

Having a cylinder about five inches in diameter, with closely fitted piston, attach a weight of 250 lbs. to the lower side of the piston, exhaust the air from the cylinder above the piston, and the weight will be raised.

The pressure of the air upon our bodies and the outward pressure of the blood against the walls of the small veins and capillaries are in equilibrium. Place the palm of the hand upon the broad opening of a receiver, called a hand glass, and exhaust the air beneath; the air pressure being removed, the flesh will protrude into the receiver, and the skin, by its redness, will give evidence of the engorgement of the blood-vessels.

233. Buoyancy.—When the water displaced by an immersed

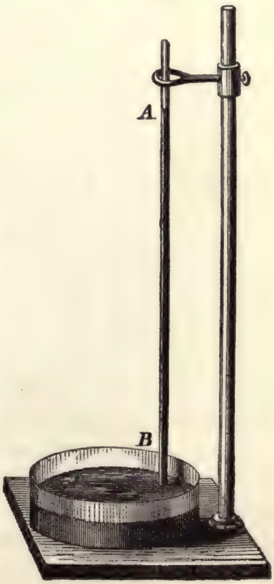
body weighs more than the body itself, it will rise to the surface and float; so too when the volume of air displaced by a body weighs more than that body, it will rise and float in the air. Attach the gas jet to a clay pipe by rubber tubing, and blow soap-bubbles; these will rise rapidly to the ceiling of the room. Blow a small bubble, and then transfer the end of the tube to the gas jet and enlarge the bubble till its specific gravity is about the same as that of the air; it will now float about the room, sometimes rising, sometimes falling, until it bursts.

Large balloons have ascended to a height of seven miles.

If a large and a small body are in equilibrium on the two arms of a balance, and the whole be set under a receiver, and the air be removed, the larger body will preponderate, showing that it is really the heaviest. Their apparent equality of weight when in the air is owing to its buoyant power, which diminishes the apparent weight of an immersed body by just the weight of the displaced fluid. Hence, the larger the body, the more weight it loses.

If the air be exhausted from a tube, four or five feet long and two inches in diameter, containing a small coin and a feather, it will be found, upon quickly inverting the tube, that the coin and the feather will fall through its length in the same time, both having the same velocity; if it were not for the obstruction of the air, all bodies would fall to the earth with the same velocity.

FIG. 160.



234. Torricelli's Experiment.—A glass tube *A B* (Fig. 160) about three feet long, and hermetically sealed at one end, is filled with mercury, and then, while the finger is held tightly on the open end, it is inverted in a cup of mercury. On removing the finger after the end of the tube is beneath the surface of the mercury, the column sinks a little way from the top, and there remains. Its height is found to be nearly thirty inches above the level of mercury in the cup. If sufficient care is taken to expel globules of air from the liquid, the space above the column in the tube is as perfect a vacuum as can be obtained. It is called the *Torricellian* vacuum, from Torricelli of Italy, a

disciple of Galileo, who, by this experiment, disproved the doctrine that *nature abhors a vacuum,* and fixed the limits of atmospheric pressure.

235. Pressure of Air Measured.—The column is sustained in the Torricellian tube by the pressure of air on the surface of mercury in the vessel; for the level of a fluid surface cannot be preserved unless there is an equal pressure on every part. Hence, the column of mercury on one part, and the column of air on every other equal part, must press equally. To determine, therefore, the pressure of air, we have only to weigh the column of mercury, and measure the area of the mouth of the tube. If this is carefully done, it is found that the weight of mercury is about 14.7 lbs. on a square inch. Therefore the atmosphere presses on the earth with a force of nearly 15 pounds to every square inch, or more than 2000 lbs. per square foot.

The specific gravity of mercury is about 13.6; and therefore the height of a column of water in a Torricellian tube should be 13.6 times greater than that of mercury, that is, about 34 feet. Experiment shows this to be true. And it was this significant fact, that *equal weights* of water and mercury are sustained in these circumstances, which led Torricelli to attribute the effect to a common force, namely, the pressure of the air.

236. Pascal's Experiment.—As soon as Torricelli's discovery was known, Pascal of France proposed to test the correctness of his conclusion, by carrying the apparatus to the top of a mountain, in order to see if less air above the instrument sustained the mercury at a less height. This was found to be true; the column gradually fell, as greater heights were attained. The experiment of Pascal also determined the relative density of mercury and air. For the mercury falls one-tenth of an inch in ascending 87.2 feet; therefore the weight of the one-tenth of an inch of mercury was balanced by the weight of the 87.2 feet of air. Therefore the specific gravities of mercury and air (being inversely as the heights of columns in equilibrium) are as $(87.2 \times 12 \times 10 =)$ 10464 : 1. In the same way it is ascertained that water is 770 times as dense as air. These results can of course be confirmed by directly weighing the several fluids, which could not be done before the invention of the air-pump.

That it is the atmospheric pressure which sustains the column of mercury may be shown thus: Place the Torricellian tube and cistern under a receiver, made for the purpose, and exhaust the air; the mercury will fall lower and lower at each stroke of the pump, until, if the pump be in good working order, the column will be nearly at the level of the mercury in the cistern.

237. Mariotte's Law.—

At a given temperature, the volume of air is inversely as the compressing force.

An instrument constructed for showing this is called *Mariotte's tube.* The end B (Fig. 161) is sealed, and A open. Pour in small quantities of mercury, inclining the tube so as to let air in or out, till both branches are filled to the zero point. The air in the short branch now has the same tension as the external air, since they just balance each other. If mercury be poured in till the column in the short tube rises to C, the inclosed air is reduced to one-half of its original volume, and the column A in the long branch is found to be 29 or 30 inches above the level of C, according to the barometer at the time. Thus, *two* atmospheres, one of mercury, the other of air above it, have compressed the inclosed air into *one-half* its volume. If the tube is of sufficient length, let mercury be poured in again, till the air is compressed to one-third of its original space; the long column, measured from the level of the mercury in the short one, is now twice as high as before; that is, *three* atmospheres, two of mercury and one of air, have reduced the same quantity of air to *one-third* of its first volume. This law has been found to hold good in regard to atmospheric air up to a pressure of nearly thirty atmospheres.

Fig. 161.

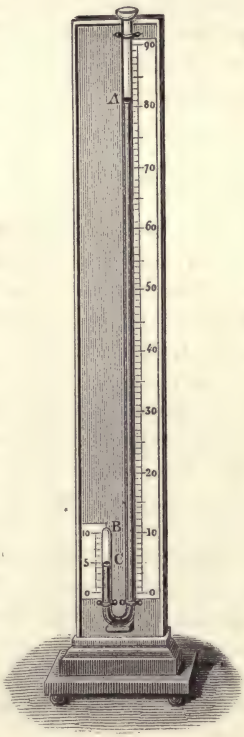


Fig. 162.

On the other hand, if the pressure on a given mass of air is diminished, its volume is found to increase according to the same law. When the pressure is *half* an atmosphere, the volume is *doubled;* when *one-third* of an atmosphere, the volume is three times as great, &c.

This may be shown by filling a tube A, closed at one end, nearly full of mercury, and inverting it in a tube-like cistern B, as in Fig. 162. Suppose that the tube, of uniform bore, contains an inch of air when the mercury is at

the same level within and without; upon raising the tube until the column within is about fifteen inches high the air will be found to occupy two inches of space. At the beginning of the experiment the mercury being at the same level in both tubes, the pressure upon the contained air was due to the atmosphere, and was about 14.7 lbs. per square inch. At the close the pressure was less by the force necessary to sustain the fifteen inches of mercury, leaving only a pressure of about one-half an atmosphere. Experiments in this case will not be satisfactory unless precautions be taken to remove the air bubbles which adhere to the glass tube. Since the tension of the inclosed air always balances the compressing force, and since the density is inversely as the volume, it follows from Mariotte's Law that, when the temperature is the same,

The tension of air varies as the compressing force; and *The tension of air varies as its density.*

This law is, however, not strictly true of gases, except when far removed from the critical point; when near the point of condensation the departure from the law is most marked. Even in the ordinary state the law is not *strictly* followed by many gases.

238. Dalton's Law.—

At a given temperature the tension of a mixture of gases is equal to the sum of the tensions of the gases taken separately.

In this law a *mixture* is spoken of and not a chemical combination. Each gas diffuses and is found equally distributed throughout the containing vessel just as though no other gas were present, differing in this respect from mechanical mixtures of liquids, such as oil and water, in which the components of the mixture arrange themselves according to their specific gravities. If a vessel A contains a cubic foot of nitrogen at a tension of ten lbs. per square inch, and a perfect vacuum B of the same capacity be connected with this, and *all the gas* be transferred to the new vessel B, the tension in the latter case will be the same as in the former, ten lbs. Now, if a cubic foot of oxygen at a tension of ten lbs. be transferred to the vessel B also, it will exert a pressure of ten lbs. just as though the nitrogen were not present, giving a total pressure of twenty lbs. for the mixture. These two illustrations have been given to prevent any misapprehension which may arise from the following frequently repeated and very concise wording of the law: "Every gas acts as a vacuum with respect to every other."

239. Laws of Mixture of Gases and Liquids.—Water and many other liquids will contain gases in solution; but under the same conditions of temperature and pressure, a given liquid

does not absorb equal quantities of different gases. For example, at the mean temperature and pressure water dissolves about .025 of its volume of nitrogen, .046 of its volume of oxygen, its own volume of carbonic dioxide, and 430 times its volume of ammonia. Mercury, on the other hand, does not dissolve any of the gases.

Experiment has determined the three following laws of the mixture of liquids and gases.

1st. *For the same gas, the same liquid, at a constant temperature, the weight of gas absorbed is proportional to the pressure.* From this it follows that the volume dissolved is constant, whatever may be the changes in pressure, or, what is the same thing, the density of the gas absorbed bears a constant ratio to that of the gas not absorbed.

2d. *The quantity of gas absorbed increases as the temperature decreases.*

3d. *The quantity of a gas which a given liquid will dissolve is independent of the kinds and quantities of other gases which may already be held in solution.*

If instead of a single gas in contact with the liquid, a mixture of several gases be used, each of these will be dissolved in the quantity due to its proportional part of the total pressure. For example, since oxygen forms only about one-fifth the volume of the air, water under ordinary conditions absorbs the same quantity of oxygen, as if the atmosphere were wholly of oxygen under a pressure of $\frac{14.7}{5}$ lbs.

The first law may be experimentally illustrated by opening a bottle of common soda water. As soon as the cork is loosened it is driven out by the tension of the confined carbonic dioxide above the liquid, and the pressure being reduced by the escape of the free gas, the absorbed gas is at once given off in bubbles, the escape of which produces foaming.

If after all the gas has seemingly escaped, a portion of the liquid be poured into a beaker and placed under the receiver of the air pump, a fresh discharge of bubbles will follow the first stroke of the pump, consequent upon the still further reduction of pressure.

A portion poured into a flask and heated will serve to illustrate the second law, the rise in temperature causing a constant rise of bubbles of gas.

240. The Barometer.—When the Torricellian tube and basin are mounted in a case, and furnished with a graduated scale, the instrument is called a barometer. The scale is divided into

inches and tenths, and usually extends from 26 to 32 inches, a space more than sufficient to include all the natural variations in the weight of the atmosphere. By attaching a vernier to the scale, the reading may be carried to hundredths and thousandths of an inch, as is commonly done in meteorological observations. By observing the barometer from day to day, and from hour to hour, it is found that the atmospheric pressure is constantly fluctuating.

As the meteorological changes of the barometer are all comprehended within a range of two or three inches, much labor has been expended in devising methods for magnifying the motions of the mercurial column, so that more delicate changes of atmospheric pressure might be noted. The inclined tube and the wheel barometer are intended for this purpose. A description of these contrivances, however, is unnecessary, as they are all found to be inferior in accuracy to the simple tube and basin.

241. Corrections for the Barometer.—

1. For *change of level in the basin.*—The numbers on the barometer scale are measured from a certain zero point, which is assumed to be the level of the mercury in the basin. If now the column falls, it raises the surface in the basin; and if it rises, it lowers it. If the basin is broad, the change of level is small, but it always requires a correction. To avoid this source of error, the bottom of the basin is made of flexible leather, with a screw underneath it, by which the mercury may be raised or lowered, till its surface touches an index that marks the zero point. This adjustment should always be made before reading the barometer.

2. For *capillarity.*—In a glass tube mercury is depressed by capillary action (Art. 200). The amount of depression is less as the tube is larger. This error is to be corrected by the manufacturer, the scale being put below the true height by a quantity equal to the depression.

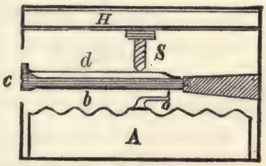
There is a slight variation in this capillary error, arising from the fact that the rounded summit of the column, called the *meniscus,* is more convex when ascending than when descending. To render the meniscus constant in its form, the barometer should be jarred before each reading.

3. For *temperature.*—As mercury is expanded by heat and contracted by cold, a given atmospheric pressure will raise the column too high, or not high enough, according to the temperature of the mercury. A thermometer is therefore attached to the barometer, to show the temperature of the instrument. By a table of corrections, each reading is reduced to the height the mercury would have if its temperature was 32° F.

4. For *altitude of station.*—Before comparing the observations of different places, a correction must be made for altitude of station, because the column is shorter according as the place is higher above the sea level.

242. The Aneroid Barometer.—This is a small and portable instrument, in appearance a little like a large chronometer. The essential part of this barometer is a flat cylindrical metallic box, shown in section at *A* (Fig. 163), whose upper surface is corrugated, so as to be yielding.

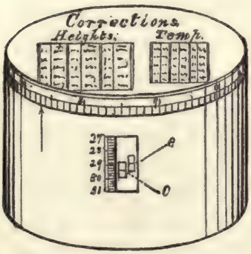
Fig. 163.



The box being partly exhausted of air, the external pressure causes the top to sink in to a certain extent; if the pressure increases, the surface descends a little more; if it diminishes, a little less.

These small movements are communicated to a lever *b* (Fig. 163) whose end *c* moves over a scale of inches on the case (Fig. 164). To insure contact of the pin *o* (Fig. 163), and also to secure a constant resistance to the motion of the lever *b* a spring *d* is attached to the lever *b* near the fulcrum, which is pressed upon by the screw *s*, whose graduated head *H* is of the same diameter as the box. The end of this spring *e* (Fig. 164) must be brought to coincide with the end of the lever, by turning the screw-head *H*. The reading of inches and tenths is taken from the scale of inches, the hundredths and thousandths being given by the screw-head. This is one of the most simple of all the aneroids in construction. A table of corrections for temperature, and reductions to the standard mercurial barometer is entered upon the cover.

Fig. 164.



243. Pressure and Latitude.—The mean pressure of the atmosphere at the level of the sea is very nearly 30 inches. But it is not the same at all latitudes. From the equator either northward or southward, the mean pressure increases to about latitude 30°, by a small fraction of an inch, and thence decreases to about 65°, where the pressure is less than at the equator, and beyond that it slightly increases. This distribution of pressures in zones is due to the great atmospheric currents, caused by heat in connection with the earth's rotation on its axis.

The amount of variation in barometric pressure is very unequal in different latitudes; and in general, the higher the latitude, the

11

greater the variation. Within the tropics the extreme range scarcely ever exceeds one-fourth of an inch, while at latitude 40° it is more than two inches, and in higher latitudes even reaches three inches.

244. Diurnal Variation.—If a long series of barometric observations be made, and the mean obtained for each hour of the day, the changes caused by weather become eliminated, and the diurnal oscillation reveals itself. It is found that the pressure reaches a maximum and a minimum twice in 24 hours. The times of greatest pressure are from 9 to 11, and of least pressure from 3 to 5, both A. M. and P. M. In tropical climates this variation is very regular, though small; but in the temperate zones the irregular fluctuations of weather conceal it in a great degree.

This daily fluctuation of the barometer is caused by the changes which take place from hour to hour of the day in the temperature, and by the varying quantity of vapor in the atmosphere.

The surface of the globe is always divided into a day and night hemisphere, separated by a great circle which revolves with the sun from east to west in twenty-four hours. The hemisphere exposed to the sun is warm, the other is cold. The time of greatest heat is not at noon, when the sun is in the meridian, but about two or three hours after; the period of greatest cold occurs about four in the morning. As the hemisphere under the sun's rays becomes heated, the air, expanding upwards and outwards, flows over upon the other hemisphere where the air is colder and denser. There thus revolves round the globe from day to day a wave of heated air, from the crest of which air constantly tends to flow towards the meridian of greatest cold on the opposite side of the the globe.

The barometer is influenced to a large extent by the elastic force of the vapor of water invisibly suspended in the atmosphere, in the same way as it is influenced by the dry air (oxygen and hydrogen). But the vapor of water also exerts a pressure on the barometer in another way. Vapor tends to diffuse itself equally through the air; but as the particles of air offer an obstruction to the watery particles, about 9 or 10 A. M., when evaporation is most rapid, the vapor is accumulated or pent up in the lower stratum of the atmosphere, and being impeded in its ascent its elastic force is increased by the reaction, and the barometer consequently rises. When the air falls below the temperature of the dew-point part of its moisture is deposited in dew, and since some time must elapse before the vapor of the upper strata can diffuse itself downwards to supply the deficiency, the barometer falls,—

most markedly at 10 P. M., when the deposition of dew is greatest.

245. The Barometer and the Weather.—The changes in the height of the barometer column depend directly on nothing else than the atmospheric pressure. But these changes of pressure are due to several causes, such as wind and changes of temperature and moisture.

The practice formerly prevailed of engraving at different points of the barometer scale several words expressive of states of weather, "fair, rain, frost, wind," &c. But such indications are worthless, being as often false as true; this is evident from the fact that the height of the column would be changed from one kind of weather to another by simply carrying the instrument to a higher or lower station.

No general system of rules can be given for anticipating changes of weather by the barometer, which would be applicable in different countries. Rules found in English books are of very little value in America.

Severe and extensive storms are almost always accompanied by a fall of the barometer while passing, and succeeded by a rise of the barometer.

246. Heights Measured by the Barometer.—Since mercury is 10464 times as heavy as air (Art. 236), if the barometer is carried up until the mercury falls one inch, it might be inferred that the ascent is 10464 inches, or 872 feet. This would be the case if the density were the same at all altitudes. But, on account of diminished pressure, the air is more and more expanded at greater heights. Besides this, the height due to a given fall of the mercury varies for many reasons, such as the temperature of the air, the temperature of the mercury, the elevation of the stations, and their latitude. Hence, the measurement of heights by the barometer is somewhat troublesome, and not always to be relied on. Formulæ and tables for this purpose are to be found in practical works on physics.

247. The Gauge of the Air-Pump.—The Torricellian tube is employed in different ways as a gauge for the air-pump, to indicate the degree of exhaustion. In Fig. 166 the gauge *G* is a tube about 33 inches long, both ends of which are open, the lower immersed in a cup of mercury, and the upper communicating with the interior of the receiver. As the exhaustion proceeds, the pressure is diminished within the tube, and the external air raises the mercury in it. A perfect vacuum would be indicated by a height of mercury equal to that of the barometer at the time.

Another kind of gauge is a barometer already filled, the basin of which is open to the receiver. As the tension of air in the receiver is diminished, the column descends, and would stand at the same level in both tube and basin, if the vacuum were perfect.

A modified form of the last, called the *siphon gauge*, is the best for measuring the rarity of the air in the receiver when the vacuum is nearly perfect. Its construction is shown by Fig. 165. The top of the column, *A*, is only 5 or 6 inches above the level of *B* in the other branch of the recurved tube. As the air is withdrawn from the open end *C*, the tension at length becomes too feeble to sustain the column; it then begins to descend, and the mercury in the two branches approaches a common level.

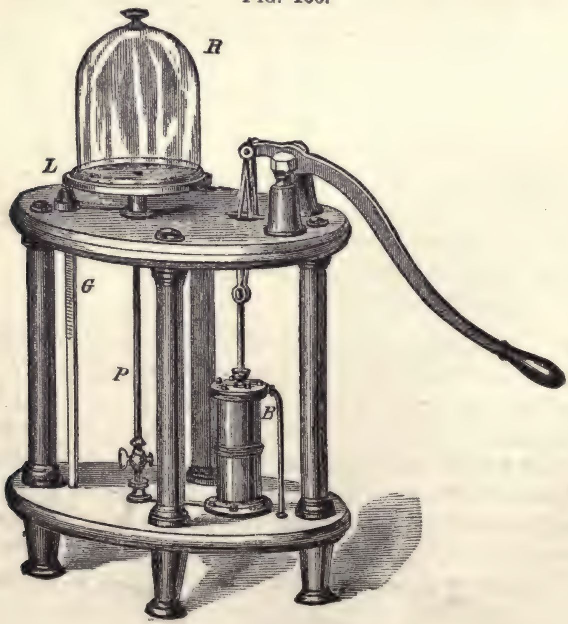
Fig. 165.

CHAPTER II.

INSTRUMENTS WHOSE OPERATION DEPENDS ON THE PROPERTIES OF AIR.

248. The Air-Pump.—This is an instrument by which nearly all the air can be removed from a vessel or receiver. It has a variety of forms, one of which is shown in Fig. 166. In the

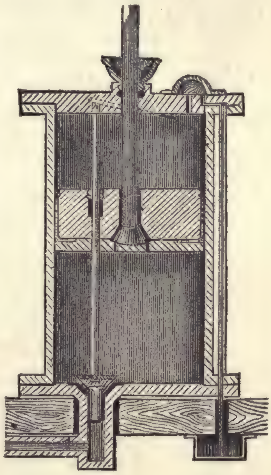
Fig. 166.



barrel B an air-tight piston is alternately raised and depressed by the lever, the piston-rod being kept vertical by means of a guide. The pipe P connects the bottom of the barrel with the brass plate L, on which rests the receiver R. The surface of the plate and the edge of the receiver are both ground to a plane. G is the gauge which indicates the degree of exhaustion. There are three valves, the first at the bottom of the barrel, the second in the piston, and the third at the top of the barrel. These all open upward, allowing the air to pass out, but preventing its return.

249. Operation.—When the piston is depressed, the air below it, by its increased tension, presses down the first valve, and opens the second, and escapes into the upper part of the barrel. When the piston is raised, the air above it cannot return, but is pressed through the third valve into the open air; while the air in the receiver and pipe, by its tension, opens the first valve, and diffuses itself equally through the receiver and barrel. Another descent and ascent only repeat the same process; and thus, by a succession of strokes, the air is nearly all removed.

FIG. 167.



The exhaustion can be made more complete if the first and second valves are opened, by the action of the piston and rod, rather than by the tension of the air. This method is illustrated by Fig. 167, a section of the barrel and piston. The first and second valves, as shown in the figure, are conical or *puppet* valves, fitting into conical sockets. The first has a long stem attached, which passes through the piston air-tight, and is pulled up by it a little way, till it is arrested by striking the top of the barrel. The second valve is a conical frustum on the end of the piston-rod. When the rod is raised, it shuts the valve before moving the piston; when it begins to descend, it opens the valve again before giving motion to the piston. The first valve is shut by a lever, which the piston strikes at the moment of its reaching the top. The oil which is likely to be pressed through the third valve is drained off by the pipe (on the right in both figures) into a cup below the pump.

250. Rate of Exhaustion.—The quantity removed, by successive strokes, and also the quantity remaining in the receiver, diminishes in the same geometrical ratio. For, of the air occupy-

ing the barrel and receiver, a barrel-full is removed at each stroke, and a receiver-full is left. If, for example, the receiver is *three* times as large as the barrel, the air occupies *four* parts before the descent of the piston; and by the first stroke *one-fourth* is removed, and *three-fourths* are left. By the next stroke, three-fourths as much will be removed as before ($\frac{1}{4}$ of $\frac{3}{4}$, instead of $\frac{1}{4}$ of the whole), and so on continually. The quantity left obviously diminishes also in the rame ratio of three-fourths. In general, if b expresses the capacity of the barrel, and r that of the receiver and connecting-pipe, the ratio of each descending series is $\frac{r}{b+r}$. With a given barrel, the rate of exhaustion is obviously more rapid as the receiver is smaller. If the two were equal, ten strokes would rarefy the air more than a thousand times. For $(\frac{1}{2})^{10} = \frac{1}{1024}$.

As a term of this series can never reach zero, a complete exhaustion can never be effected by the air-pump; but in the best condition of a well-made pump, it is not easy to discover by the gauge that the vacuum is not perfect.

Fig. 168.

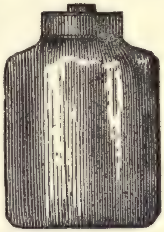
251. Sprengel's Pump.—This apparatus is too slow in its action for ordinary lecture illustration, but gives a much better vacuum than any piston pump. The length *a b* (Fig. 168) must be more than 30 inches, and the diameter of the tube should be quite small, about $\frac{1}{10}$ inch. Mercury from the funnel *F* falls down the tube *a b* in drops, which carry air before them from the receiver, which is connected with the exhaust branch *B* by suitable tubing.

Fig. 169.

252. The Air Condenser.—While the air-pump shows the tendency of air to *dilate* indefinitely, as the compressing force is removed, another useful instrument, the *condenser*, exhibits the indefinite compressibility of air. Like the pump, it consists of a barrel and piston, but its valves, one in the piston and one at the bottom of the barrel, open downward. Fig. 169 shows the exterior of the instrument. If it be screwed upon the top of a strong receiver (Fig. 170), with a stop-cock connecting them, air may be forced in, and then secured by shutting the stop-cock.

When the piston is depressed, its own valve is shut by the increased tension of the air beneath it, and the lower one opened by the same force. When the piston is raised, the lower valve is kept shut by the condensed air in the receiver, and that of the piston is opened by the weight of the outer air, which thus gets admission below the piston.

Fig. 170.

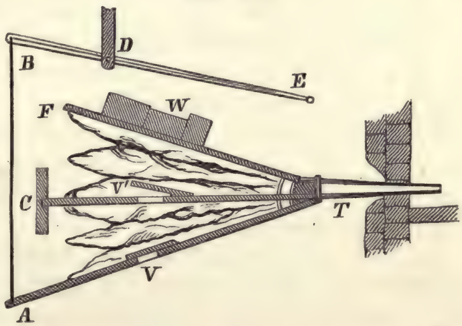


The quantity of air in the receiver increases at each stroke in an arithmetical ratio, because the same quantity, a barrel-full of common air, is added every time the piston is depressed. A small Mariotte's tube is attached to the receiver, to show how many atmospheres have been admitted.

253. Experiments with the Air Condenser.—If the receiver be partly filled with water, and a pipe from the stop-cock extend into it, then when the condenser has been used and removed, and the stop-cock opened, a jet of water will be thrown to a height corresponding to the tension of the inclosed air. A gas-bag being placed in the condenser, then filled and shut, will become flaccid when the air around it is compressed. A thin glass bottle, sealed, will be crushed by the same force. By these and other experiments may be shown the effects of increased tension.

254. The Bellows.—The simple or *hand-bellows* consists of two boards or lids hinged together, and having a flexible leather round the edges, and a tapering tube through which the air is driven out. In the lower board there is a hole with a valve lying on it, which can open inward. On separating the lids, the air by its pressure instantly lifts the valve and fills the space between them ; but when they are pressed together, the valve shuts, and the air is compelled to escape through the pipe. The stream is intermittent, passing out only when pressure is applied.

Fig. 171.

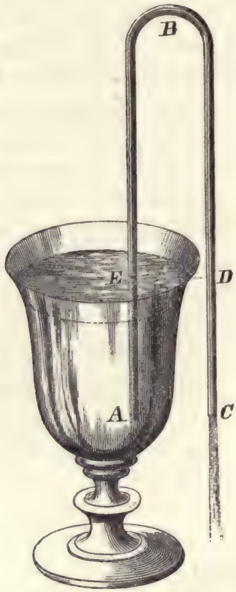


The *compound bellows,* used for forges where a constant stream is needed, are made with two compartments. The partition, C T (Fig. 171) is fixed, and has in it a valve V' opening upward. The lower lid has also a valve V opening upward, and the upper one is loaded with weights.

The pipe T is connected with the upper compartment. As the lower lid is raised by the rod A B, which is worked by the lever E B, the air in the lower part is crowded through V' into the upper part, whence it is by the weights pressed through the pipe T in a constant stream. When the lower lid falls, the air enters the lower compartment by the valve V.

255. The Siphon.—If a bent tube A B C (Fig. 172) be filled, and one end immersed in a vessel of water, the liquid will be discharged through the tube so long as the outer end is lower than the level in the vessel. Such a tube is called a *siphon*, and is much used for removing a liquid from the top of a reservoir without disturbing the lower part. The height of the bend B above the fluid level must be less than 34 feet for water, and less than 30 inches for mercury. The reasons for the motion of the water are, that the atmosphere is able to sustain a column higher than E B, and that C B is longer than E B. The two pressures on the highest cross-section B of the tube are unequal.

FIG. 172.



For the pressure at B towards the right is equal to the atmospheric pressure, which call a, minus the weight of the column E B, which call b; or $P = a - b$. The pressure towards the left at B is equal to a minus the weight of the column C B, greater than E B, and this weight we may call $b + c$; or $P' = a - (b + c)$. The difference of these pressures will determine the motion at B.

$$P - P' = (a - b) - \{a - (b + c)\} = c,$$

and this excess of pressure c causes a flow in the direction E B C.

The excess of pressure at any other point of the siphon might have been discussed in the same general way. In no case will water flow if the short arm exceeds 34 feet in length, and practically it must be less than this.

If the tube is small, it may be filled by suction, after the end A is immersed. If it is large, it may be inverted and filled, and then secured by stop-cocks, till the end is beneath the water.

256. Siphon Fountain.—In order that the flow may be maintained, it is not necessary that the tube should contain noth-

ing but liquid. Air may collect in large quantities at the highest point and still not wholly stop the action.

Into a flask F (Fig. 173) fit an air-tight cork, through which pass two tubes, one b entering several inches into the flask and terminating in a fine jet a, and the other $d\ c$ ending at the cork. Through the tube $d\ c$ pour water till the flask is filled to the jet a, when inverted as in the figure. Place the end of the tube $a\ b$ in a beaker of water H, and let the end of a rubber tube lead from d to a pail upon the floor. The water in the flask will flow out through the tube $c\ d$, and when the tension of the air in F has been sufficiently lowered, the pressure of the atmosphere upon the water in the beaker H will force it up the tube $b\ a$ and out through the jet. The action will continue, as in any other form of siphon, so long as water is supplied to the short arm.

FIG. 173.

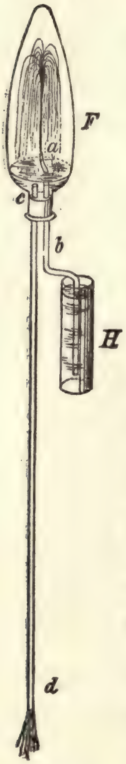
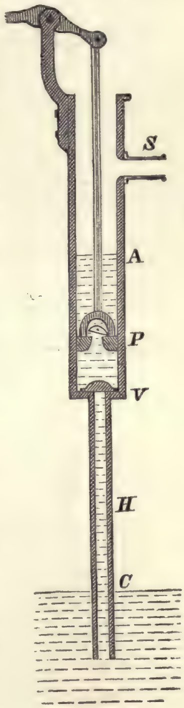


FIG. 174.



257. The Suction Pump.—The section (Fig. 174) exhibits the construction of the common suction pump. By means of a lever, the piston P is moved up and down in the tube $A\ V$. In the piston is a valve opening upward, and at the top of the pipe $V\ C$ is another valve, shown at V, also opening upward. The latter valve must be at a less height than 34 feet above the water C, the practical limit being about 29 feet, depending somewhat upon the weight of the valves. When the piston P is raised, its valve is kept shut by the pressure of the atmosphere above. The air below the piston in the barrel $A\ V$ is rarefied and presses less and less upon the valve V until at last its tension, together with the weight of the valve, is less than the tension of the air in the pipe $V\ C$ and the valve opens, the air passing through from below. Now the tension of the air in $V\ C$ being less than that of the atmosphere, a column of water will be forced up the pipe to a height such that the tension of the air in the pipe together with the weight of the column of water shall equal the pressure of the external air.

When the upward motion of P ceases, the valve V closes by its own weight. When P descends, on the return stroke, the air between it and V is compressed till its tension is greater than that of the atmosphere and the weight of the valve combined, when the valve in P is raised and the compressed air escapes. The piston being raised again, the water rises still higher, till at length it passes through the valve, and the piston dips into it; after this the water above P is lifted to the discharge spout S, while that below P is forced to follow the piston in its upward motion by the pressure of the atmosphere, as before.

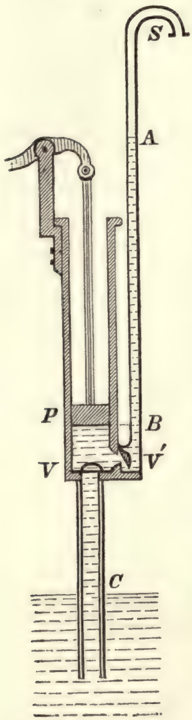
258. Calculation of the Force.—For simplicity the weight of piston, rod, and valve, and the resistances of friction will be neglected. Call the atmospheric pressure 15 lbs. per square inch. Suppose the water to have been raised to the point H, and call the downward pressure of the column $H\ C$, m lbs. per square inch, and the tension of the rarefied air in the pipe r lbs. per square inch. Call the area of the piston Q square inches. Now the tension of the inclosed air plus the weight of the water column equals the atmospheric pressure, or $r + m = 15$ lbs., whence $r = 15 - m$.

The total pressure upon the top of the piston is $Q \times 15$ lbs. Upon the lower side of the piston the pressure is $Q \times r$ lbs. or $Q\ (15 - m)$; the difference of these is to be overcome by the lifting force, $F = Q \times 15$ lbs. $- Q\ (15 - m)$ lbs. $= Q \times m$ lbs., that is to say, the force required is equal to the weight of a column of water whose cross-section equals the area of the piston and whose height is $H\ C$.

Suppose the water to be above the piston, at A. Call the pressure downward of the column $A\ P$, m' lbs. per square inch, and the pressure downward of the column $P\ C$, n' lbs. per square inch. The pressure upon the upper side of P is $Q \times (15 + m')$ lbs., and the pressure upon the lower side is $Q \times (15 - n')$ lbs. according to the principle of transmitted pressure (Art. 173). The force required is the difference of these two pressures; whence $F = Q \times (15 + m')$ lbs. $- Q \times (15 - n')$ lbs. $= Q \times (m' + n')$ lbs., or is equal to the weight of a column whose cross-section equals the area of P and height $A\ C$, as in the previous case. From this investigation we learn that only the area of the piston and height of water in the pump above the surface of the cistern need be considered, the diameter of the pipe $V\ C$ not entering the calculation. If $d =$ the diameter of the piston in decimals of a foot, then $\frac{1}{4}\pi d^2 =$ its area; $\frac{1}{4}\pi d^2 h =$ the cubic feet of water, h being the height in feet; and $\frac{1}{4}\pi d^2 h \times 62.5 =$ the *pounds* to be lifted. The atmosphere has acted simply to transmit force, and has not lessened the work in any way.

259. The Forcing Pump.—The piston of the forcing pump (Fig. 175) is solid, and the upper valve V' opens into the side pipe $V' S$. In the ascent of the piston, the water is raised as in the suction pump; but in its descent, a force must be applied to press the water which is above V into the side pipe through V'.

FIG. 175.

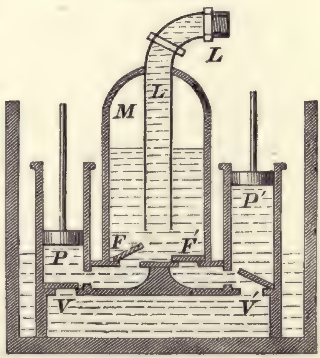


Let the height $P\ C = h$, and the height $B\ A$, above the level of the piston $= h'$. The force expended at any instant during the upward motion of the piston is $\frac{1}{4}\,\pi\, d^2\, h \times 62.5$ lbs., and as h is greatest at the end of the upward stroke this force is increasing. On the downward stroke the force is $\frac{1}{4}\,\pi\, d^2\, h' \times 62.5$ lbs., since the column $P\ V$ balances the column $B\ V'$, leaving only $B\ A = h'$ to act; as $B\ A$ is greatest at the end of the down stroke this force is also increasing.

The piston is only one of many contrivances for producing rarefaction of air in a pump-tube; but since it is the most simple and most easily kept in repair, the piston-pump is generally preferred to any other.

260. The Fire-Engine.—This machine generally consists of one or more forcing pumps, with a regulating air-vessel, though the arrangement of parts is exceedingly varied. Fig. 176 will illustrate the principles of its construction. As the piston, P, ascends, the water is raised through the valve, V, by atmospheric pressure. As P descends, the water is driven through F into the air-vessel, M, whence by the condensed air it is forced out without interruption through the hose-pipe, L. The piston P' operates in the same way by alternate movements. The piston-rods are attached to a lever (not represented), to which the strength of several men can be applied at once by means of hand-bars called *brakes*.

FIG. 176.



The air-vessel may be attached to any kind of pump, whenever it is desired to render the stream constant.

261. Hero's Fountain.—The condensation in the air-vessel, from which water is discharged, may be produced by the weight of a column of water. An illustration is seen in Hero's fountain, Fig. 177. A vertical column of water from the vessel, *A*, presses into the air-vessel, *B*, and condenses the air more or less, according to the height of *A B*. From the top of this vessel an air-tube conveys the force of the compressed air to a second air-vessel, *C*, which is nearly full of water, and has a jet-pipe rising from it. Since the tension of air in *C* is equal to that in *B*, a jet will be raised which, if unobstructed, would be equal in height to the compressing column, *A B*.

This plan has been employed to raise water from a mine in Hungary, and hence called "the Hungarian machine."

The principle of its application for this purpose may be understood from the annexed diagram.

FIG. 177.

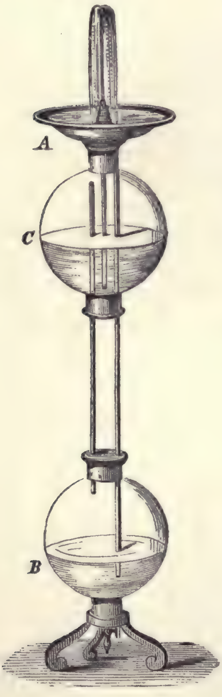
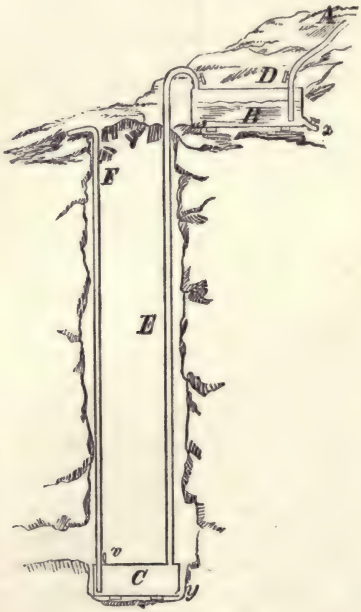


FIG. 178.

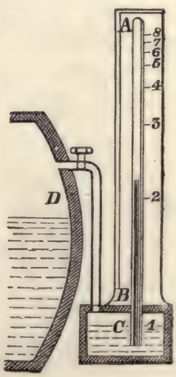


Let *A* represent a reservoir, or water supply, situated on high ground at an elevation above the mouth of the shaft greater than the depth of shaft to be drained. From this reservoir a pipe *D* (Fig. 178) passes to the *bottom* of a large and strong air chamber

B. From the *top* of the air chamber a pipe *E* passes to the *top* of a much smaller chamber *C*, at the bottom of the shaft, from the *bottom* of which passes the discharge pipe *F*, having a valve at *v*. Suppose the necessary valves to be supplied. Let all pipes and both chambers be filled with air only. Open the valve *y*, which will allow water from the mine to flow into *C*, driving out the air through *E* into *B* and out through the waste pipe *x*, which must also be open. Now close *y* and *x* and open *D*, which will permit water to flow from *A* into *B*, compressing the air in *B*, which pressure will be communicated through the air-pipe *E* to the surface of the water in *C*, driving it out through *F*. When *B* is full, or nearly full, of water, close *D*, open *x* and *y*, and thus allow water to flow into *C* and out of *B*. When *C* is full and *B* is empty, repeat the action as at first. For a shaft 100 feet deep, the air chamber *B* should be at least four times the capacity of *C*. If the height of the reservoir *A* above the mouth of the shaft be less than the depth of the shaft to be drained, the water must be raised by successive lifts.

262. Manometers.—These are instruments for measuring the tension of gases or vapors. The open manometer or "open mercurial gauge," as applied to the steam boiler, consists simply of a thick glass tube, standing vertical, both ends open, the lower end dipping into mercury contained in a closed cistern ; a pipe connects the space above the mercury in the cistern with the steam space in the boiler. When the tension of the steam is equal to one atmosphere the pressure upon the mercury in the cistern will be balanced by the pressure of the air, transmitted through the open upper end of the tube. As the steam pressure increases, the mercury will rise in the tube, at a pressure of two atmospheres standing at about 30 inches, at three atmospheres at 60 inches, and so on. The pressure of steam is always given as so many pounds above one atmosphere ; a boiler carrying 30 lbs. of steam, really has 45 lbs. internal pressure, 15 lbs. of which is counterbalanced by the pressure of the external air.

FIG. 179.



For high pressure a very long open mercurial gauge would be required ; in such cases the closed manometer, or closed mercurial gauge may be used. This differs from the former in having the glass tube *A B* closed at the top, as represented in Fig. 179. In this instrument the theoretical graduation is determined by Mariotte's law, in the following manner: Having closed

communication with the boiler D, and opened communication with the atmosphere, both above the mercury in the cistern and in the tube, the level of the mercury will be the same in both, and the tension of the air in the tube will be one atmosphere. The tube being supposed of uniform bore throughout, the *volume* of the compressed air will be proportioned to its height; the *tension* will be equal to that of the steam diminished by the weight of the column of mercury. Call the length of the tube $C\,A$ measured from the level of the mercury in the cistern l; the total pressure in lbs. p, and the height in inches of the column of mercury $C\,2$ by h. The tension of the air t in the space $A\,2 = l - h$ is given by the equation

$$t = p - \frac{h}{30} \times 15 = p - \frac{h}{2}.$$

From Mariotte's Law we have

$$15 : p - \frac{h}{2} :: l - h : l.$$

Reducing,

$$h^2 - (2\,p + l)\,h = (30 - 2\,p)\,l$$

$$\text{whence } h = \frac{2\,p + l}{2} \pm \sqrt{(30 - 2\,p)\,l + \left(\frac{2\,p + l}{2}\right)^2}$$

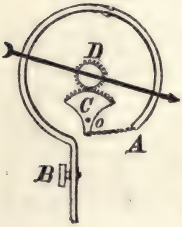
Suppose the tube to be 40 inches long, and let it be required to determine the graduation for pressure of 45 lbs. In this case we have to determine h from the values $l = 40$, $p = 60$, which substituted above give

$$h = \frac{120 + 40}{2} - \sqrt{(30 - 120)\,40 + \left(\frac{120 + 40}{2}\right)^2}.$$

Whence $h = 27.1$ inches.

The lower sign of the radical in the value of h is used, as the upper would give an impossible result. As the uniformity of the bore of the tube can not be assured, graduation by actual trial is the only accurate method. Variations caused by changes of temperature of the inclosed air must be corrected by tables for the purpose. As the figures crowd together near the top of the tube, as shown in the diagram, it has been proposed to substitute a tapering, conical tube, for the cylindrical tube, giving it such proportions as to practically correct these inequalities.

Fig. 180.

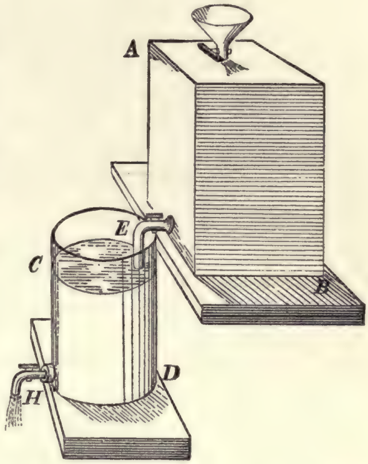


A metallic gauge, called from the inventor, a Bourdon Gauge, or some modification of it, is in very common use. It consists of a flattened tube, bent as in Fig. 180, the closed end A being connected with a toothed sector C. When steam is admitted through the stop-cock at B.

the curved tube *tends* to straighten as the pressure rises, and the motion of the end A in the direction of the arrow turns the sector C about its axis o, and by the teeth gives motion to the pinion D, which carries the index. These gauges are graduated by comparison with a standard mercurial gauge.

263. Apparatus for Preserving a Constant Level.—Let $A\ B$ (Fig. 181) be a reservoir which supplies a liquid to the vessel $C\ D$; and suppose it is desired to preserve the level at the point C in the vessel, while the liquid is discharged from it irregularly or at intervals. So long as the mouth of the pipe E is submerged in the liquid in $C\ D$, no air can enter the reservoir $A\ B$, and hence no liquid can flow from it; but when the liquid is drawn from $C\ D$ so that the level C falls, air will bubble up through the pipe E, displacing liquid in $A\ B$ till the end of the pipe E is again closed; this action will be repeated as often as the level in $C\ D$ falls below C. The pipe E should be of greater cross-section than the pipe H, or else there must be a great head of water in $A\ B$, so that E may supply liquid faster than H can discharge it.

FIG. 181.



CHAPTER III.

THE ATMOSPHERE—ITS HEIGHT, AND MOTIONS.

264. Virtual Height of the Atmosphere.—When two fluid columns are in equilibrium with each other, their heights are inversely as their specific gravities (Art. 194). The specific gravity of mercury is 10464 times that of the air at the ocean level. Therefore, if the air had the same density in all parts, its height would be found by the proportion,

$$1 : 10464 :: 2.5 : 26160 \text{ feet},$$

which is almost five miles. Hence, the quantity of the entire atmosphere of the earth is pretty correctly conceived of when we imagine it having the density of that which surrounds us, and reaching to the height of five miles.

265. Decrease of Density.—But the atmosphere is very far from being throughout of uniform density. The great cause of inequality is the decreasing weight of superincumbent air at increasing altitudes. The law of diminution of density, arising from this cause, is the following:

The densities of the air decrease in a geometrical as the altitudes increase in an arithmetical ratio. For, let us suppose the air to be divided into horizontal strata of equal thickness, and so thin that the density of each may be considered as uniform throughout. Let a be the weight of the whole column from the top to the earth, b the weight of the whole column above the lowest stratum, c that of the whole column above the second, &c. Then the weight of the lowest stratum is $a - b$, and the weight of the second is $b - c$, &c. Now the *densities* of these strata, and therefore their *weights* (since they are of equal thickness), are as the compressing forces; or,

$$a - b : b - c :: b : c;$$
$$\therefore a\,c - b\,c = b^2 - b\,c;\ \therefore a\,c = b^2;$$
$$\therefore a : b :: b : c;$$

in the same way, $b : c :: c : d$;

that is, the *weights* of the entire columns, from the successive strata to the top of the atmosphere, form a geometrical series; therefore, the *densities* of the successive strata, varying as the compressing forces, also form a geometrical series. If, therefore, at a certain distance from the earth, the air is twice as rare as at the surface of the earth, at twice that distance it will be four times as rare, at three times that distance eight times as rare, &c.

By barometric observations at different altitudes, it is found that at the height of three and a half miles above the earth the air is one-half as dense as it is at the surface. Hence, making an arithmetical series, with $3\frac{1}{2}$ for the common difference, to denote heights, and a geometrical series, with the ratio of $\frac{1}{2}$, to denote densities, we have the following:

Heights, $3\frac{1}{2}$, 7, $10\frac{1}{2}$, 14, $17\frac{1}{2}$, 21, $24\frac{1}{2}$, 28, $31\frac{1}{2}$, 35.
Densities, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$, $\frac{1}{64}$, $\frac{1}{128}$, $\frac{1}{256}$, $\frac{1}{512}$, $\frac{1}{1024}$.

According to this law, the air, at the height of 35 miles, is at least a thousand times less dense than at the surface of the earth. It has, therefore, a thousand times less weight resting upon it; in other words, only one-thousandth part of the air exists above that height.

266. Actual Height of the Atmosphere.—The foregoing law, founded on that of Mariotte, cannot, however, be applicable except to moderate distances. If it were strictly true, the atmosphere would be unlimited. But that is impossible on a revolving

body, since the centrifugal force must at some distance or other equal the force of gravity, and thus set a limit to the atmosphere; and that limit in the case of the earth is more than 20,000 miles high. The actual height of the atmosphere is doubtless far below this; for there can be none above the point where the repellency of the particles is less than their weight; and the repellency diminishes just as fast as the density, while the weight diminishes very slowly. The highest portions concerned in reflecting the sunlight are about 45 miles above the earth. But there is reason to believe that the air extends much above that height, probably 100 or 200 miles from the earth.

267. The Motions of the Air.—The air is never at rest. When in motion, it is called *wind.* The equilibrium of the atmosphere is disturbed by the unequal heat on different parts of the earth. The air over the hotter portions becomes lighter, and is therefore pressed upward by the cooler and heavier air of the less heated regions. And the motions thus caused are modified as to direction and velocity by the rotation of the earth on its axis.

268. The Trade Winds.—The most extensive and regular system of winds on the earth is known by the name of the *trade winds,* so called on account of their great advantage to commerce. They are confined to a belt about equal in width to the torrid zone, but whose limits are four or five degrees further north than the tropics.

In the northern half of this trade-wind zone the wind blows continually from the northeast, and in the southern half from the southeast. As these currents approach each other, they gradually become more nearly parallel to the equator, while between them there is a narrow belt of calms, irregular winds, and abundant rains.

The oblique directions of the trade winds are the combined effects of the heat of the torrid zone and the rotation of the earth. The cold air of the northern hemisphere tends to flow directly south, and crowd up the hot air over the equator. In like manner, the cold air of the southern hemisphere tends to flow directly northward. So that if the earth were at rest, there would be *north* winds on the north side of the equator, and *south* winds on the south side. But the earth revolves on its axis from west to east, and the air, as it moves from a higher latitude to a lower, has only so much eastward motion as the parallel from which it came. Therefore, since it really has a less motion from the west than those regions over which it arrives, it has relatively a motion *from the east.* This motion from the east, compounded with the motion from the north on the north side of the equator, and with that

from the south on the south side, constitutes the northeast and southeast tradewinds.

The limits of this system move a few degrees to the north during the northern summer, and to the south during the northern winter, but very much less than might be expected from the changes in the sun's declination.

In certain localities within the tropics the wind, owing to peculiar configurations of coast and elevations of the interior, changes its direction periodically, blowing six months from one point, and six months from a point nearly opposite. The *monsoons* of southern India are the most remarkable example.

269. The Return Currents.—The air which is pressed upward over the torrid zone must necessarily flow away northward and southward towards the higher latitudes, to restore the equilibrium. Hence, there are south winds in the upper air on the north side of the equator, and north winds on the south side. But these upper currents are also oblique to the meridians, because, having the easterly motion of the equator, they move faster than the parallels over which they successively arrive, so that a motion from the west is combined with the others, causing southwest winds in the northern hemisphere, and northwest in the southern. These motions of the upper air are discovered by observations made on high mountains, and in balloons, and by noticing the highest strata of clouds. It is to be borne in mind that although the atmosphere is more than 100 miles high, yet the lower half does not extend beyond three and a half miles above the earth (Art. 265).

270. Circulation Beyond the Trade Winds.—The upper part of the air which flows away from the equator cannot wholly retain its altitude, because of the diminishing space on the successive parallels. About latitude 30°, it is so much accumulated that it causes a sensible increase of pressure (Art. 243), and begins to descend to the earth. It is probable that some of the descending air still retains its oblique motion towards higher latitudes (for the prevailing winds of the northern temperate zone are from the southwest, and of the southern temperate zone from the northwest), while a part joins with the lower air which is moving towards the equator. Only so much of the rising equatorial mass can flow back to the polar regions as is needed to supply the comparatively small area within them. On account of the successive descent of the air returning from the equator, there is much less distinctness and regularity in the general circulation outside of the torrid zone than within it. Besides this, various local causes, such as mountain ranges, sea-coasts, and ocean cur-

rents, clear and cloudy skies, &c., mingle their effects with the more general circulation, and modify it in every possible way.

271. Land and Sea Breezes.—These are limited circulations over adjoining portions of land and water, the wind blowing from the water to the land in the day time, and in the contrary direction by night. When the sun begins to shine each day, it heats the land more rapidly than the water. Hence the air on the land becomes warmer and lighter than that on the water, and the surface current sets toward the land. By night the flow is reversed, because the land cools most rapidly, and the air above it becomes heavier than that over the water. These effects are more striking and more regular in tropical countries, but are common in nearly all latitudes.

272. A Current Through a Medium.—There are some phenomena relating to currents moving through a fluid, either of the same or a different kind, which belong alike to hydraulics and pneumatics; a brief account of these is presented here.

If a stream is driven through a medium, it carries along the adjoining particles by *friction* or *adhesion*. The experiment of Venturi illustrates this kind of action, as it takes place between the particles of water. A reservoir filled with water has in it an inclined plane of gentle ascent, whose summit just reaches the edge of the reservoir. A stream of water is driven up this plane with force sufficient to carry it over the top; but in doing so, it takes out continually some part of the water of the reservoir, and will in time empty it to the level of the lowest part of the stream. A stream of air through air produces the same effect, as may be shown by the flame of a lamp near the stream always bending toward it. In like manner, water through air carries air with it; when a stream of water is poured into a vessel of water, air is carried down in bubbles; and cataracts carry down much air, which as it rises forms a mass of foam on the surface. The strong wind from behind a high waterfall is owing to the condensation of air brought down by the back side of the sheet.

273. Ventilators.—If the stream passes across the end of an open tube, the air within the tube will be taken along with the stream and thus a partial vacuum formed, and a current established. It is thus that the wind across the top of a chimney increases the draught within. To render this effect more uniformly successful, by preventing the wind from striking the interior edge of the flue, appendages, called *ventilators*, are attached to the chimney top. A simple one, which is generally effectual, consists of a conical frustum surrounding the flue as in

Fig. 182, so that the wind, on striking the oblique surface, is thrown over the top in a curve, which is convex upward. The same mechanical contrivance is much used for the ventilation of public halls and the holds of ships. A horizontal cover may be supported by rods, at the height of a few inches, to prevent the rain from entering.

FIG. 182.

274. A Stream Meeting a Surface.—Though the moving fluid may be elastic, yet, when it meets a surface, it tends to *follow* it, rather than to rebound from it. This effect is partly due to adhesion, and partly to the resistance of the medium in which the stream moves. It will not only follow a plane or concave surface, but even one which is convex, provided the velocity of the current is not too great, or the curvature too rapid. A stream of air, blown from a pipe upon a plane surface, will extinguish the flame of a lamp held in the direction of the surface beyond its edge, while, if the lamp be held elsewhere near the stream, the flame will point toward the stream, according to Art. 272. Hence, snow is blown away from the windward side of a tight fence, and from around trees.

275. Diminution of Pressure on a Surface.—When a stream is thus moving along a surface, the fluid pressure on that surface is slightly diminished. This is proved by many experiments. If a curved vane be suspended on a pivot, and a stream of air be directed tangentially along the surface, it will move toward the stream, and may be made to revolve rapidly by repeating the blast at each half revolution. What is frequently called the *pneumatic paradox* is a phenomenon of the same kind. A stream of air is blown through the centre of a disk, against another light disk, which, instead of being blown off, is forcibly held near to it by the means. The pressure is diminished by all the radial streams along the surface contiguous to the other disk, and the full pressure on the outside preponderates. Another form of the experiment is to blow a stream of air through the bottom of a hemispherical cup, in which a light sphere is lying loosely. The sphere cannot be blown out, but, on the contrary, is held in, as may be seen by inverting the cup, while the blast continues. It appears to be for a reason of the same sort that a ball or a ring is sustained by a jet of water. It lies not on the *top*, but on the *side* of the jet, which diminishes the pressure on that side of the ball, so that the air on the outside keeps it in contact. The tangential force of the jet

causes the body to revolve with rapidity. A ball can be sustained a few inches high by a stream of air.

276. Vortices where the Surface Ends.—As a current reaches the termination of the surface along which it was flowing, a *vortex* or whirl is likely to occur in the surrounding medium behind the edge of the surface. Vortices are formed on water, whose flow is obstructed by rocks; and often when the obstructing body is at a distance below the surface, the whirl which is established there is communicated to the top, so that the vortex is seen, while its cause is out of sight. There is a depression at the centre, caused by the centrifugal force; and if the rotation is rapid, a spiral tube is formed, in which the air descends to great depths. These are called *whirlpools*. In a similar manner whirls are produced in the air, when it pours off from a surface. The eddying leaves on the leeward side of a building in a windy day often indicate such a movement, though it may have no permanency, the vortex being repeatedly broken up and reproduced.

277. Vortices by Currents Meeting.—But vortices are also formed by counteracting currents in an open medium. When an aperture is made in the middle of the bottom of a vessel, as the water runs toward it, the filaments encounter each other, and usually, though not invariably, they establish a rotary motion, and form a whirlpool. Vortices are a frequent phenomenon of the atmosphere, sometimes only a few feet in diameter, in other instances some rods or even miles in width. The smaller ones, occurring over land, are called *whirlwinds;* over water, *water-spouts*. They probably originate in currents which do not exactly oppose each other, but act as a *couple* of forces, tending to produce rotation (Art. 56).

The burning of a forest sometimes occasions whirlwinds, which are borne away by the wind, and maintain their rotation for miles. As the pressure in the centre is diminished by the centrifugal force, substances heavier than air, as leaves and spray, are likely to be driven up in the axis, and floating substances, as cloud, will for the same reason descend. The rising spray and the descending cloud frequently mark the progress of a vortex in the air, as it moves over a lake or the ocean. Such a phenomenon is called a water-spout.

PART IV.

ACOUSTICS.

CHAPTER I.

NATURE AND PROPAGATION OF SOUND.

278. Sound. — Vibrations. — The impression which the mind receives through the organ of hearing is called *sound.* But the same word is constantly used to signify that *progressive vibratory movement* in a medium by which the impression is produced, as when we speak of the velocity of sound.

This is one of the several modes of motion mentioned in Art. 4. The vibrations constituting sound are comparatively slow, and are often perceived by sight and by feeling as well as by hearing. For these reasons, the true nature of sound is investigated with far greater ease than that of light, electricity, &c. It is not difficult to discover that *vibrations* in the medium about us are essential to hearing; and these vibrations are always traceable to the body in which the sound originates. A body becomes a source of sound by producing an impulse or a series of impulses on the surrounding medium, and thus throwing the medium itself into motion. A single sudden impulse causes a *noise,* with very little continuance; an irregular and rapid succession of impulses a *crash,* or *roar,* or *continued noise* of some kind; but if the impulses are rapid and perfectly equidistant, the effect is a *musical sound.* In most cases of the last kind the impulses are vibrations of the body itself; and whatever affects these vibrations is found to affect the sound emanating from it; and if they are destroyed, the sound ceases.

If we rub a moistened finger along the edge of a tumbler nearly full of water, or draw a bow across the strings of a viol, we can procure sounds which remain undiminished in intensity as long as the operation by which they are excited is continued. In both cases the vibrations are visible; those of the tumbler are plainly seen as crispations on the water to which they are commu-

nicated ; the string appears as a broad shadowy surface. If a wire or light piece of metal rests against a bell or glass receiver, when ringing, it will be made to rattle. If sand be strewed on a horizontal plate while a bow is drawn across its edge, the sand will be agitated, and dance over the surface, till it finds certain places where vibrations do not exist. Near an organ-pipe the tremor of the air is perceptible, and pipes of the largest size jar the seats and walls of an edifice. Every species of sound may be traced to impulses or vibrations in the sounding body.

279. Sonorous Bodies.—Two qualities in a body are necessary, in order that it may be sonorous. It must have a form favorable for vibratory movements, and sufficient strength of elasticity.

The favorable *forms* are in general rods and plates, rather than very compact masses, like spheres and cubes; because the particles of the former are more free to receive lateral movements than those of the latter, which are constrained on every side. But even a thin lamina may have a form which allows too little freedom of motion, such as a spherical shell, in which the parts mutually support each other. If the shell be divided, the hemispheres are bell-shaped and very sonorous.

The *elasticity* of some materials is too imperfect for continued vibration ; thus lead, in whatever form, has no sonorous quality. In other cases, where the elasticity is nearly *perfect,* yet it is a *feeble* force, and hence the vibrations are slow and inaudible. Thus india-rubber is quite elastic, but its force is feeble, and occasions but little sound.

280. Air as a Medium of Sound.—There must not only be a vibrating body, as a *source* of sound, but a medium for its *communication* to the organ of hearing. The ordinary medium is air. Let a bell mounted with a hammer and mainspring, so as to continue ringing for several minutes, be placed on a thick cushion under the receiver of an air-pump. The cushion, made of several thicknesses of woolen cloth, is necessary to prevent communication through the metallic parts of the instrument. As the process of exhaustion goes on, the sound of the bell grows fainter, and at length ceases entirely. From this experiment we learn that sound cannot be propagated through a vacant space, even though it be only an inch or two in extent ; and also that air conveys sound more feebly as it is more rare. The latter is proved by the faintness of sounds on the tops of high mountains. Travelers among the Alps often observe that at great elevations a gun can be heard only a small distance. The fact that meteoric bodies are

sometimes heard when passing over at the height of 40 or 50 miles does not conflict with the above statements; for the velocity of meteors is vastly greater than any other velocities which occur within the earth's atmosphere. On the other hand, when air has more than the natural density, it conveys sound with more intensity, and therefore to a greater distance. In a diving-bell sunk to a considerable depth a whisper is painfully loud.

281. Velocity of Sound in Air.—Sound occupies an appreciable time in passing through air. This is a fact of common observation. The flash of a distant gun is seen before the report is heard. Thunder usually follows lightning after an interval of many seconds; but if the electric discharge is quite near, the lightning and thunder are almost simultaneous. If a person is hammering at a distance, the perceptions of the blows received by the eye and the ear do not generally agree with each other: or if in any case they do agree, it will be observed that the first stroke seen is inaudible, and the last one heard is invisible; for it requires just the time between two strokes for the sound of each to reach us.

A long column of infantry, marching to the music of a single band, will have a vertical wave-like motion, since each rank steps to the music, and a given beat reaches the different ranks in succeeding periods of time.

Many careful experiments were made in the eighteenth century to determine the velocity of sound; but as the temperature was not recorded, they have but little value. During the present century, the velocity has been determined by several series of observations in different countries, and all reduced for temperature to the freezing-point. The agreement between them is very close, and the mean of all is 1090 feet per second at 32° F.

282. Velocity as Affected by the Condition of the Air and the Quality of the Sound.—

Temperature affects the velocity of sound; the latter is increased about one foot (1.11 ft.) for each degree Fahrenheit of rise in the temperature. Therefore, in most New England climates, the velocity of sound varies more than 100 feet during the year on this account. Probably the celebrated experiments of Derham, in London, 1708, who made the velocity 1142 feet, were performed in the heat of summer.

Wind of course affects the velocity of sound by the addition or subtraction of its own velocity, estimated in the same direction, because it transfers the medium itself in which the sound is conveyed. This modification, however, is only slight, for

sound moves ten times faster than wind in the most violent hurricane.

But other changes in the condition of the air produce little or no effect. Neither pressure, nor moisture, nor any change of weather, alters the *velocity* of sound, though they may affect its *intensity*, and therefore the distance at which it can be heard. Falling snow and rain *obstruct* sound, but do not *retard* it.

All *kinds* of sound—the firing of a gun—the blow of a hammer—the notes of a musical instrument, or of the voice, however high or low, loud or soft, are conveyed at the same rate. That sounds of different pitch are conveyed with the same velocity was conclusively proved by Biot, in Paris, who caused several airs to be played on a flute at one end of a pipe more than 3000 feet long, and heard the same at the other end distinctly, and without the slightest displacement in the order of notes, or intervals of silence between them.

283. The Calculated Velocity.—For several years there was a large unexplained difference between the calculated velocity of sound and the actual velocity as determined by experiment. While the latter is, as already stated, 1090 feet per second at the freezing-point, calculation gave 916 feet. The difference was at length explained by La Place, who ascertained that it arises from the heat developed in the air by the compression which it undergoes.

The velocity of sound is directly proportional to the square root of the elasticity and inversely proportional to the square root of the density; but according to Mariotte's law, the elasticity varies as the density, and hence the velocity in air is independent of the density, *the temperature being constant.*

But it is a well-known fact that when air is compressed, a part of its latent heat becomes sensible, *and raises its temperature.* If the condensation is gradual, the heat is radiated or conducted off, especially if in contact with other bodies ; but the heat developed in the propagation of sound has little opportunity to escape, and, though without continuance, it augments the elasticity of the air, so as to add 174 feet to the velocity of sound in it.

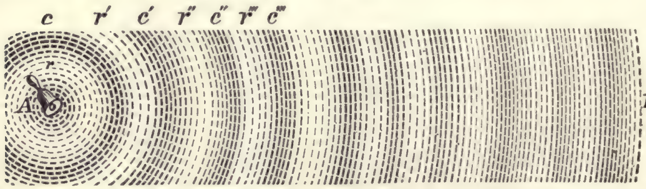
284. Diffusion of Sound.—Sound produced in the open air tends to spread equally in all directions, and will do so whenever the original impulses are alike on every side. But this is rarely the case. In firing a gun, the first impulse is given in one direction, and the sound will have more intensity, and be heard further in that direction than in others. It is ascertained by experiment, that a person speaking in the open air can be equally well heard at the distance of 100 feet directly before him, 75 feet on the right

and left, and 30 feet behind him; and therefore an audience, in order to hear to the best advantage, should be arranged within limits having these proportions. But, as will be seen hereafter, this rule is not applicable to the interior of a building.

Sound is also heard in certain directions with more intensity, and therefore to a greater distance, if an obstacle prevents its diffusion in other directions. On one side of an extended wall sound is heard further than if it spread on both sides; still further, in an angle between two walls; and to the greatest distance of all, when confined on four sides, and limited to one direction, as in a long tube. The reason in these several cases is obvious; for a given force can produce a given amount of motion; and if the motion is prevented from spreading to particles in some directions, it will reach more distant ones in those directions in which it does spread. Speaking-tubes confine the movement to a slender column of air, and therefore convey sound to great distances, and are on this account very useful in transmitting messages and orders between remote parts of manufacturing edifices and public houses.

285. Nature of Acoustic Waves.—The vibrations of a medium in the transmission of sound are of the kind called *longitudinal;* that is, the particles vibrate longitudinally with regard to the movement of the sound; whereas, in water-waves, the particle-motion is partly transverse to the wave-motion (Art. 221). If, for example, sound is passing from A to B (Fig. 183), the

FIG. 183.



particles just about A are (at the moment represented) in a state of condensation; around this condensed centre is a rarefied portion, then a condensed portion, &c., as marked by the letters r, c, r', c', r'', &c. From r to c the particles are advancing; so likewise from r' to c', and from r'' to c''. But from c to r', from c' to r'', &c., they are rebounding. The condensed wave near B has advanced from A, and others have followed it at equal intervals; and between these waves of condensation are waves of rarefaction, which in like manner spread outward from the centre A. And yet no one particle has any other motion than a small vibration back and forth in the line, near its original place of rest.

The distance from a particle which has completed one vibration to the particle next in order which is just commencing its vibration is called a wave length, and all the intermediate particles, which represent every possible phase of vibration, constitute the wave; or we may define a wave length to be the distance between any two particles in consecutive like phases. The *amplitude* is the distance through which a particle vibrates.

In water-waves we distinguish carefully between the motion of the *wave* and the motion of the *water* which forms the wave; so here, the wave-motion is totally different from the motion of the air itself. The wave, i. e. the state of condensation and subsequent rarefaction, travels swiftly forward; but the masses of air, which suffer these condensations and rarefactions, simply tremble in the line of that motion.

286. Intensity of Sound.—Since the motion is propagated in all directions alike, the entire system of waves around the point where sound originates consists of spherical strata of air alternately condensed and rarefied. As the quantity set in motion in these successive layers increases with the square of the distance, the amount of motion communicated to each particle must diminish in the same ratio. Hence, the intensity of sound varies inversely as the square of the distance.

Intensity increases with the amplitude of the vibrations, and is proportional to the square of the amplitude, or what is the same thing, it is proportional to the square of the maximum velocity of the particle vibrating. To obtain a loud tone from a piano its keys must be struck with great force, thus increasing the amplitude of vibration of the strings.

Intensity depends upon the density of the air in which the sound is produced, but not upon that of the air through which it is transmitted. A sound which could be heard in water at a distance of 23 feet would be audible in air at only 10 feet. The report of a cannon, fired upon a mountain side, heard by a person in the rare air of the summit, would have the same intensity as the same report heard in the valley below; but a gun fired in the rare air of the summit might not be heard in the valley, while a report in the valley would be heard distinctly upon the summit, the intensity depending upon the density of the medium in which the sound is produced, as stated above.

Intensity is modified by motion of the air. In still air sound is more perfectly transmitted than when air currents exist. In case of winds sound is more intense, for a given distance, in the direction of the wind than in the contrary direction.

Sound is strengthened by sympathetic vibrations of other

bodies than that which first produced the pulses. A vibrating string produces a sound scarcely audible; but when it vibrates upon a sounding box, the sympathetic vibrations of the latter are communicated to the air and a loud sound results. A vibrating tuning fork held in the hand can not be heard; the same fork caused to vibrate over the mouth of a cylinder closed at one end, and of a length equal to one-fourth of the wave length corresponding to the pitch of the fork will give a very loud sound. Savart's resonator illustrates this fact very satisfactorily.

A *ray* of sound is any one of the radii of the sphere whose centre is the source of sound. The vibratory motion is propagated along each of the rays.

287. Other Gaseous Bodies, as Media of Sound.—Let a spherical receiver, having a bell suspended in it, be exhausted of air, till the bell ceases to be heard; then fill it with any gas or vapor instead of air, and the bell will be heard again. By means of an organ-pipe blown by different gases, it can be learned with what velocity sound would move in each kind of gas experimented upon, because the pitch of a given pipe depends upon the velocity of the waves, as will be seen hereafter.

From such experiments the following velocities at temperature 32° Fahrenheit have been deduced:

Carbonic acid, 856 ft. per second.

Oxygen, 1040 ft. per second.

Carbonic oxide, 1106 ft. per second.

Hydrogen, 4163 ft. per second.

It has been stated (Art. 283) that the velocity in gases varies directly as the square root of the elasticities and inversely as the square root of the densities; hence for the same pressures the velocities should be inversely proportional to the square root of the densities. Oxygen being 16 times as heavy as hydrogen, the velocity of sound in the latter should be four times as great as in the former, which conclusion is confirmed by the facts given above. Momentary development of heat by compression produces, in all gaseous bodies, the effect of increasing the velocity of sound.

288. Liquids as Media.—Many experimenters have determined the circumstances of the propagation of sound in water. Franklin found that a person with his head under water could hear the sound of two stones struck together at a distance of more than half a mile. In 1826, Colladon made many careful experiments in the water of Lake Geneva. The results of these and other trials are principally the following:

1. Sounds produced in the air are very faintly heard by a per-

son in water, though quite near; and sounds originating under water are feebly communicated to the air above, and in positions somewhat oblique are not heard at all.

2. Sounds are conveyed by water with a velocity of 4700 feet per second, at the temperature of 47° F., which is more than four times as great as in air. The calculated and the observed velocity of sound in water agree so nearly with each other, that there appears to be no appreciable effect arising from heat developed by compression.

Calculated velocities are as follows:

Seine water, at	59° F. = 4174	ft. per	second.
" "	86° F. = 5013	"	"
" "	140° F. = 5657	"	"

Solution of calcic chloride, at 73.4° F. = 6493 ft. per second.

Sulphuric ether at 32° F. = 3801 ft. per second.

Hence sound travels with different velocities in different liquids; the velocity is greater in the liquid of greater density; the velocity is increased by increase of temperature.

3. Sounds conveyed in water to a distance, lose their sonorous quality. For example, the ringing of a bell gives a succession of short sharp strokes, like the striking together of two knife-blades. The musical quality of the sound is noticeable only within 600 or 700 feet. In air, it is well known that the contrary takes place; the blow of the bell-tongue is heard near by, but the continued musical note is all that affects the ear at a distance.

4. Acoustic *shadows* are formed; that is, sound passes the edges of solid bodies nearly in straight lines, and does not turn around them except in a very slight degree. In this respect, sound in water resembles light much more than it does sound in air.

To enable the experimenter to hear distant sounds without placing himself under water, Colladon pressed down a cylindrical tin tube, closed at the bottom, thus allowing the acoustic pulses in the water to strike perpendicularly on the sides of the tube. In this way, the faintest sounds were brought out into the air. It appears to be true of sound as of light, that it cannot pass from a denser to a rarer medium at large angles of incidence, but suffers nearly a total reflection.

289. Solids as Media.—Solid bodies of high elastic energy are the most perfect media of sound which are known. An iron rod—as, for instance, a lightning-rod—will convey a feeble sound from one extremity to the other, with much more distinctness than the air. If the ears are stopped, and one end of a long wire is held between the teeth, a slight scratch or blow on the remote

end will sound very loud. The sound in this case travels through the wire and the bones of the head to the organ of hearing. The sound of earthquakes and volcanic eruptions is transmitted to great distances through the solid earth. By laying the ear to the ground, the tramp of cavalry may be heard at a much greater distance than through the air.

290. Velocity in Solids.—Structure.—The velocity of sound in cast iron was estimated by Biot to be about 11000 feet per second—ten times greater than in air. He obtained this result by experiments on the aqueduct pipes in Paris. A blow upon one end was brought to an observer at the other end, 3000 feet distant, both by the iron and also by the air within it. The velocity in air being known, and the difference of time observed, the velocity in iron is readily calculated; thus, suppose the temperature of the air to have been 41° F., the length of the pipe 3300 feet, and the observed interval of time $2\frac{7}{10}$ seconds, then $\frac{3300}{1100} - \frac{3300}{x} = 2\frac{7}{10}$, in which 1100 is the velocity of sound in air at the given temperature, and x the velocity in the pipe: from this we get $x = 11000$.

The following table, according to Wertheim, is taken from Tyndall:

Name of Metal.	At 20° C.	At 100° C.	At 200° C.
Lead	4030	3951	——
Gold	5717	5640	5691
Silver	8553	8658	8127
Copper	11666	10802	9690
Iron	16822	17386	15483
Iron wire	16130	16728	——
Steel wire	16023	16443	——

As a rule the velocity in metals decreases with rise of temperature, but iron and silver are shown above to be exceptions to this general rule between the limits 20° C. and 100° C.

In one important particular solids differ from fluids, namely, in the fixed relations of the particles among themselves. These relations are usually different in different directions; hence, sound is likely to be transmitted more perfectly in some directions through a given solid than in others. The scratch of a pin at one end of a stick of timber seems loud to a person whose ear is at the other end. The sound is heard more perfectly in the direction of the grain than across it. In crystallized substances it is unquestion-

ably true that the vibrations of sound move with different speed and with different intensity in the line of the axis, and in a line perpendicular to it.

The velocity in woods along the fibre is from about 11000 feet to 16000 feet; across the annual rings from 4500 feet to 6000; across the fibre, in the direction of the rings from about 2500 feet to 4500 feet, all of which velocities are approximate and depend upon the wood selected.

291. Mixed Media.—In all the foregoing statements it has been supposed that the medium was homogeneous; in other words, that the material, its density, and its structure, continue the same, or nearly the same, the whole distance from the source of sound to the ear. If abrupt changes occur, even a few times, the sound is exceedingly obstructed in its progress. When the receiver is set over the bell on the pump plate, the sound in the room is very much weakened, though the glass may not be one-eighth of an inch in thickness, and is an excellent conductor of sound. The vibrations of the internal air are very imperfectly communicated to the glass, and those received by the glass pass into the air again with a diminished intensity. If a glass rod extended the whole distance from the bell to the ear, the sound would arrive in less time, and with more loudness, than if air occupied the whole extent. For a like reason, walls, buildings, or other intervening bodies, though good conductors of sound themselves, obstruct the progress of sound in the air. This explains the fact mentioned in Art. 288, that sound in air is heard faintly in water, and *vice versa.* When the texture of a substance is loose, having many alternations of material, it thereby becomes unfit for transmitting sound. It is for this reason that the bell-stand, in the experiment just referred to, is set on a cushion made of several thicknesses of loose flannel, that it may prevent the vibrations from reaching the metallic parts of the pump. The waves of sound, in attempting to make their way through such a substance, continually meet with new surfaces, and are reflected in all possible directions, by which means they are broken up into a multitude of crossing and interfering waves, and are mutually destroyed. A tumbler, nearly filled with water, will ring clearly; but if filled with an effervescing liquid, it will lose all its sonorous quality, for the same cause as before. The alternate surfaces of the liquid and gas, in the foam, confuse the waves, and deaden the sound.

CHAPTER II.

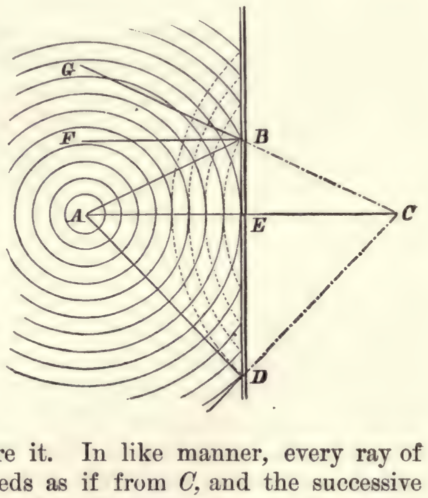
REFLECTION, REFRACTION, AND INFLECTION OF SOUND.

292. Reflection of Sound.—Sound is reflected from surfaces in accordance with the common law of reflection in the case of elastic bodies; that is,

The angle of incidence equals the angle of reflection, and the two angles are on opposite sides of the perpendicular to the reflecting surface.

Suppose sound to emanate from A (Fig. 184), and meet the plane surface $B\ D$. The particles of air in the ray $A\ B$ vibrate back and forth in that line, and those contiguous to B will, after striking the surface, rebound on the line $B\,G$, as an elastic ball would do (Art. 101), and propagate their motion along that line. The angle of incidence $A\ B\ F$ equals the angle of reflection $F\ B\ G$, and the two angles are on opposite sides of $F\,B$, which is perpendicular to the reflecting surface $B\ D$. If $G\ B$ be produced backward, it will meet the perpendicular $A\ E$ at C, as far behind $B\ D$ as A is before it. In like manner, every ray of sound after reflection proceeds as if from C, and the successive waves are situated as represented by the dotted lines in the figure. From the point E the reflection is directly back in the line $E\ A$.

Fig. 184.



293. Echoes.—When sound is so distinctly reflected from a surface that it seems to come from another source, it is called an *echo*. Broad and even surfaces, such as the walls of buildings and ledges of rock, often produce this effect. According to the law (Art. 292), a person can hear the echo of his own voice only by standing in a line which is perpendicular to the echoing surface. In order that one person may hear the echo of another's voice,

they must place themselves in lines making equal angles with the perpendicular.

The interval of time between a sound and its echo enables one to judge of the distance of the surface, since the sound must pass over it twice. Thus, if at the temperature of 74° the echo of the speaker's voice reaches him in two seconds after its utterance, the distance of the reflecting body is about 1130 feet, and in that proportion for other intervals. And he can hear a distinct echo of as many syllables as he can pronounce while sound travels twice the distance between himself and the echoing surface.

The ear can recognize about nine successive sounds in one second; two sounds separated by less than one-ninth of a second blend and produce confusion; therefore the distance from the speaker to the reflecting surface at temperature 32° F. must not be less than $\frac{1090}{9} \div 2 = 60.5$ ft. in order that an echo of a sharp sound may be heard. For articulate sounds at ordinary temperatures the distance may be about 112.5 feet.

294. Simple and Complex Echoes.—When a sound is returned by one surface, the echo is called *simple;* it is called *complex* when the reflection is from two or more surfaces at different distances, each surface giving one echo. Thus, a cannon fired in a mountainous region is heard for a long time echoed on all sides, and from various distances.

A complex echo may also be produced by two parallel walls, if the hearer and the source of sound are both situated between them. The firing of a pistol between parallel walls a few hundred feet apart has been known to return from 30 to 40 echoes before they became too faint to be heard. The rolling of thunder is in part the effect of reverberation between the earth and the clouds. This is made certain by the observed fact that the report of a cannon, which in a level country and under a clear sky is sharp and single, becomes in a cloudy day a prolonged roar, mingled with distant and repeated echoes. But the peculiar inequalities in the reverberations of thunder are doubtless due in part to the irregularly crinkled path of the electric spark. A discharge of lightning occupies so short a time, that the sound may be considered as starting from all points of its track at once. But that track is full of large and small curves, some convex and some concave to the ear, and at a great variety of distance; and all points which are at equal distances would be heard at once. Hence, the original sound comes to the hearer with great irregularity, loud at one instant and faint at another. These inequalities are prolonged and intensified by the echoes which take place between the clouds and the earth.

295. Concentrated Echoes.—The divergence of sound from a plane surface continues the same as before, that is, in spherical waves, whose centre is at the same distance behind the plane as the real source is in front. But concave surfaces in general produce a concentrating effect. A sound originating in the centre of a hollow sphere will be reflected back to the centre from every point of the surface. If it emanates from one focus of an ellipsoid, it will, after reflection, all be collected at the other focus. So, if two concave paraboloids stand facing each other, with their axes coincident, and a whisper is made at the focus of one, it will be plainly heard at the focus of the other, though inaudible at all points between. In the last case the sound is twice reflected, and passes from one reflector to the other in parallel lines. All these effects are readily proved from the principle that the angles of incidence and reflection are equal.

The speaking-trumpet and the ear-trumpet have been supposed by many writers to owe their concentrating power to multiplied reflections from the inner surface. But a part of the effect, and sometimes the whole, is doubtless due to the accumulation of force in one direction, by preventing lateral diffusion, till the intensity is greatly increased.

Concave surfaces cause all the curious effects of what are called *whispering galleries*, such as the dome of St. Paul's, in London. In many of these instances, however, there seems to be a continued series of reflections from point to point along the smooth concave wall, which all meet simultaneously (if the curves are of equal length) at the opposite point of the dome; for the whisperer places his mouth, and the hearer his ear, close to the wall, and not in a focus of the curve. The *Ear of Dionysius* was probably a curved wall of this kind in the dungeons of Syracuse. It is said that the words, and even the whispers, of the prisoners were gathered and conveyed along a hidden tube to the apartment of the tyrant. The sail of a ship when spread, and made concave by the breeze, has been known to concentrate and render audible to the sailors the sound of a bell 100 miles distant. A concave shell held to the ear concentrates such sounds as may be floating in the air, and is suggestive of the murmur of the ocean.

296. Resonance of Rooms.—If a rectangular room has smooth, hard walls, and is unfurnished, its reverberations will be loud and long-continued. Stamp on the floor, or make any other sudden noise, and its echoes passing back and forth will form a prolonged musical note, whose pitch will be lower as the apartment is larger. This is called the *resonance* of the room. Now, let furniture be placed around the walls, and the reverberations

will be weakened and less prolonged. Especially will this be the case if the articles be of the softer kinds, and have irregular surfaces. Carpets, curtains, stuffed seats, tapestry, and articles of dress have great influence in destroying the resonance of a room. The appearance of an apartment is not more changed than is its resonance by furnishing it with carpet and curtains. The blind, on entering a strange room, can, by the sound of the first step, judge with tolerable accuracy of its size and the general character of its furniture.

The reason why substances of loose texture do not reflect sound well, is essentially the same as what has been stated (Art. 291) for their not transmitting well ; they are not homogeneous—the waves are reflected in all directions by successive surfaces, interfere with each other, and are destroyed.

297. Halls for Public Speaking.—In large rooms, such as churches and lecturing halls, all echoes which can accompany the voice of the speaker syllable by syllable, are useful for increasing the volume of sound ; but all which reach the hearers sensibly later, only produce confusion. It is found by experiment that if a sound and its echo reach the ear within *one-sixteenth* of a second of each other, they seem to be one. Hence, this fraction of time is called the *limit of perceptibility.* Within that time an echo can travel about 70 feet more than the original sound, and yet appear to coincide with it. If an echoing wall, therefore, is within 35 feet of the speaker, each syllable and its echo will reach every hearer within the limit of perceptibility. The distance may, however, be increased to 40 or even 50 feet without injury, especially if the utterance is not rapid. Walls intended to aid by their echoes should be smooth, but not too solid; plaster on lath is better than plaster on brick or stone ; the first echo is louder, and the reverberations less. Drapery behind the speaker deprives him of the aid of just so much echoing surface. A lecturing hall is improved by causing the wall behind the speaker to change its direction, on the right and left of the platform, at a very obtuse angle, so as to exclude the rectangular corners from the room. The voice is in this way more reinforced by reflection, and there is less resonance arising from the parallelism of opposite walls. Paneling, and any other recesses for ornamental purposes, may exist in the reflecting walls without injury, provided they are not curved. The ceiling should not be so high that the reflection from it would be delayed beyond the limit of perceptibility. Concave surfaces, such as domes, vaults, and broad niches, should be carefully avoided, as their effect generally is to concentrate all the sounds they reflect. An equal diffusion of sound throughout the

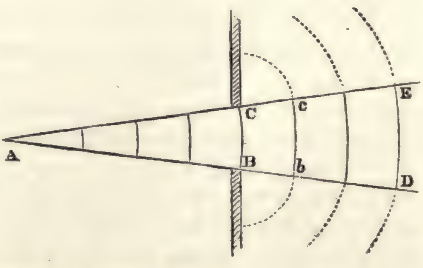
apartment, not concentration of it to particular points, is the object to be sought in the arrangement of its parts.

As to distant parts of a hall for public speaking, the more completely all echoes from them can be destroyed, the more favorable is it for distinct hearing. It is indeed true that if a hearer is within 35 feet of a wall, however remote from the speaker, he will hear a syllable, and its echo from that wall, as one sound; but to all the audience at greater distances from the same wall, the echoes will be perceptibly retarded, and fall upon subsequent syllables, thus destroying distinctness. The distant walls should, by some means, be broken up into small portions, presenting surfaces in different directions. A gallery may aid in effecting this; and the seats of the gallery and of the lower floor may rise rapidly one behind another, so that the audience will receive directly much of the sound which would otherwise go to the remote wall, and be reflected. Especially should no large and distant surfaces be *parallel* to nearer ones, since it is between parallel walls that prolonged reverberation occurs.

298. Refraction of Sound.—It has been ascertained by experiment that sound, like light, may be *refracted*, or bent out of its rectilinear course by entering a substance of different density. If a large convex lens be formed of carbonic acid gas, by inclosing it in a sphere of thin india-rubber, a feeble sound, like the ticking of a watch, produced on one side, will be concentrated to a focal point on the other. In this case, the several diverging rays of sound are refracted toward each other on entering the sphere, and still more on leaving it, so that they are converged to a focus.

299. Inflection of Sound.—If air-waves are allowed to pass through an opening in an obstructing wall, they are not entirely confined within the radii of the wave-system produced through the opening, but spread with diminished intensity in lateral directions. The particles near the edges of the opening, as *B* and *C* (Fig. 185) may be considered as sources of sound; and if they be made centres of concentric spheres, whose radii are equal to the length of the wave, *B b*, or *C c*, and its multiples, then these spherical surfaces will represent the lateral systems of waves which are diffused on every side of the direct beam,

Fig. 185.



B D, C E. But the sound is in general more feeble as the distance from *B D*, or *C E*, is greater, and in certain points is destroyed by interference. This spreading of sound in lateral directions is called the *inflection* of sound.

What is true of all sides of an opening is of course true whenever sound passes by the side of an obstacle. Instead of being limited by lines almost straight drawn from the source, as light is in the formation of a shadow, it bends round the edge, and is heard, though more feebly, behind the intervening body. It has been already noticed (Art. 288) that in water there is little or no inflection of sound.

CHAPTER III.

MUSICAL SOUNDS AND MODES OF PRODUCING THEM.

300. The Vibrations in Musical Sounds.—When the impulses of a sounding body upon the air are equidistant, and of sufficient frequency, they produce what is termed a musical sound. In most cases these impulses are the isochronous vibrations of the body itself, but not necessarily so; it is found by experiment that blows or pulses, of any species whatever, if they are more than about 15 or 20 per second, and possess the property of *isochronism*, cause a musical tone. For example, the snapping of a stick on the teeth of a metallic wheel would seem as unlikely as anything to produce a musical sound; but when the wheel is in rapid motion, the succession causes a pure musical note. Equidistant *echoes* often produce a musical sound, as when a person stamps on the floor of a rectangular room, finished, but unfurnished (Art. 296). So, on a walk by the side of a long baluster fence, a sudden sharp sound, like the blow of a hammer on a stone, brings back a tone more or less prolonged, resembling the chirp of a bird. It is occasioned by successive equidistant echoes from the balusters of the fence. A flight of steps will sometimes produce the same effect, the tone being on a lower key than that from the fence, as it should be.

301. The Pitch of Musical Sounds.—What is called the *pitch* of a musical sound, or its degree of acuteness, is owing entirely to its rate of vibration. Other qualities of sounds are due to other and often unknown circumstances; but rapidity of vibration is the only condition on which the pitch depends. In com-

paring one musical sound with another, if the number of vibrations per second is greater, the sound is more acute, and is said to be of a *higher pitch;* if the vibrations are fewer per second, the sound is graver, or of a *lower pitch.*

302. The Monochord or Sonometer.—If a string of uniform size and texture is stretched on a box of thin wood, by means of a pulley and weight, the instrument is called a *monochord,* and is useful for studying the laws of vibrations in musical sounds. The sound emitted by the vibrations of the whole length of the string is called its *fundamental* sound.

If the string be drawn aside from its straight position, and then released, one component of the force of tension urges every particle back towards its place of rest; but the string passes beyond that place, on account of the momentum acquired, and deviates as far on the other side; from which position it returns, for the same reason as before, and continues thus to vibrate till obstructions destroy its motion. By the use of a bow, the vibrations may be continued as long as the experimenter chooses.

The pitch of the fundamental sound of musical strings is found by experience to depend on three circumstances; the *length* of the string—its *weight* or quantity of matter—and its *tension.* The tone becomes more acute as we increase the tension, or diminish either the length or the weight. The operation of these several circumstances may be seen in a common violin. The pitch of any one of the strings is raised or lowered by turning the screw so as to increase or lessen its tension; or, the tension remaining the same, higher or lower notes are produced by the same string, by applying the fingers in such a manner as to shorten or lengthen the string which is vibrating; or, both the tension and the length of the string remaining the same, the pitch is altered by making the string larger or smaller, and thus increasing or diminishing its weight.

A string is said to make a *single vibration* in passing from the extreme limit on one side to the extreme limit on the other; a *double vibration* is the motion across and back again to the original position. Independently of calculation, it is easy to see that, with a given weight per inch, and a given tension, the string will vibrate *slower,* if *longer,* since there is more matter to be moved, and only the same force to move it; and for a similar reason, the length and tension being given, it will also vibrate *slower,* if *heavier.* On the other hand, if length and weight are given, it will vibrate *faster,* if the *tension* is *greater;* because a greater force will move a given quantity at a swifter rate.

303. Time of a Single Vibration.—The mathematical formula for the time of a single vibration is

$$T = l\left(\frac{w}{g\,t}\right)^{\frac{1}{2}},$$

in which T is the time, in seconds, of a vibration ; $l =$ length of the string ; $w =$ the weight of one unit of the string; $t =$ the tension, and $g =$ the force of gravity. In applying the formula l and g must be in the same unit, either both in feet or both in inches, and also w and t in the same unit, both in ounces or both in pounds.

The constant factor, g, being omitted, the variation may be expressed thus :

$$T \propto \frac{l\sqrt{w}}{\sqrt{t}};\ \text{that is,}$$

The time of a vibration varies as the length of the string multiplied by the square root of its weight per inch, and divided by the square root of its tension.

As the *distance* of the string from its quiescent position does not form an element of the algebraic expression for the time of a vibration, it follows that the time is independent of the amplitude. Hence, as in the pendulum, the vibrations of a string, fixed at both ends, are performed in equal times, whether the amplitude of the vibrations be greater or smaller. It is on this account that the pitch of a string does not alter, when left to vibrate till it stops. The excursions from side to side grow less, and therefore the sound more feeble, till it ceases ; but the *rate* of vibration, and therefore the pitch, remains the same to the last. This property of *isochronism,* independent of extent of excursion, is common to sounding bodies generally, and is owing to what may be called *the law of elasticity, that the restoring force, acting on any particle, varies directly as its distance from the place of rest.* For example, each particle of the string, if removed twice as far from its place of rest, is urged back by a force twice as great, and therefore returns in the same time.

304. The Number of Vibrations in a Given Time.—The greater is the length of one vibration, the less will be the number of vibrations in a given time ; that is, if N represents the number, $N \propto \frac{1}{T}$; but as $T \propto \frac{l\sqrt{w}}{\sqrt{t}}$, $\therefore N \propto \frac{\sqrt{t}}{l\sqrt{w}}$. If t and w are constant, $N \propto \frac{1}{l}$; if l and t are constant, $N \propto \frac{1}{\sqrt{w}}$; and if l and w are constant, $N \propto \sqrt{t}$; that is,

1. *The number of vibrations varies inversely as the length.*

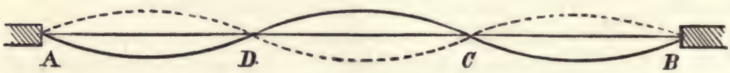
2. *The number of vibrations varies inversely as the square root of the weight of the string.*

3. *The number of vibrations varies as the square root of the tension.*

Thus, the number of vibrations in a second may be doubled, either by *halving* the length of the string or by making its weight *one-fourth as great*, or, finally, by making its tension *four times as great*.

305. Vibrations of a String in Parts.—The monochord may be made to vibrate in parts, the points of division remaining at rest; and this mode of vibration may even coexist with the one already described. Of course the sound produced by the parts will be on a higher pitch, since they are shorter, while the tension and the weight per inch remain unaltered. It is a noticeable fact that the parts are always such as will exactly measure the whole without a remainder. Hence the vibrating parts are either halves, thirds, fourths, or other aliquot portions. The sounds produced by any of these modes of vibration are called *harmonics*, for a reason which will appear hereafter. Suppose a string (Fig. 186) to

FIG. 186.



be stretched between *A* and *B*, and that it is thrown into vibration in three parts. Then while *A D* makes its excursion on one side, *D C* will move in the opposite direction, and *C B* the same as *A D*; and when one is reversed, the others are also, as shown by the dotted line. In this way *D* and *C* are kept at rest, being urged toward one side by one portion of string, and toward the opposite by the next portion. But the string may at the same time vibrate as a whole; in which case *D* and *C* will have motion to each side of their former places of rest, while relatively to them the three portions will continue their movements as before. The points *C* and *D* are called *nodes;* the parts *A D*, *D C*, and *C B*, are called *ventral segments.* By a little change in the quickness of the stroke, the bow may be made to bring from the monochord a great number of harmonic notes, each being due to the vibrations of certain aliquot parts of the string. By confining a particular point, however, at the distance of $\frac{1}{2}$, $\frac{1}{3}$, or other simple fraction of the whole from the end, the particular harmonic belonging to that mode of division may be sounded clear, and unmingled with the others.

306. Longitudinal Vibrations of Strings.—A string may

vibrate in the direction of its length, in consequence of the elastic force of its molecules. Such vibrations can be produced by rubbing the stretched string quickly with resined leather in the direction of its length.

The fundamental note thus produced is of much higher pitch than that of the same string caused to vibrate transversely, owing to the great molecular elasticity as compared with the elasticity due to tension of the string between its supports.

By experimenting as in the case of transverse vibrations it may be shown that the number of vibrations in a given time is inversely proportional to the length of the string.

By altering the tension within the limits of elasticity of the substance no change of pitch is produced, longitudinal vibrations differing in this respect from transverse.

Changing the thickness or weight, the material being the same, does not alter the pitch, another difference to be noted between transverse and longitudinal vibrations.

Two wires of different material but of the same length will give different notes. The time required for a pulse to run from one end to the other and back again is the same as the time of a complete vibration of any one molecule ; hence in a long wire the vibrations must be slower than in a short one, since it will take the pulse a longer time to travel the length of the long wire than to move the length of a short one.

Now if two wires of the same length but of different materials be used, that which transmits the sound pulse with the greatest velocity will give the highest note, since the number of vibrations per second due to this greater velocity of transmission will be greater. If two wires of different materials be so adjusted as to length as to give the same note, the ratio of their lengths is the ratio of the velocities of transmission of sound in the two substances.

307. Vibrations of a Column of Air.—When a musical sound is produced by a pipe of any kind, it is the column of inclosed air which must be regarded as the sounding body. A condensed wave is caused, by some mode of excitation, to travel back and forth in the pipe, followed by a rarefied portion ; and these waves affect the surrounding air much in the same way as do the alternate excursions of a string. That it is the air, and not the pipe itself, which is the source of sound, is proved by using pipes of various materials—the most elastic and the most inelastic—as glass, wood, paper, and lead ; if they are of the same form and size, the tone in each case has the same pitch.

In order to examine the manner in which the air-columns in

pipes perform their vibrations, it is convenient to consider them in three classes:

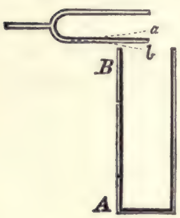
1st. Pipes which are closed at one end and open at the other.

2d. Those which are closed at both ends.

3d. Those which are open at both ends.

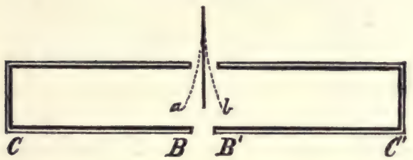
308. One End of the Pipe Closed.—Suppose the column of air in the pipe *B A* (Fig. 187) to be of such length as to respond to the motions of the fork placed above it, in which case its length would be almost exactly one-fourth the length of the sound wave produced by the fork; then while the prong moves from *a* to *b* the condensed pulse travels from *B* to *A* and back again to *B*, having been reflected at *A*, reaching *B* just as the prong begins its excursion back again to *a*, producing a rarefaction which in like manner travels to *A* and back again to *B* by the time the prong has reached *a*, ready to begin the next vibration. These successive condensations and rarefactions passing down and up the air column produce the sound or note peculiar to the pipe in use.

FIG. 187.



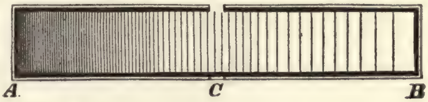
309. Both Ends of the Pipe Closed.—If two equal pipes, like that used in the last paragraph, be placed with their open ends near together as in Fig. 188, and a vibrating prong or reed communicates motion to the air columns as before, while the reed moves from *a* to *b* a condensed pulse will pass from *B′* to *C′* and back again, while at the same time a rarefied pulse will travel from *B* to *C* and back again; while the reed moves from *b* to *a* the condensation passes down *B C* and back, while the rarefaction moves down *B′ C′* and back again. Each separate pipe in this case gives the same note as though it alone were used. Now if we join the pipes at *B B′* leaving only a small orifice through which to communicate motion to the air column, the effect is in no way changed, and the note will still be the same. Thus in Fig. 189, while the condensed pulse moves from *B* to *C*, the point of rarefaction runs from *A* to *C*, where they pass each other; hence, at the middle of the pipe there is no change of density, since every degree of condensation is at that point met by an equal degree of rarefaction of the other half of the general wave. At the ex-

FIG. 188.



tremities, A and B, there is alternately a *maximum* of condensation and of rarefaction, each being reflected and returning, to meet again at C. Fig. 189 shows the air in a state of condensation at A, and of rarefaction at B. At all points between the centre and the ends there is alternate condensation and rarefaction, but in a less degree according to the distance from the ends.

FIG. 189.

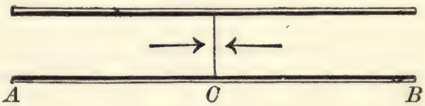


On the other hand, the *excursions* of the particles are greatest at C, and nothing at A and B, where all motion is prevented by the fixed stoppers by which the pipe is closed. Between the ends and the centre, the amplitude of vibration is greater, as the distance from the centre is less.

The pitch of such a pipe will be lower, as the pipe is longer, because the waves have a greater distance to travel between the successive reflections, and hence there will be a smaller number per second. So also, lowering the temperature lowers the pitch, since the wave then travels more slowly, and suffers fewer reflections in a second.

310. Both Ends of the Pipe Open.—When both ends of a pipe are open, it may still produce a musical tone, by having a node in the centre of it, thus forming two pipes like the one first described. When the vibration is established in such a pipe, the pulses from the ends move simultaneously toward C (Fig. 190), and again from it after reflection. Thus C is a fixed point, where the greatest condensation and rarefaction occur alternately, like A in Fig. 187. It therefore has the same pitch as $A\ C$ alone, stopped at C and open at A. If a solid partition be inserted at C, it causes no change of pitch.

FIG. 190.



Such a pipe can produce no sound, except by the formation of at least one node.

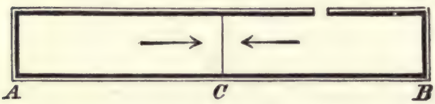
311. The First Kind of Pipe is the Elementary Form.—In comparing with each other the three kinds of pipe which have been described, it is observable that the *second* kind (stopped at both ends), and the *third* kind (open at both ends), are both double pipes of the *first* kind (open at one end, and stopped at the other). For, if two pipes of the first kind be placed with their open ends together, as we have seen, they form one of the second kind, and there is no change of pitch. Again, if

the two be placed with the closed ends in contact, they form a pipe of the third class; since the partition may remain or be removed, without affecting the mode of vibration. Hence, a pipe open at both ends, and one of the same length closed at both ends, each yields the same fundamental note as a pipe of half their length, open only at one end.

312. Vibrations of a Column of Air in Parts.—The same is true of a column of air as of a string, that it may vibrate in parts; and also that two or more modes of vibration may coexist in the same column.

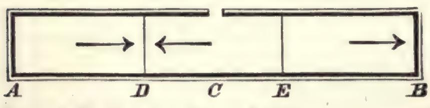
The *second* and *third* kinds of pipe can divide so that the whole and the vibrating segments have the ratios of 1 : $\frac{1}{2}$: $\frac{1}{3}$: &c.; these ratios in the closed pipe are shown in Figs. 188, 191, and 192; and in the open pipe in Figs. 190 and 193. In Fig. 191 the pipe is divided into two equal parts, in each of which the vibrations take place in the same manner as in the whole, Fig. 188. Condensations run simultaneously from A and B to the middle point C, and thence back to A and B. When C is condensed, A and B are rarefied; and when A and B are condensed, C is rarefied. Those three points have no amplitude, but the greatest changes in density. But the points midway between have the greatest amplitude, and no change of density. As the waves run over the parts in half the time that they would over the whole, the pitch is raised accordingly. In this mode of vibrating, the opening where the vibrations are excited cannot be at C, where the node is formed.

FIG. 191.



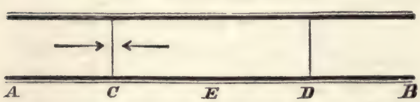
In Fig. 192 are shown *three* vibrating segments. B and D are condensed at one moment, A and E at another.

FIG. 192.



In the third kind, as already stated (Art. 310), there must be at least one node. When there are two, it is apparent by Fig. 193 that they must be one-fourth of the length from each end, in order that the three parts may vibrate in unison; for the middle part is a complete segment, like the pipe A B (Fig. 189), while the ends are half segments, like the pipe A B (Fig. 187). If there were three nodes,

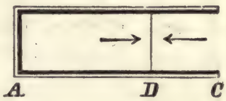
FIG. 193.



there would be two complete segments between them, and two half segments at the ends. It is evident that the lengths of the half segments, being $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{6}$, &c., are as 1, $\frac{1}{2}$, $\frac{1}{3}$, &c., of the whole pipe; therefore the rates of vibration (being inversely as the lengths) are as the numbers 1, 2, 3, &c.

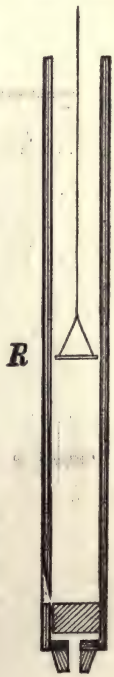
The length of the elementary form of pipe is one-fourth the length of the fundamental sound wave which it produces. In this form of pipe nodes may divide the pipe into segments having the ratios 1 : $\frac{1}{3}$: $\frac{1}{5}$, &c. The simplest division is by one node, a third of the length from the open end, as in Fig. 194. Then *C D*, a half segment, and *A D*, a complete segment, have the same rate of vibration. If there were two nodes, one must be a fifth from the open end, while the other divides the remainder into two complete segments. Therefore, in the several modes of vibration of the first kind of pipe, the half segments, being 1, $\frac{1}{3}$, $\frac{1}{5}$, &c., of the whole length, the rates of vibration in them are as the odd numbers 1, 3, 5, &c.

Fig. 194.



313. Position of Nodes Determined Experimentally.—The position of nodes in pipes may be readily shown by introducing a ring *R* (Fig. 195) over which is stretched a thin membrane. If this ring, suspended from a cord, be lowered into the open upper end of a sounding pipe of glass, or one which has one transparent face, it will make a rattling or fluttering noise, or will show that it is vibrating by the motion of grains of sand sprinkled upon it; this vibration decreases in intensity as the disc is gradually lowered, until, when the disc reaches the place of a node, the vibration ceases and the sand remains at rest. Thus the place of each of any number of nodes may be determined experimentally. Fine silica powder in a sounding pipe held horizontally will also mark the segments and nodes very beautifully.

Fig. 195.



314. Modes of Exciting Vibrations in Pipes.—There are two methods of making the air-column in a pipe to vibrate: one by a stream of air blown across an orifice in the pipe, the other by an elastic plate called a reed. A familiar example of the first is the *flute*. A stream of air from the lips is directed across the *embouchure,* so as just to strike the opposite edge; this causes a wave to move through the tube. The stream of air, like a spring,

vibrates so as to keep time with the movement of the wave to and fro, while at each pulse it renews that movement, and makes the sound continuous. For higher notes, the stream must be blown more swiftly, that by its greater elastic force, it may be able to conform to the more rapid vibration of the column. A large proportion of the pipes of an organ are made to produce musical tones essentially in the same way as the flute, and are called *mouth-pipes*.

FIG. 196.

Fig. 196 shows the construction of the mouth-pipe of an organ; *o b* is the mouth; and as the stream of air issues from the channel *i*, it starts a wave in the pipe, and then the stream itself vibrates laterally past the lip *b*, keeping time with the successive returns of the wave in the pipe. The pipe is attached to the wind-chest by the foot *P*.

The *clarinet* is an example of vibrations in an air-column by a reed. In that instrument the reed is often made of wood; when the air is blown past its edge into the tube, the reed is thrown into vibration, and by it the column of air. The strength of elasticity in the reed should be such that its vibrations will keep time with the excursions of the wave in the column. What are called the reed pipes of the organ are constructed on the same principle, but the reeds are metallic. An example is seen in Fig. 197, which represents a model of the reed pipe, made to show the vibrations through the glass walls at *E*. A chimney, *H*, is usually attached, sometimes of a form (as in the figure) to increase the loudness of the sound, and sometimes of a different form, for softening it. The air in an open tube may also be thrown into vibration by a burning jet of gas, as in Fig. 198. The note depends upon the size of the flame and the length of the tube. By varying the position of the jet in a long tube a series of nodes in the ratio of 1 : 2 : 3 : 4, &c., can be obtained.

FIG. 197.

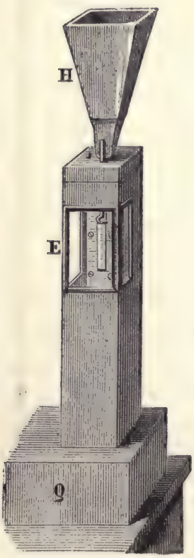


FIG. 198.

If a gauze diaphragm be inserted in a tube open at both ends, about two inches in diameter and two feet long,

at a point three or four inches from the end, and this diaphragm be heated red hot by a Bunsen burner, upon removing the burner and depressing the tube so as to cause a current of air to pass through it, a very loud note will be produced.

315. Vibrations of Rods and Laminæ.—A plate of metal called a reed is much used for musical purposes in connection with a column of air, as already stated. In parlor organs the sound is produced by the action of vibrating reeds upon air currents, just as in the case of the musical toy called harmonicon. Except in such connection, the sounds of wires and laminæ are generally too feeble to be employed in music. But their vibrations have been much studied, on account of the interesting phenomena attending them.

Such vibrations afford a convenient mode of determining the velocity of sound in solids. A rod, held firmly by a clamp in the middle, and rubbed about half way between the middle and the end by a leather pad well resined, will give a note due to longitudinal vibrations of the rod. While the rod gives out its fundamental note the *ends* vibrate freely, being neither compressed nor extended : but at the centre, held by the clamp, there is no vibration, but a maximum effect of alternate compression when the two pulses meet, and extension when they again travel toward the ends. The middle of the rod is a node. The time of a complete vibration is the time that a pulse would require to travel twice the length of the rod. If the note given is due to 512 vibrations per second, and the length of the rod be x feet, then a length of $2x$ feet is passed over 512 times in a second, and the velocity in the substance of the rod is $512 \times 2x$ feet per second.

316. Wires.—If one end of a steel wire is fastened in a vise and vibrated, while a thin blade of sunlight falls across it, the path of the illuminated point may be traced. It is not ordinarily a circular arc about the fixed point as a centre, but some irregular figure ; and frequently the point describes two systems of ellipses, the vibrations passing alternately from one system to the other several times before running down. If the structure of the wire were the same in every part across its section, and if the fastening pressed equally on every point around it, the orbit of each particle would be a series of ellipses, whose major axes are on the same line. If, moreover, there was no obstruction to the motion, and the law of elasticity could obtain perfectly, it would vibrate in the same elliptic orbit forever, the force toward the centre being directly as the distance. It is easy to cause the wire, in the experiment just described, to vibrate also in parts ; in which case each atom, while describing the elliptic orbit, will perform severa

smaller circuits, which appear as waves on the circumference of the larger figure.

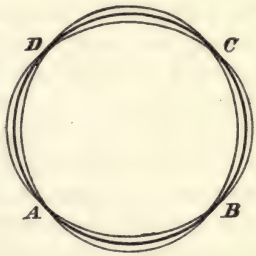
317. Chladni's Plates.—If a square plate of glass or elastic metal, of uniform thickness and density, be fastened by its centre in a horizontal position, and a bow be drawn on its edge, it will emit a pure musical tone; and by varying the action of the bow, and touching different points of the edge with the finger, a variety of sounds may be obtained from it. The plate necessarily vibrates in parts; and the lowest pitch is produced when there are two nodal lines parallel to the sides, and crossing at the centre, thus dividing the plate into four square ventral segments. The position of the nodal lines, and the forms of the segments, are beautifully exhibited by sprinkling writing-sand on the plate. The particles will dance about rapidly till they find the lines of rest, where they will presently be collected. For every new tone the sand will show a new arrangement of nodal lines; and as two or more modes of vibration may coexist in plates, as well as in strings and columns of air, the resultant nodes will also be rendered visible. Again, by fastening the plate at a different point, still other arrangements will take place, each distinguishable by the position of its nodal lines and the pitch of its musical note. The form of the plate itself may also be varied, and each form will be characterized by its own peculiar systems. Chladni, who first performed these interesting experiments, delineated and published the forms of *ninety* different systems of vibration in the square plate alone.

If a fine light powder, as lycopodium (the pollen of a species of fern), be scattered on the plate, it is affected in a very different manner from heavy sand. It will gather into rounded heaps on those portions of the segments which have the greatest amplitude of vibration; the particles which compose the heaps performing a continual circulation, down the sides of the heaps, along the plate to the centre, and up the axis. If the vibration is violent, the heaps will be thrown up from the plate in little clouds over the portions of greatest motion. The cause of this singular effect was ascertained by Faraday, who found that in an exhausted receiver the phenomenon ceased. It is due to a circulation of the air, which lies in contact with a vibrating plate. The air next to those parts which have the greatest amplitude is at each vibration thrown upward more powerfully than elsewhere, and surrounding particles press into its place, and thus a circulation is established; and a fine light powder is more controlled by these atmospheric movements than by the direct action of the plate.

318. Bells.—If a thin plate of metal takes the form of a cylinder or bell, its fundamental note is produced when each ring of the

material changes from a circle to an ellipse, and then into a second ellipse, whose axis is at right angles to that of the former, as seen in Fig. 199. It thus has four ventral segments and four nodal lines, the latter lying in the plane of the axis of the bell or cylinder. If the rings which compose the bell were all detached from one another, they would have different rates of vibration according to their diameter, and hence would produce tones of various pitch; but, being bound together by cohesion, they are compelled to keep the same time, and hence give but one fundamental tone. But a bell, especially if quite thin, may be made to emit a series of harmonic sounds by dividing up into a greater number of segments. It is obvious that the number of nodes must always be *even*, because two successive segments must move in opposite directions in one and the same instant; otherwise the point between them could not be kept at rest, and therefore would not be a node. Besides the principal tone of a church-bell, one or two subordinate sounds on a different pitch may usually be detected. A glass bell, suitably mounted for the lecture-room, will yield *ten* or *twelve* harmonics, by means of a bow drawn on its edge.

FIG. 199.



319. The Voice.—The vocal organ is complex, consisting of a cavity called the *larynx*, and a pair of membranous folds like valves, having a narrow opening between them; this opening, called the *glottis*, admits the air to the larynx from the wind-pipe below. The edges of these valves are thickened into a sort of cord, and for this reason the apparatus is called the *vocal cords*. In the act of breathing, the folds of the glottis lie relaxed and separate from each other, and the air passes freely between them, without producing vibration. But in the effort to form a vocal sound, they approach each other, and become tense, so that the current of air throws them into vibration. These vibrations are enforced by the consequent vibrations in the air of the larynx above; and thus a fullness of sound is produced, as in many musical instruments, in which a reed, and the air of a cavity, perform synchronous vibrations, and emit a much louder sound than either could do alone. If two pieces of thin india-rubber be stretched across the end of a tube, with their edges parallel, and separated by a narrow space, as represented in Fig. 200, the arrangement will give an idea of the larynx and glottis of the vocal

FIG. 200.

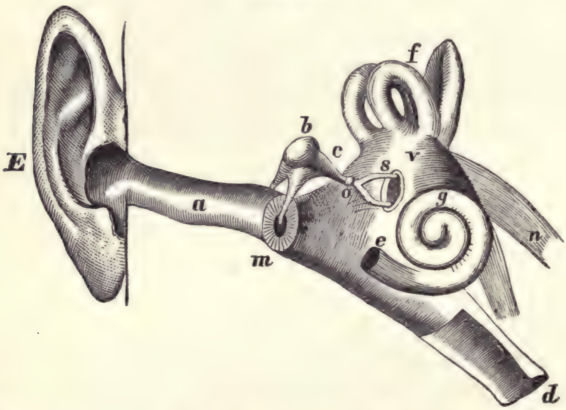
organ. If air be forced through, a sound is produced, whose pitch depends on the size of the tube and the tension of the valves.

The natural key of a person's voice depends on the length and weight of the vocal cords, and the size of the larynx. The yielding nature of all the parts, and the ability, by muscular action, to change the form and size of the cavity and the tension of the valves, give great variety to the pitch, and the power of adjusting it with precision to every shade of sound within certain limits. No instrument of human contrivance can be brought into comparison with the organ of voice. After the voice is formed by its appropriate organ, it undergoes various modifications, by means of the palate, the tongue, the teeth, the lips, and the nose, before it is uttered in the form of articulate speech.

320. The Organ of Hearing.—The principal parts of the ear are the following:

1. The *outer ear*, *E a* (Fig. 201), terminating at the membrane of the tympanum, *m*.

FIG. 201.



2. The *tympanum*, a cavity separated from the outer ear by a membrane, *m*, and containing a series of four very small bones (*ossicles*), *b*, *c*, *o* and *s*, severally called, on account of their form, the *hammer*, the *anvil*, the *ball*, and the *stirrup*. The figure represents the walls of the tympanum as mostly removed, in order to show the internal parts. This cavity is connected with the back part of the mouth by the *Eustachian tube*, *d*.

3. The *labyrinth*, consisting of the *vestibule*, *v*, the *semicircular canals*, *f*, and the *cochlea*, *g*. The latter is a spiral tube, winding two and a half times round. The parts of the labyrinth are excavated in the hardest bone of the body. The figure shows only its exterior. There are two orifices through the bone which sepa-

rates the labyrinth from the tympanum, the round orifice, *e*, passing into the cochlea, and the oval orifice, *s*, leading to the vestibule. These orifices are both closed by a thin membrane. The ossicles of the tympanum form a chain which connects the centre of the membrane, *m*, with that which closes the oval orifice. The labyrinth is filled with a liquid, in various parts of which float the fibres of the auditory nerve.

By the form of the outer ear, the waves are concentrated upon the membrane of the tympanum, thence conveyed through the chain of bones to the membrane of the labyrinth, and by that to the liquid within it, and thus to the auditory nerve, whose fibres lie in the liquid.

CHAPTER IV.

MUSICAL SCALES—THE RELATIONS OF MUSICAL SOUNDS.

321. Numerical Relations of the Notes.—To obtain the series of notes which compose the common scale of music, it is convenient to use the monochord. Calling the sound, which is given by the whole length of the string, the *fundamental*, or *key note*, of the scale, we measure off the following fractions of the whole for the successive notes, namely: $\frac{8}{9}$, $\frac{4}{5}$, $\frac{3}{4}$, $\frac{2}{3}$, $\frac{3}{5}$, $\frac{8}{15}$, $\frac{1}{2}$. If the whole, and these fractions, are made to vibrate in order, the ear will recognize the sounds as forming the series called the *gamut*, or *diatonic scale*.

Now as the number of vibrations varies inversely as the length of the string, the number of vibrations of these notes respectively, expressed in fractions of the number of vibrations of the whole string, which we will call 1, will be 1, $\frac{9}{8}$, $\frac{5}{4}$, $\frac{3}{7}$, $\frac{3}{2}$, $\frac{5}{3}$, $\frac{15}{8}$, 2.

If the whole string vibrates 128 times per second, $\frac{8}{9}$ of the string would give $\frac{9}{8}$ of 128 vibrations, or 144 vibrations; $\frac{4}{5}$ of the string would give $\frac{5}{4}$ of 128 vibrations per second, or 160 vibrations. To express the *relative* number of vibrations in the series above, reduce the fractions to a common denominator and compare their numerators, and we have

24, 27, 30, 32, 36, 40, 45, 48.

Sounds whose vibrations per second bear to each other the ratios of the series above are not arbitrarily chosen to form the scale, but they are demanded by the ear. The notes corresponding to the series are named according to their place in the series; thus a note whose vibrations are $\frac{30}{24}$ or the vibrations of the fundamental, is called the third, one whose vibrations are $\frac{36}{24}$ is

the fifth, and that whose vibrations are $\frac{48}{24}$, or twice as many as those of the fundamental, is the *eighth* or octave.

322. Relations of the Intervals.—An *interval* is the relative pitch of two sounds, and its numerical value is expressed by a fraction whose numerator is the number of vibrations per second of the higher sound, and whose denominator is the number of vibrations of the lower or graver sound, or by any fraction equal to this.

In examining the relation of each two successive numbers in the foregoing series, we find three different ratios. Thus,

27 : 24, 36 : 32, and 45 : 40, is each as 9 : 8.
30 : 27 . . . . . and 40 : 36, ... 10 : 9.
32 : 30 . . . . . and 48 : 45, ... 16 : 15.

Therefore, of the seven successive intervals, in the diatonic scale, there are three equal to $\frac{9}{8}$, two equal $\frac{10}{9}$, and two others equal to $\frac{16}{15}$. Each of the first five is called a *tone;* each of the last two is called a *semitone.*

323. Repetition of the Scale.—The eighth note of the scale so much resembles the first in sound, that it is regarded as a repetition of it, and called by the same name. Beginning, therefore, with the half string, where the former series closed, let us consider the sound of that as the fundamental, and take $\frac{8}{9}$ of it for the second, $\frac{4}{5}$ of it for the third, &c.; we then close a second series of notes on the quarter-string, whose sound is also considered a repetition of the former fundamental. Each fraction of the string used in the second scale is obviously half of the corresponding fraction of the whole string, and therefore its note an octave above the note of that. This process may be repeated indefinitely, giving the *second octave, third octave,* &c. Ten or eleven octaves comprehend all sounds appreciable by the human ear; the vibrations of the extreme notes of this entire range have the ratio of $1 : 2^{10}$, or $1 : 2^{11}$; that is, 1 : 1024, or 1 : 2048. Hence, if 16 vibrations per second produce the lowest appreciable note, the highest varies from 16,000 to 33,000. It was ascertained by Dr. Wollaston that the highest limit is different for different ears; so that when one person complains of the piercing shrillness of a sound, another maintains that there is no sound at all. The lowest limit is indefinite for a different reason; the sounds are heard by all, but some will recognize them as low musical tones, while others only perceive a rattling or fluttering noise. Few musical instruments comprehend more than six octaves, and the human voice has only from one to three, the male voice being in pitch an octave lower than the female.

324. Modes of Naming the Notes.—There is one system

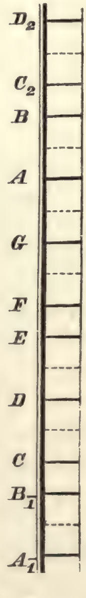
of names for the notes of the scale, which is fixed, and another which is movable. The first is by the seven letters, *A, B, C, D, E, F, G*. The notes of the second octave are expressed by the same letters, in some way distinguished from the former. The best method is to write by the side of the letter the numeral expressing that index of 2, which corresponds to the octave: as A_2, A_3, &c., in the octaves above; $A_{\bar{1}}$, $A_{\bar{2}}$, in those below.

Any note may be designated as *C*, but this letter is usually assigned to that note which is due to 256 vibrations per second, or middle *C* of the piano-forte.

The second mode of designation is by the syllables, *do, re, mi, fa, sol, la, si*. These express merely the *relations* of notes to each other, *do* always being the fundamental, *re* its second, *mi* its third, &c. In the natural scale, *do* is on the letter *C*, *re* on *D*, &c.; but by the aid of interpolated notes, the scale of syllables may be transferred, so as to begin successively with every letter of the fixed scale.

325. The Chromatic Scale.—Let the notes of the diatonic scale be represented (Fig. 202) by the horizontal lines, *C, D*, &c.; the distance from *C* to *D* being a tone, from *D* to *E* a tone, *E* to *F* a semitone, &c. It will be observed that the fundamental, *C*, is so situated that there are *two* whole tones above it, before a semitone occurs, and then *three* whole tones before the next semitone. *C* is therefore the letter to be called by the syllable *do*, in order to bring the first semitone between the 3d and 4th, and the other semitone between the 7th and 8th, as the figure represents them. Now, that we may be able to transfer the scale of relations to every part of the fixed scale (which is necessary, in order to vary the character of music, without throwing it beyond the reach of the voice), the whole tones are bisected, and two semitone intervals occupy the place of each. The dotted lines in the figure show the places of the interpolated notes, which, with the original notes of the diatonic scale, divide the whole into a series of semitones. This is called the *chromatic* scale. The interpolated note between *C* and *D* is written *C* ♯ (*C* sharp), or *D* ♭ (*D* flat), and so of the others. As the whole tones lie in groups of *twos* and *threes*, so the new notes inserted are grouped in the same way. This explains the arrangement of the *black keys* by twos and threes alternately in the key-board of the organ and piano-forte. The white keys compose the diatonic scale, the white and black keys together, the chromatic scale. It is obvious

Fig. 202.



that on the chromatic scale any one of the twelve notes which compose it may become *do,* or the fundamental note, since the required series, 2 tones, 1 semitone, 3 tones, 1 semitone, can be arranged to succeed each other, at whatever note we begin the reckoning. This change, by which the fundamental note is made to fall on different letters, is called the *transposition* of the scale.

326. Chords and Discords.—When two or more sounds, meeting the ear at once, form a combination which is agreeable, it is called a *chord;* if disagreeable, a discord. The disagreeable quality of a discord, if attended to, will be perceived to consist in a certain roughness or harshness, however smooth and pure the simple sounds which are combined. On examining the combinations, it will be found that if the vibrations of two sounds are in some very simple relations, as 1 : 2, 1 : 3, 2 : 3, 3 : 4, &c., they produce a chord; and the lower the terms of the ratio, the more perfect the chord. On the other hand, if the numbers necessary to express the relations of the sounds are large, as 8 : 9, or 15 : 16, a discord is produced. It appears that concordant sounds have *frequent* coincidences of vibrations. If, in two sounds, there is coincidence at every vibration of each, then the pitch is the same, and the combination is called *unison.* If every vibration of one coincides with every alternate vibration of the other, the ratio is 1 : 2, and the chord is the *octave,* the most perfect possible. The *fifth* is the next most perfect chord, where every second vibration of the lower meets every third of the higher, 2 : 3. The *fourth,* 3 : 4, the *major third,* 4 : 5, the *minor third,* 5 : 6, and the *sixth,* 3 : 5, are reckoned among chords; while the *second,* 8 : 9, and the *seventh,* 8 : 15, are harsh discords. What is called the *common chord* consists of the 1st, 3d, and 5th, combined, and is far more used in music than any other. *Harmony* consists of a succession of chords, or rather, of such a succession of combined sounds as is pleasing to the ear; for discords are employed in musical composition, their use being limited by special rules. Many combinations, which would be too disagreeable for the ear to dwell upon, or to finish a musical period, are yet quite necessary to produce the best effect; and without the relief which they give, perfect harmony, if long continued, would satiate.

327. Temperament.—This is a term applied to the small errors introduced into the notes, in tuning an instrument of fixed keys, in order to adapt the notes equally to the several scales. If the tones were all equal, and if semitones were truly half tones, no such adjustment of notes would be needed; they would all be exactly correct for every scale. Representing the notes in

the scale whose fundamental is *C* by the numbers in Art. 321, we have,

$$\begin{array}{ccccccccccc} C, & D, & E, & F, & G, & A, & B & C_2, & D_2, & E_2, & \&c. \\ 24, & 27, & 30, & 32, & 36, & 40, & 45, & 48, & 54, & 60, & \&c. \end{array}$$

Now suppose we wish to make *D*, instead of *C*, our key-note; then it is obvious that *E* will not be exactly correct for the second on the new scale. For the fundamental to its second is as 8 to 9; and 8 : 9 :: 27 : 30.375, instead of 30. Therefore, if *D* is the key-note, we must have a new *E*, slightly above the *E* of the original scale. So we find that *A*, represented by 40, will not serve to be the 5th in the new scale; since 2 : 3 :: 27 : 40.5, which is a little higher than *A* (= 40). After adding these and other new notes, to render the intervals all exactly right for the new key of *D*, if we proceed in the same manner, and make *E* (= 30) our key-note, and obtain its second, third, &c., exactly, we shall find some of them differing a little, both from those of the key of *C*, and also of the key of *D*. Using in this way all the twelve notes of the chromatic scale in succession for the fundamental, it appears that several different *E's*, *F's*, *G's*, &c., are required, in order to make each scale perfect. In instruments, whose sounds cannot be modified by the performer, like the organ and piano-forte, as it is considered impossible to insert all the pipes or strings necessary to render every scale perfect, such an *adjustment* is made as to distribute these errors equally among all the scales. For example, *E* is not made a perfect *third* for the key of *C*, lest it should be too *imperfect* for a *second* in the key of *D*, and for its appropriate place in other scales. It is this equal distribution of errors among the several scales which is called *temperament*. The errors, when thus distributed, are too small to be observed by most persons; whereas, if an instrument was tuned perfectly for any one scale, all others would be intolerable.

The word temperament, as above explained, has no application except to instruments of fixed keys, as the organ and piano-forte; for, where the performer can control and modify the notes as he is playing, he can make every key perfect, and then there are no errors to be distributed. The flute-player can roll the flute slightly, and thus humor the sound, so as to cause the same fingering to give a precisely correct *second* for one scale, a correct *third* for another, and so on. The player on the violin does the same, by touching the string in points slightly different. The organs of the voice, especially, can be adjusted to make the intervals perfect on *every* scale. In these cases there is no *tempering*, or dividing of errors among different scales, but a *perfect adjustment* to each scale, by which all error is avoided.

328. Harmonics.—The fact has been mentioned that a string, or a column of air, may vibrate in parts, even while vibrating as a whole. It only remains to show the musical relations of the sounds thus produced. When a string vibrates in parts, it divides into halves, thirds, fourths, or other *aliquot* parts. Now, a half-string produces an *octave* above the whole, making the most perfect chord with it. The third of a string being two-thirds of the half-string, produces the *fifth* above the octave, a very perfect chord. The quarter-string gives the *second octave ;* the fifth part of it, being $\frac{4}{5}$ of the quarter, gives the *major third* above the second octave ; and the sixth part, being $\frac{2}{3}$ of the quarter, gives the *fifth* above the second octave. Thus, all the simpler divisions, which are the ones most likely to occur, are such as produce the best chords ; and it is for this reason that the sounds are called *harmonics.* The same is true of air-columns and bells. The Æolian harp furnishes a beautiful example of the harmonics of a string. Two or more fine smooth cords are fastened upon a box, and tuned, at suitable intervals, like the strings of a violin ; and the box is placed in a narrow opening, where a current of air passes. Each string at different times, according to the intensity of the breeze, will emit a pure musical note ; and, with every change, will divide itself in a new mode, and give another pitch, while it will frequently happen that the vibrations of different divisions will coexist, and their harmonic sounds mingle with each other.

329. Overtones.—But the parts into which a sounding body divides do not always harmonize with the whole. For instance, $\frac{1}{9}$ or $\frac{1}{11}$ of a string is discordant with the fundamental. The word *harmonics* is not, therefore, applicable except to a very few of the many possible sounds which a body may produce. The word *overtone* is used to express in general any sound whatever, given by a part of a sounding body. A string may furnish 20 or 30 overtones, but only a small number of them would be harmonics.

The presence of these overtones may be determined by means of the resonator devised by Helmholtz and modified by König, which can be adjusted to respond to a great variety of notes ; if on drawing out the cylinder a tone is produced, it must be that the same tone exists in the compound sound under investigation.

330. Timbre, or Quality of Tone.—Even when the pitch of two sounding bodies is the same, the ear almost always distinguishes one sound from the other by certain qualities of tone peculiar to each. Thus, if the same letter be sounded by a flute

and the string of a piano, each note is easily distinguished from the other. Two church-bells may be upon the same key, and yet one be agreeable, and the other harsh to the ear.

As a result of his researches Helmholtz decides that the timbre is determined by the overtones which accompany the primary tones. If these overtones could be eliminated, leaving only the pure, simple tones, a note of given pitch sounded by a flute would not be distinguishable from a note of the same pitch sounded by a violin.

A long monochord can, by varying the mode of exciting the vibrations, be made to yield a great variety of sounds, while there is perceived in them all the same fundamental undertone which determines the pitch. If the string be struck at the *middle*, then no node can be formed at that point; hence, the mixed sound will contain no overtones of the $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{8}$, $\frac{1}{10}$, or other *even* aliquot parts of the string; for all such would require a node at the middle. But if struck at *one-third* of its length from the end, then the overtones, $\frac{1}{2}$, $\frac{1}{4}$, &c., may exist, but not those of $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{9}$, or any other parts whose node would fall at $\frac{1}{3}$ of the length from the end.

For reasons which are mostly unknown, some sounding bodies have their fundamental accompanied by harmonic overtones, and others by overtones which are discordant. And this is one cause of the agreeable, or unpleasant, quality of the sounds of different bodies.

331. Communication of Vibrations.—The acoustic vibrations of one body are readily communicated to others, which are near or in contact. We have already noticed that the vibrations of a reed will excite those of a column of air in a pipe. If two strings, which are adapted to vibrate alike, are fastened on the same box, and one of them is made to sound, the other will sound also more or less loudly, according to the intimacy of their connection. The vibrations are communicated partly through the air, and partly through the materials of the box. So, if a loud sound is uttered near a piano-forte, several strings will be thrown into vibration, whose notes are heard after the voice ceases. The noticeable fact in all such experiments is, that the vibrations thus communicated from one body to another cause sounds which *harmonize* with each other, and with the original sound. For the rate of vibration will either be identical, or have those simple relations which are expressed by the smallest numbers. Let a person hold a pneumatic receiver or a large tumbler before him, and utter at the mouth of it several sounds of different pitch; and he will probably find some one pitch which will be distinctly rein-

forced by the vessel. That particular note, which the receiver by its size and form is adapted to produce, will not be called forth by a sound that would be discordant with it. The melodeon, seraphine, and instruments of like character, owe their full and brilliant notes to reeds, each of which has its cavity of air adapted to vibrate in unison with it. It sometimes happens that the second body, vibrating as a whole, would not harmonize with the first, and yet will give the same note by some mode of division. Thus it is that all the various sounds of the monochord, and of the strings of the viol, are reinforced by the case of thin wood upon which they are stretched. The plates of wood divide by nodal lines into some new arrangement of ventral segments for every new sound emitted by the string. In like manner, the pitch of the tuning-fork, and all the rapid notes of a music-box, are rendered loud and full by the table, in contact with which they are brought. The extended material of the table is capable of division into a great variety of forms, and will always give a sound in unison with the instrument which touches it.

332. One System of Vibrations Controlling Another.—If two sounding bodies are nearly, but not precisely on the same key, they will sometimes, when brought into close contact, be made to harmonize perfectly. The vibrations of the more powerful will be communicated to the other, and control its movements so that the discordance, which they produce when a few inches apart, will cease, and concord will ensue. Two diapason pipes of an organ, tuned a quarter-tone or even a semitone from unison, so as to jar disagreeably upon the ear, when one inch or more asunder, will be in perfect unison, if they are in contact through their whole length. Even the slow oscillations of two watches will influence each other; if one gains on the other only a few beats in an hour, then, if they are placed side by side on the same board, they will beat precisely together.

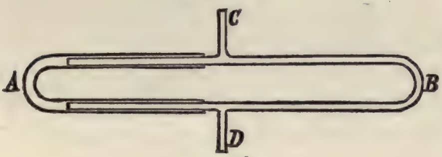
333. Crispations of Fluids.—Among the numerous acoustic experiments illustrating the communication of vibrations, none are more beautiful than those in which the vibrations of glass rods are conveyed to the surface of a fluid. Let a very shallow pan of glass or metal be attached to the middle of a thin bar of wood, three or four feet long, and resting near its ends on two fixed bridges; let water be placed in the pan, and a long glass rod standing in it, or on the wood, be vibrated longitudinally, by drawing the moistened fingers down upon it; the liquid immediately shows that the vibrations are communicated to it. The surface is covered with a regular arrangement of heaps, called *crispations*, which vary in size with the pitch of sound, which is produced

by the same vibration. If the pitch is higher, they are smaller, and may be readily varied from three or four inches in diameter to the fineness of the teeth of a file. Crispations of the same character are also formed in clusters on the water in a large tumbler or glass receiver, when the finger is drawn along its edge; every ventral segment of the glass produces a group of hillocks by the side of it on the surface of the water.

334. Interference of Waves of Sound.—Whenever two sounds are moving through the air, every particle will, at a given instant, have a motion which is the resultant of the two motions which it would have had if the sounds were separate. These motions may conspire, or they may oppose each other. The word *interference* is used in scientific language to express the *resultant effect*, whatever it may be.

If two sound waves of equal intensity and of the same length move together so that any phase of one is coincident with the like phase of the other the resultant sound has greater intensity than either of the components; if, however, any phase of one is coincident with the opposite phase of the other, entire extinction of the sound results, and we have silence. To illustrate this experimentally, take two pieces of tubing and bend them into the form shown in Fig. 203, the branch A being large enough to slide over the other, and insert a whistle at D. The sound waves travel to the ear placed at C by two different routes, starting in the same phase at the point D. If the branches A and B are of equal length, or differ by one or more whole wave lengths, the waves will meet at C in the same phase and produce a sound of greater intensity than that of either alone; but if the branch A be drawn out or pushed in till the route A differs in length from B by half a wave length, or by any odd multiple of half a wave length, opposite phases will meet at C and destroy each other, producing silence. A much more simple experiment to show the same effect is the following: The two prongs of a tuning-fork always vibrate in opposite directions, one producing a condensation in the direction in which the other produces rarefaction, thus destroying each other's effect by interference, and hence the almost total absence of sound when the fork is held free in the hand. In such case, if the sound waves from one prong be intercepted by slipping over it without contact a paper cylinder, the

FIG. 203.



sound is augmented. Hold a vibrating fork so that either the back face of a prong, or the side faces of the two prongs, are parallel to the ear, and the sound will be distinctly audible ; turn the fork about its axis 45° from either of these positions, and silence results. During one entire rotation of the fork there will be four positions of maximum intensity and four other positions of total extinction of the sound. If the fork be rotated over the mouth of a resonating jar, the effect is much more striking.

The *beats,* which are frequently heard in listening to two sounds, indicate the points of maximum condensation produced by the union of the condensed parts of both systems of waves. And the sounds are considered discordant when these beats are just so frequent as to produce a disagreeable fluttering or rattling. If too near or too far apart for this, they are regarded practically as concordant. And when the beats are too close to be perceived separately, yet the peculiar adjustment of condensations of one system with those of the other, according as *one* wave measures *two,* or *two* waves measure *three,* or *four* measure *five,* &c., is at once distinguished by the ear, and recognized as the chord of the *octave,* the *fifth,* the *third,* &c. When a sound and its octave are advancing together, there are instants in which any given particle of air is impressed with two opposite motions, and other alternate moments when both motions are in the same direction. For the waves of the highest sound are half as long as those of the lowest ; hence, while every *second* condensation of the former coincides with *every* condensation of the latter, the alternate ones of the former must be at the points of greatest rarefaction of the latter ; and this cannot occur without opposite movements of the particles.

335. Number and Length of Waves for Each Note.—Though the vibrations of any musical note are too rapid to be counted, yet the number may be ascertained in several ways.

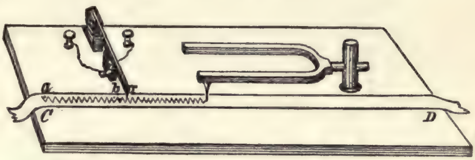
If an elastic slip of metal be clamped at one end so that the other end may rest against a toothed wheel, and the wheel be revolved with different velocities, musical notes of different pitch will be produced. If a uniform velocity be maintained for a given time, sixty seconds for instance, and the number of revolutions of the toothed wheel be read from an indicator suitably connected with the wheel, then the number of teeth upon the wheel multiplied by the number of revolutions gives the number of impulses communicated to the air in the given time ; this product, divided by the seconds in the time, sixty in the case supposed, gives the number of vibrations per second. The instrument is called Savart's Wheel.

Another method of determining the number of vibrations is by means of the *siren,* invented by De La Tour. In this instrument the pulses are produced by puffs of air, in rapid succession, caused by revolving a disc, perforated around its circumference by numerous holes which pass in front of an air jet. The calculation is similar to that given for Savart's wheel.

In the method given above it is difficult in practice to maintain a constant velocity. A graphic representation of the vibrations, devised by Duhamel, is without this objection. Without giving details of the construction, the principle of the method may be given as follows:

Support the vibrating rod—a tuning-fork for example—above a table, as in Fig. 204; to one prong of the fork attach a fine steel point or style, which shall very lightly touch the strip of paper *D C,* which has been coated with a film of lamp black; place at *I* an electric style, which is in connection with a pendulum beating seconds, which shall make a dot upon the paper at each beat of the pendulum. If the paper be moved in the direction *D C* while the fork is kept vibrating, an undulating line will be traced upon the film, and the number of undulations between any two consecutive *second's* dots, as *b a,* gives the number of vibrations of the fork per second.

FIG. 204.



In these ways it is ascertained that the numbers corresponding to the letters of the scale are the following:

$$\begin{cases} C, & D, & E, & F, & G, & A, & B, & C_2, \\ 128, & 144, & 160, & 170\frac{2}{3}, & 192, & 213\frac{1}{3}, & 240, & 256, \end{cases}$$

The highest note of the above series, C_2, 256, is the middle *C* of the piano-forte.

There is not, however, a perfect agreement of pitch in different countries, and among different classes of musicians. Accordingly, *C,* which is given above as corresponding to 128 vibrations per second, has several values, varying from 127 to 131.

To find the *length* of acoustic waves for any given pitch, we have only to divide the velocity of sound in one second by the number of vibrations which reach the ear in the same length of time. For example, at the temperature of 60° F. sound travels in air 1121 feet per second; therefore the length of waves of middle *C* on the piano $= 1121 \div 256 = 4.4$ feet nearly, a wave length being the distance through which sound moves during one vibra-

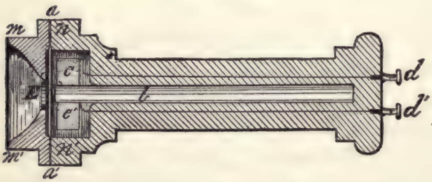
tion (Par. 285). The waves of the lowest musical note are about 70 feet long; and of the highest, less than half an inch.

336. Doppler's Principle.—Thus far the hearer and the source of sound have been supposed not to change their relative positions. When a sounding body approaches the ear, the tone perceived is higher than that due to the number of vibrations per second, since more vibrations per second reach the ear than if the body had remained at rest with respect to the hearer. Suppose the sound to be middle *C*, and the sounding body and the hearer to remain relatively stationary, then 256 vibrations per second will be communicated to the ear: if now the sounding body approach at the rate of 66 feet per second there will be perceived, in addition to the 256 vibrations, $\frac{66}{4.4} = 15$ vibrations per second, and the pitch will be that due to 271 vibrations per second instead of 256. If the sounding body recede from the hearer the opposite effect will be produced.

337. Acoustic Vibrations Visibly Projected.—The vibrations of heavy tuning-forks can be magnified and rendered distinctly visible to an audience by projecting them on a screen. The fork being constructed with a small metallic mirror attached near the end of one prong, a sunbeam reflected from the mirror will exhibit all the movements of the fork greatly enlarged on a distant wall; and if the fork is turned on its axis, the luminous projection will take the form of a waving line. And by the use of two forks, all the phenomena of interference may be rendered as distinct to the eye as they are to the ear.

338. The Telephone.—This instrument for reproduction of sound at a distance by means of electric currents is shown in section in Fig. 205 in which *a a′* is a disc or diaphragm of thin soft iron, the circumference of which is firmly clamped between the mouth guard *m m′* and the case *n n′*, upon the centre of which the sound waves from the mouth impinge, as at *E*, and communicate to it vibrations corresponding to the simple or composite sounds uttered. These vibrations of the disc cause a continual variation in the distance of the disc from the end of a bar magnet *b*. Around the end of the magnet nearest to the diaphragm *a a′* is a

FIG. 205.



helix $c\ c'$ of fine insulated copper wire, the ends of which are connected with binding posts d, d', From these posts are carried wires to another precisely similar instrument at the station with which communication is to be held. When a word is spoken into the instrument at E, the vibrations communicated to the disc $a\ a'$ cause variations in the magnetic intensity of the bar b, and these variations cause electric currents to flow in the helix $c\ c'$ and thence through the connecting wires to the helix in the instrument held to the ear of the listener, and these currents in the last named helix produce variations in the magnet of the receiving instrument, causing precisely the same vibrations in its diaphragm as were originally set up in the first. The vibrations of the diaphragm are transmitted through the air to the ear; and though no *sound* has been transmitted from one station to the other, the words spoken into one instrument are distinctly delivered by the other. The sound vibrations are the cause of electric currents, as is explained elsewhere, and these in turn finally produce sound vibrations again.

To such perfection of action have these instruments been brought, that not only can the spoken words be heard, but the peculiar characteristics of voice are so faithfully reproduced that by these the speaker may be recognized.

PART V.

OPTICS.

CHAPTER I.

MOTION AND INTENSITY OF LIGHT.

339. Definitions.—Light is supposed to consist of exceedingly minute and rapid vibrations in a medium or ether which fills space; which vibrations, on reaching the retina of the eye, cause vision, as the vibrations of the air cause hearing, when they impinge on the tympanum of the ear, and as thermal vibrations produce a sensation of warmth, when they fall on the skin; the difference between light and heat is solely a difference in wave length, waves longer than those of the extreme red, or shorter than the extreme violet, producing no effect upon the optic nerve.

Bodies, which of themselves are able to produce vibrations in the ether surrounding them, are said to *emit* light, and are called *self-luminous*, or simply *luminous;* those, which only *reflect* light, are called *non-luminous.* Most bodies are of the latter class. A *ray* of light is a line, along which light is propagated; a *beam* is made up of many parallel rays; a *pencil* is composed of rays either diverging or converging; and is not unfrequently applied to those which are parallel.

A substance, through which light is transmitted, is called a *medium;* if objects are clearly seen through the medium, it is called *transparent;* if seen faintly, *semi-transparent;* if light is discerned through a medium, but not the objects from which it comes, it is called *translucent;* substances which transmit no light are called *opaque,* though all are more or less translucent when cut in sufficiently thin laminæ.

340. Light Moves in Straight Lines.—So long as the medium continues uniform, the line of each ray is perfectly

straight. For an object cannot be seen through a bent tube; and if three discs have each a small aperture through it, a ray cannot pass through the three, except when they are exactly in a straight line. The shadow which is projected through space from an opaque body proves the same thing; for the edges of the shadow, taken in the direction of the rays, are all straight lines.

From every point of a luminous surface light emanates in all possible directions, when not prevented by the interposition of an opaque body. Thus, a candle is seen by night at the distance of one or two miles; and within that limit, no space so small as the pupil of the eye is destitute of rays from the candle. A point from which light emanates is called a *radiant*. If light from a radiant falls perpendicularly on a circular disk, the pencil is a cone; if on a square disk, it is a square pyramid, &c., the illuminated surface in each case being the base, and the radiant the vertex.

341. The Velocity of Light.—It has been ascertained by several independent methods, that light moves at the rate of about 186,300 *miles per second.*

One method is by means of the eclipses of Jupiter's satellites. The planet Jupiter is attended by four moons which revolve about it in short periods. These small bodies are observed, by the telescope, to undergo frequent eclipses by falling into the shadow which the planet casts in a direction opposite to the sun. The exact moment when the satellite passes into the shadow, or comes out of it, is calculated by astronomers. But sometimes the earth and Jupiter are on the same side, and sometimes on opposite sides of the sun; consequently, the earth is, in the former case, the whole diameter of its orbit, or about one hundred and eighty-five millions of miles nearer to Jupiter than in the latter. Now it is found by observation, that an eclipse of one of the satellites is seen about sixteen minutes and a half sooner when the earth is nearest to Jupiter, than when it is most remote from it, and consequently, the light must occupy this time in passing through the diameter of the earth's orbit, and must therefore travel at the rate of about 186,868 miles per second, according to this determination.

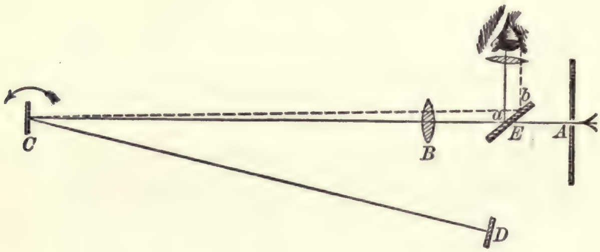
Another method of estimating the velocity of light, wholly independent of the preceding, is derived from what is called the *aberration of the fixed stars.* The apparent place of a fixed star is altered by the motion of its light being combined with the motion of the earth in its orbit. The place of a luminous object is determined by the direction in which its light meets the eye. But the direction of the impulse of light on the eye is modified by the

motion of the observer himself, and the object appears forward of its true place. The stars, for this reason, appear slightly displaced in the direction in which the earth is moving; and the velocity of the earth being known, that of light may be computed in the same manner as we determine one component, when the angles and the other component are known.

342. Determination of the Velocity of Light by Experiment.—The velocity of light has been determined also by experiment, in a manner somewhat analogous to that employed by Wheatstone for ascertaining the velocity of electricity. The method adopted by Foucault is essentially the following:

Through an aperture *A*, in a shutter (Fig. 206), a beam of light is admitted, which passing through an inclined transparent

Fig. 206.



glass mirror, *E*, and through a lens of very long focus, *B*, falls upon a mirror *C*, and is reflected to a mirror *D*; the mirror *D* again reflects the beam back to *C*, whence it is returned through the lens *B* to the glass mirror *E*, is reflected, and finally enters the eye. The mirror *C* is so mounted as to rotate with great velocity upon an axis, perpendicular to the plane of the paper in the case supposed.

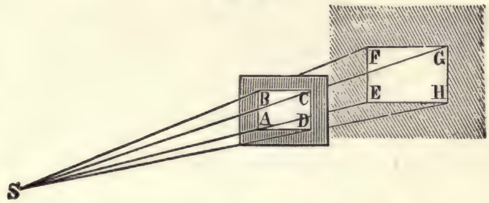
If the mirror *C* rotate slowly, in the direction of the arrow, the beam will alternately disappear and reappear at the point *a*; but if the velocity be increased to 30 or more revolutions per second the impression on the eye becomes persistent and the beam is seen without interruptions, appearing stationary at *a*. If now the speed of the mirror *C* be increased to from 300 to 600 revolutions per second, the change of position of the mirror *C*, while light is passing from it to *D* and back again, is sufficient to return the reflected beam to some point *b*, the distance from *a* depending upon the velocity of rotation. From the displacement at *b*, the angular motion of the mirror at *C*, while the beam traverses the distance from *C* to *D* and back again, can be determined; and knowing the rate of rotation, this fraction of one turn gives

the time which the light required to traverse double the distance $C\ D$, and hence its velocity. Such is an outline of the mode of experimenting, all details for securing precision having been omitted.

By this method the velocity given in Art. 341—186,300 miles per second—was determined by A. A. Michelson, U. S. N. The distance between the revolving mirror C and the mirror D was 2000 feet.

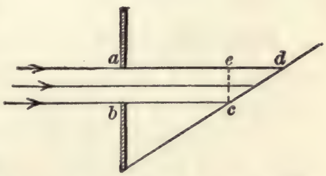
343. Loss of Intensity by Distance.—The *intensity* of light varies *inversely as the square of the distance.* In Fig. 207, suppose light to radiate from S, through the rectangle $A\ C$, and fall on $E G$, parallel to $A\ C$. As $S\ A\ E$, $S\ B\ F$, &c., are straight lines, the triangles, $S\ A\ B$, $S\ E\ F$, are similar, as also the rectangles, $A\ C$, $E\ G$; therefore, $A\ C : E\ G :: A\ B^2 : E\ F^2 :: S\ A^2 : S\ E^2$. But the same quantity of light, being diffused over $A\ C$ and $E\ G$, will be more intense, as the surface is smaller. Hence, the intensity of light at E : intensity at $A :: A\ C : E\ G :: S\ A^2 : S\ E^2$, which proves the proposition. This demonstration is applicable to every kind of emanation in straight lines from a point.

Fig. 207.



If the surface receiving the light be oblique to the axis of the beam, the intensity of illumination is proportional to the sine of the angle which the rays make with the surface. Let Fig. 208 represent a section through the axis of a beam passing through an orifice $a\ b$ and falling upon the inclined surface at $c\ d$. Now because of the obliquity, the surface $c\ d$ is greater than the section at right angles to the beam represented by $e\ c$, and hence is less intensely illuminated at any point. But surface $e\ c$: surface $c\ d$ :: line $e\ c$: line $c\ d$ :: sine $e\ d\ c$: sine $c\ e\ d$.

Fig. 208.



Calling the illumination upon any point of the right section $e\ c$ unity or u, the illumination upon any point of $c\ d$ will be $u \times \frac{e\ c}{c\ d} = u \times \text{sine}\ e\ d\ c$.

344. Brightness the Same at all Distances.—The *bright-*

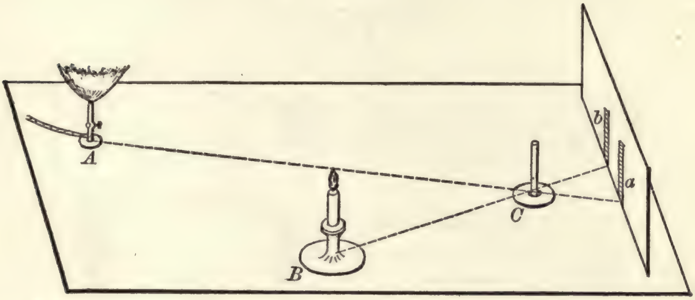
ness of an object is the quantity of light which it sheds, as compared with the apparent area from which it comes. Now the *quantity* (or intensity), as has just been shown, varies inversely as the square of the distance. The apparent *area* of a given surface also diminishes in the same ratio, as we recede from it. Hence the brightness is constant. For illustration, if we remove to *three* times the distance from a luminous body, we receive into the eye nine times less light, but the body also appears nine times smaller, so that the relation of light to apparent area remains the same.

345. Loss of Intensity by Absorption.—In a uniform medium, while the distance increases *arithmetically*, the intensity diminishes *geometrically*. Imagine the medium to be divided by parallel planes into strata of equal thickness; and suppose the first stratum to diminish the intensity by $\frac{1}{n}$ of the whole. Then the intensity of the light which reaches the second stratum is $1 - \frac{1}{n} = \frac{n-1}{n}$. But on account of the uniformity of the medium, every stratum produces the same effect, that is, it transmits to the next, $\frac{n-1}{n}$ of that which falls upon it. Therefore, $\frac{n-1}{n}$ of $\frac{n-1}{n}$, or $\frac{(n-1)^2}{n^2}$, leaves the second stratum, $\frac{(n-1)^3}{n^3}$, the third, and so on, in a geometrical series. For example, if a piece of colored glass is $1\frac{1}{4}$ inch thick, and each quarter of an inch absorbs $\frac{3}{5}$ of the light which falls upon it, then about *one-hundredth* of what enters the first surface will escape from the last. For $\left(\frac{2}{5}\right)^5 = .01$ nearly.

346. Photometers.—These are instruments designed for the measurement of the relative intensities of light. We cannot determine by the eye alone how many times more intense one light is than another, though we can judge with tolerable accuracy when two surfaces are *equally* illuminated. Photometers are, therefore, generally constructed on the plan of determining the ratio of intensities of two lights, by means of our ability to decide when they illuminate two surfaces equally. It is sufficient to mention Rumford's method. Let the two unequal lights be placed at A and B (Fig. 209) so that the shadows of an opaque rod C shall fall side by side upon a screen as at a and b. The portion of the screen upon which the shadow a falls receives light only from the candle B and none from the gas flame A; the por-

tion b is illuminated by A alone. The opaque body thus secures for each light a portion of the screen which it alone illuminates.

FIG. 209.



Now move either light towards or from the screen until the two portions a and b are equally illuminated by their respective lights, and then measure the distances from A to b, $= m$, and from B to a, $= n$.

B at distance n illuminates the screen as intensely as A at distance m.

Calling B' the illumination by B at distance n, and A' that of A at distance m, we have $A' = B'$. If B were moved back to distance m, then, according to Art. 343, its power of illuminating, or intensity, would be $\frac{n^2}{m^2} \times B' = B'' = \frac{n^2}{m^2} A'$; hence,

$$B'' : A' :: n^2 : m^2;$$

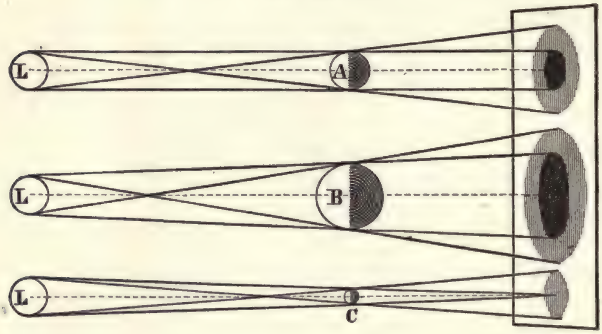
or, *the intensities vary directly as the squares of those distances from the screen at which equal illumination is obtained.*

347. Shadows.—When a luminous body shines on one which is opaque, the space beyond the latter, from which the light is excluded, is called a *shadow*. The same word, as commonly used, denotes only the *section* of a shadow made by a surface which crosses it. Shadows are either *total* or *partial*. If tangents are drawn on all the corresponding sides of the two bodies, the space inclosed by them beyond the opaque body is the total shadow; if other tangents are drawn, crossing each other between the bodies, the space between the total shadow and the latter system of tangents is the partial shadow, or *penumbra*. In case the bodies are spheres, as in Fig. 210, the total shadow will be a cylinder, or conical frustum, each of infinite length, or a complete cone, according to the relative size of the spheres. But, in every case, the penumbra and inclosed total shadow will form an increasing frustum. It is obvious that the shade of the penumbra grows

gradually deeper from the outer surface to the total shadow within it.

Every shadow cast by the sun has a penumbra bordering it, which gives to the shadow an ill-defined edge; and the more

FIG. 210.



remote the sectional shadow is from the opaque body which casts it, the broader will be the partial shadow on the edge.

If instead of a luminous body of sensible magnitude, the source of light be a point, then no penumbra will be formed. The electric arc between carbon points casts a sharply-defined shadow of a hair upon a screen placed at a great distance.

CHAPTER II.

REFLECTION OF LIGHT.

348. Radiant and Specular Reflection.—Light is said to be *reflected* when, on meeting a surface, it is turned back into the same medium. In ordinary cases of reflection, the light is diffused in all directions, and it is by means of the light thus scattered from a body that it becomes visible, when it sheds no light of its own. This is called *radiant reflection.* It is produced by unpolished surfaces. But when a surface is highly polished, a beam of light falling on it is reflected in some particular direction; and, if the eye is placed in this reflected beam, it is not the reflecting surface which is seen, but the original object, apparently in a new position. This is called *specular reflection.* It is, however, generally accompanied by some degree of radiant reflection, since the reflector itself is commonly visible in all directions. Ordinary

mirrors are not suitable for accurate experiments on reflection, because light is modified by the glass through which it passes. The *speculum* is therefore used, which is a reflector made of solid metal, and accurately ground to any required form, either *plane*, *convex*, or *concave*. The word *mirror* is, however, much used in optics for every kind of reflector.

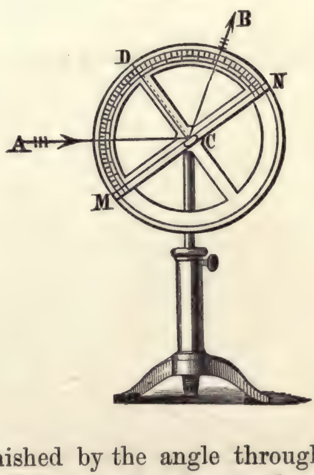
Optical experiments are usually performed on a beam of light admitted through an aperture into a darkened room; the direction of the beam being regulated by an adjustable mirror placed outside. An instrument consisting of a plane speculum moved by a clock, in such a manner that the reflected sunbeam shall remain stationary at all hours of the day, is called a *heliostat*.

349. The Law of Reflection.—When a ray of light is incident on a mirror, the angle between it and a perpendicular to the surface at the point of incidence, is called the *angle of incidence;* and the angle between the reflected ray and the same perpendicular, is called the *angle of reflection*. The law of reflection found to be universally true is the following:

The incident ray, the reflected ray, and the normal to the surface are in the same plane, and the normal bisects the angle which these rays make with each other.

This is well shown by attaching a small mirror to the centre of a graduated semicircle perpendicular to its plane. Let $M\ D\ N$ (Fig. 211) be the semicircle, graduated from D both ways to M and N, and mounted so that it can be revolved on its centre, and clamped in any position. Let the small mirror be at C, with its plane perpendicular to $C\ D$; then a ray from the heliostat, as $A\ C$, passing the edge at a particular degree, will be seen after reflection to pass the corresponding degree in the other quadrant. By revolving the semicircle, any angle of incidence may be tried, and the two rays are always found to be in the same plane with $C\ D$, and equally inclined to it.

FIG. 211.



As the mirror revolves, the reflected ray revolves twice as fast.

For $A\ C\ D$ is increased or diminished by the angle through which the mirror turns; therefore $D\ C\ B$ is also increased or diminished by the same; hence $A\ C\ B$, the angle between the two

rays, is increased or diminished by the sum of both, or twice the same angle.

It follows from the law of reflection, that a ray which falls on a mirror perpendicularly, retraces its own path after reflection. It is obvious, also, that the complements of the angles of incidence and reflection are equal, i. e. $A\,C\,M = B\,C\,N$. The law of reflection is applicable to curved as well as to plane mirrors ; the radius of curvature at any point being the perpendicular with which the incident and reflected rays make equal angles.

Radiant reflection forms no exception to the foregoing law, though the incident rays are in one and the same direction, and the reflected rays are scattered every way. For the minute cavities and prominences which constitute the roughness of the general surface are bounded by small surfaces lying at all inclinations; and each one reflecting the rays which meet it in accordance with the law, those rays are necessarily thrown off in all possible directions.

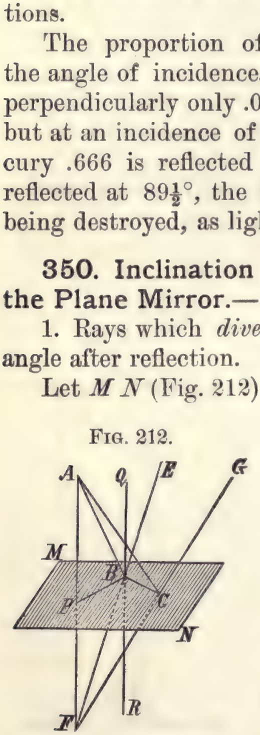
The proportion of the incident light reflected varies with the angle of incidence. When light strikes the surface of water perpendicularly only .018 is reflected, the rest entering the water ; but at an incidence of 89½° .721 is reflected. In the case of mercury .666 is reflected at perpendicular incidence, while .721 is reflected at 89½°, the non-reflected rays entering the metal and being destroyed, as light.

350. Inclination of Rays to each other not altered by the Plane Mirror.—

1. Rays which *diverge* before reflection, diverge at the same angle after reflection.

Let $M\,N$ (Fig. 212) be a plane mirror, and $A\,B$, $A\,C$, any two rays of light falling upon it from the radiant A, and reflected in the lines $B\,E$, $C\,G$. Draw the perpendicular $A\,P$, and produce it indefinitely, as to F, behind the mirror; also produce the reflected rays back of the mirror. Let $Q\,R$ be perpendicular to the mirror at the point B ; it is therefore parallel to $A\,F$, and the plane passing through $A\,F$ and $Q\,R$, is that which includes the ray $A\,B$, $B\,E$. Therefore $E\,B$, when produced back of the mirror, intersects $A\,P$ produced. Let F be the point of intersection. $B\,A\,F = A\,B\,Q$, and $A\,F\,B = E\,B\,Q$; but $A\,B\,Q = E\,B\,Q$ (Art. 349) ; $\therefore B\,A\,F = A\,F\,B$, and $A\,B = F\,B$. If P and B be joined, $P\,B$ being in

Fig. 212.



the plane $M N$ is perpendicular to $A F$, and therefore bisects it. Hence, the reflected ray meets the perpendicular $A F$ as far behind the mirror, as the incident ray does in front. In the same way it may be proved that $A C = C F$, and that $C G$, when produced back of the mirror, meets $A F$ at the same point F.

Now, since the triangles $A C B$ and $F C B$, have their sides respectively equal, their angles are equal also; hence $B A C = B F C$. Therefore any two rays diverge at the same angle after reflection as they did before reflection.

Since the reflected rays seem to emanate from F, that point is called the *apparent radiant;* A is the *real radiant.*

2. Rays which *converge* before reflection, converge at the same angle after reflection. Let $E B$, $G C$, be incident rays converging toward F, and let $B A$, $C A$, be the reflected rays. It may be proved as before, that A and F are in the same perpendicular, $A F$, and equidistant from P, and that $E F G = B A C$.

The point F, to which the incident rays were converging, is called the *virtual focus;* A is the *real focus.*

3. Rays which are *parallel* before reflection are parallel after reflection.

It has been proved in case 1, that F, the intersection of the reflected rays, is as far behind the mirror, as A, the intersection of incident rays, is before it. Now, if the incident rays are parallel, A is at an infinite distance from the mirror. Therefore F is at an infinite distance behind it, and the reflected rays are parallel.

In all cases, therefore, rays reflected by a plane mirror retain the same inclination to each other which they had before reflection.

351. Spherical Mirrors.—A *spherical mirror* is one which forms a part of the surface of a sphere, and is either convex or concave. The *axis* of such a mirror is that radius of the sphere which passes through the middle of the mirror. In the practical use of spherical mirrors, it is found that the light must strike the surface very nearly at right angles; hence, in the following statements, the mirror is supposed to be a very small part of the whole spherical surface, and the rays nearly coincident with the axis.

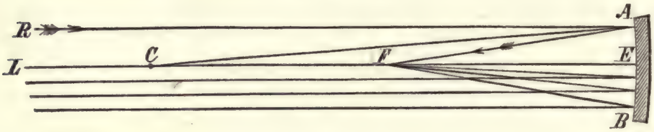
It is sufficient to trace the course of the rays on one side of the axis, since, on account of the symmetry of the mirror around the axis, the same effect is produced on every side.

352. Converging Effect of a Concave Mirror.—

1. *Parallel* rays are converged to the *middle* point between the centre and surface, which is therefore called the *focus of parallel rays* or the *principal focus.* Let $R A$, $L E$ (Fig. 213), be parallel

rays incident upon the concave mirror $A\ B$, whose centre of concavity is C. The ray $L\ E$, passing through C, and therefore

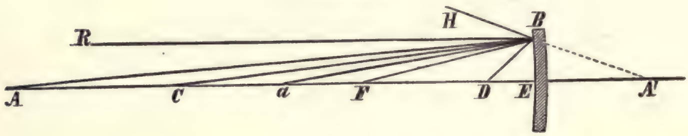
FIG. 213.



perpendicular to the mirror at E, is reflected directly back. Join $C\ A$, and make $C\ A\ F = R\ A\ C$; then $R\ A$ is reflected in the line $A\ F$, and the two reflected rays meet at F. $R\ A\ C = A\ C\ F$, $\therefore A\ C\ F = F\ A\ C$, and $A\ F = C\ F$, and as A and E are very near together, $E\ F = A\ F = F\ C$; that is, the focus of parallel rays is at the middle point between C and E.

2. *Diverging* rays, falling on a given concave mirror, are reflected *converging, parallel, or less diverging,* according to the degree of divergency in the original pencil. Let C (Fig. 214) be the centre of concavity, and F the focus of parallel rays. Then,

FIG. 214.



rays diverging from any point, A, beyond C, will be converged to some point, a, between C and F, since the angles of incidence and reflection are less than those for parallel rays. Rays diverging from C are reflected back to C; those from points between C and F, as a, are converged to points beyond C, as A; those diverging from F become parallel; and those from points between F and the mirror, as D, *diverge* after reflection, but at a less angle than before, and seem to flow from A'. To prove, in the last case, that the angle of divergence, A', after reflection, is less than the angle D, the divergence before reflection, observe that the angle A' is less than the exterior angle $H\ B\ C$, or its equal, $D\ B\ C$ (Art. 349); and $D\ B\ C$ is less than the exterior, $A'\ D\ B$; much more, then, is A' less than $A'\ D\ B$.

3. *Converging* rays are made to *converge more.* The rays $H\ B$, $A\ E$, converging to A', are reflected to D, nearer the mirror than F is. And it has been shown that the angle D is larger than A', hence the convergency is increased.

From the three foregoing cases, it appears that the *concave*

mirror always tends to produce *convergency;* since, when it does not actually produce it, it diminishes divergency.

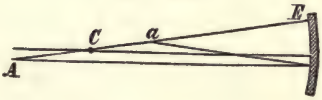
The principal focus can be determined practically by receiving the sun's rays upon the mirror, parallel to its axis, and finding the point at which a sharp image of the sun is formed. The distance of this image from the surface is one-half the radius of curvature.

353. Conjugate Foci.—When light radiates from A, it is reflected to a; when it radiates from a, it meets at A. Any two such interchangeable points are called *conjugate foci.* If the radius of the mirror and the distance of one focus from the mirror are given, the distance of its conjugate focus may be determined. Let the radius $= r$; the distance $A\,E = m$; and $a\,E = n$. As the angle $A\,B\,a$ is bisected by $B\,C$, $A\,B : a\,B :: A\,C : a\,C$; that is, since $B\,E$ is very small, $A\,E : a\,E :: A\,C : a\,C$, or, $m : n :: m - r : r - n$.

$$\therefore\ m = \frac{n\,r}{2\,n - r};\ \text{and}\ n = \frac{m\,r}{2\,m - r}.$$

If A is not on the axis of the mirror, as in Fig. 215, let a line be drawn through A and C, meeting the mirror in E; this is called a *secondary* axis, and the light radiating from A will be reflected to a on the same secondary axis, for $A\,E$ is perpendicular to the mirror, and will be reflected directly back; and if $A\,E$ and $C\,E$ are given, $a\,E$ may be found as before.

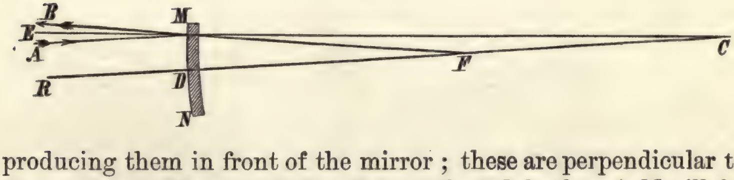
Fig. 215.



354. Diverging Effect of a Convex Mirror.—

1. *Parallel* rays are reflected diverging from the *middle point* between the centre and surface. Let C (Fig. 216) be the centre of convexity of the mirror $M\,N$, and draw the radii, $C\,M$, $C\,D$,

Fig. 216.

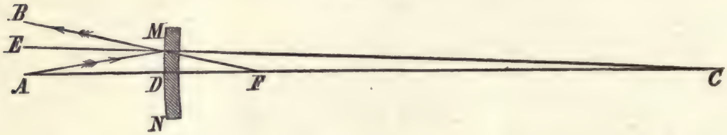


producing them in front of the mirror; these are perpendicular to the surface. The ray $R\,D$ will be reflected back; $A\,M$ will be reflected in $M\,B$, making $B\,M\,E = A\,M\,E$. Produce the reflected ray back of the mirror, and it will meet the axis in F, midway from C to D; for $F\,C\,M = A\,M\,E$, and $F\,M\,C = B\,M\,E$; therefore the triangle $F\,C\,M$ is isosceles, and $C\,F = F\,M$, and as

M is very near D, $C\,F = F\,D$. Hence the rays, after reflection, diverge as if they radiated from a point in the middle of $C\,D$, which is the apparent radiant.

2. *Diverging* rays have their divergency increased. Let $A\,D$, $A\,M$ (Fig. 217), be the diverging rays; $D\,A$, $M\,B$, the reflected

FIG. 217.



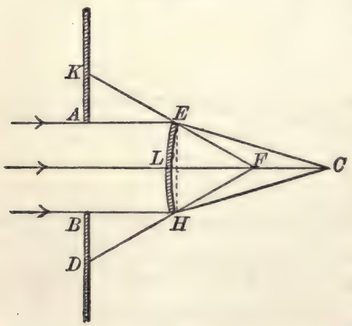
rays; these when produced meet at F, which is the apparent radiant. $M\,A\,F$ is the divergency of the incident rays, and $A\,F\,B$ of the reflected rays. Now the exterior angle, $A\,F\,B$, is greater than $C\,M\,F$, or $B\,M\,E$, or $A\,M\,E$. But $A\,M\,E$, being exterior, is greater than $M\,A\,F$; much more, then, is $A\,F\,B$ greater than $M\,A\,F$.

3. *Convergent* rays are at least rendered less convergent, and may become parallel or divergent, according to the degree of previous convergency. The two first effects are shown by Figs. 216 and 217, reversing the order of the rays. And it is easy to perceive that rays converging to C, will diverge from C after reflection; if to a point more distant than C, they will diverge afterward from a point between C and F (Fig. 216), and *vice versa.*

The general effect, therefore, of a *convex* mirror, is to produce *divergency.*

A and F (Fig. 217) are called conjugate foci, being interchangeable points; for rays from A move after reflection as though from F, and rays converging to F are by reflection converged to A. Conjugate foci, in the case of the convex mirror, are in the same axis either principal or secondary, as they are in the concave mirror, and for the same reason, viz., that every axis is perpendicular to the surface.

FIG. 218.



Their relative positions may be determined by the formula, easily deduced, as in Art. 353,

$$m = \frac{n\,r}{2\,n + r}.$$

To determine the radius of curvature experimentally: Through a circular opening in a screen whose diameter is greater than $E\,H$ (Fig. 218), receive the sun's rays upon the mirror, parallel to the axis, and move the screen so

that the diameter $K D$ of the illuminated circle is twice the chord $E H$ of the mirror; then measure

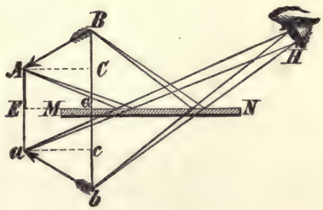
$$D H = F H = F L = \tfrac{1}{2} \text{ Rad.}$$

To render this method more accurate cover all of the mirror, except a small central circle, with some opaque covering, and use only the exposed portion as above.

355. Images by Reflection.—An optical image consists of a collection of focal points, from which light either really or apparently radiates. When rays are converged to a focus they do not stop, but cross, and diverge again, as if originally emanating from the focal point. A collection of such points, arranged in order, constitutes a *real image.* When rays are reflected diverging, they proceed *as though* they emanated from a point behind the mirror. A collection of such imaginary radiants forms an *apparent* or *virtual image.* The images formed by *plane* and *convex* mirrors are always apparent; those formed by *concave* mirrors may be of either kind.

356. Images by a Plane Mirror.—When an object is before a plane mirror, its image is at the *same distance behind* it, of the *same magnitude,* and *equally inclined* to it. Let $M N$ (Fig. 219) be a plane mirror, and $A B$ an object before it, and let the position of the object be such that the reflected rays may enter the eye placed at H. From A and B let fall upon the plane of the mirror the perpendiculars $A E$, $B G$, and produce them, making $E a = A E$, and $G b = B G$. Now, since the rays from A will, after reflection, radiate as if from a (Art. 350), and those from B, as if from b, and the same of all other points, therefore the image and object are equally distant from the mirror. $A C$, $a c$, parallel to the mirror, are equal; as $B G = b G$, and $A E = a E$, therefore, by subtraction, $B C = b c$; also the right angles at C and c are equal. Therefore $A B = a b$, and $B A C = b a c$; that is, the object and image are of equal size, and equally inclined to the mirror.

Fig. 219.



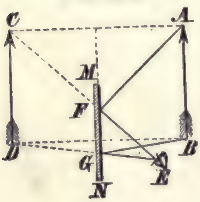
It appears from the demonstration, that the object and its image are comprehended between the same perpendiculars to the plane of the mirror; and this image will appear in the same position whatever may be the position of the eye.

The object and image obviously have to each other twice the inclination that each has to the mirror. Hence, in a mirror inclined 45° to the horizon, a horizontal surface appears vertical, and one which is vertical appears horizontal.

357. Symmetry of Object and Image.—All the three dimensions of the object and image are respectively *equal,* as shown above, but one of them is *inverted* in position, namely, that dimension which is perpendicular to the mirror. Hence, a person and his image face in opposite directions; and trees seen in a lake have their tops downward. Those dimensions which are parallel to the mirror are not inverted. In consequence of the inversion of *one* dimension alone, the object and its image are not *similar,* but *symmetrical* forms; and one could not coincide with the other if brought to occupy the same space. The image of a *right* hand is a *left* hand, and all relations of right and left are reversed. It is for this reason that a printed page, seen in a mirror, is like the type with which it was printed.

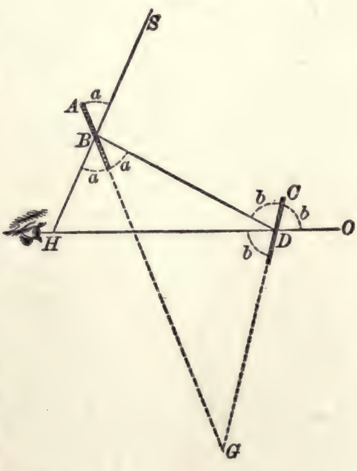
358. The Length of Mirror Requisite for Seeing an Object.—If an object is parallel to a mirror, the length of mirror occupied by the image is to the length of the object as the reflected ray to the sum of the incident and reflected rays. Let $A\ B$ (Fig. 220) be the length of the object, $C\ D$ that of the image, and $F\ G$ that of the space occupied on the mirror; then, by similar triangles, $F\ G : C\ D :: E\ F : E\ C$. But $C\ D = A\ B$, and $C\ F = A\ F$; $\therefore F\ G : A\ B :: E\ F : A\ F + F\ E$. If the eye is brought nearer the mirror, the space on the mirror occupied by the image is diminished, because $E\ F$ has to $A\ F + F\ E$ a less ratio than before. The same effect is produced by removing the object further from the mirror. The length of mirror necessary for a person to see himself is equal to half his height, because in that case, $E\ F : A\ F + F\ E :: 1 : 2$, which ratio will not be altered by change of distance.

Fig. 220.



359. Displacement of Image by Two Reflections.—If an image is seen by light reflected from two mirrors in a plane perpendicular to their common section, its angular deviation from the object is equal to twice the inclination of the mirrors. Let $A\ B$, $C\ D$ (Fig. 221) be two plane

Fig. 221.



mirrors inclined at the angle $A\ G\ C$. If an eye at H sees the star S in the direction O, the angle $S\ H\ O = 2\ A\ G\ C$.

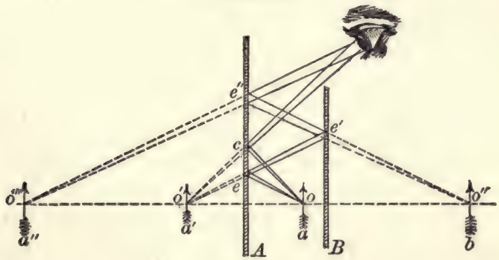
For the exterior angle $C\ D\ B = b = a + G$, or $2\ b = 2\ a + 2\ G$, and $B\ D\ O = 2\ b = 2\ a + H$; hence $2\ a + H = 2\ a + 2\ G$; therefore $H = 2\ G$.

This principle is employed in the construction of *Hadley's quadrant*, and the *sextant*, used at sea for measuring angular distances. The angles measured are twice as great as the arc passed over by the index which carries the revolving mirror; hence, in the quadrant, an arc of 45° is graduated into 90°; and, in the sextant, an arc of 60° is graduated into 120°.

360. Multiplied Images by Two Mirrors.—

1. *Parallel Mirrors.* The series of images is *infinite* in number, and arranged *in a straight line,* perpendicular to the mirrors. The object a, between the parallel mirrors, A and B (Fig. 222), has an image at a', as far behind A as a is in front of it. To

FIG. 222.



avoid confusion, a pencil from only one point o is drawn, once reflected at c, and entering the eye as though it came from o'. The rays reflected by A diverge as though they emanated from a'; hence, the light reflected from A upon B may be regarded as proceeding from a *real* object at a', whose image will be b, as far back of B as a' is in front of B. The light reflected from B to A again diverges as though it really came from b, and regarding b as a *real* object as before *its* image would be formed at a'' as far behind A as b is in front of it. The pencil which enters the eye seems to proceed from o''', having been reflected from e'', as though it came from o'', its reflection in this case having been from e' as though it came from o', though it was really reflected from e after having emanated from o. The pencil which would enter the eye from a third image at the left of a'' may be traced through all its reflections in like manner. As light is absorbed and scattered at each reflection the number of such images is limited.

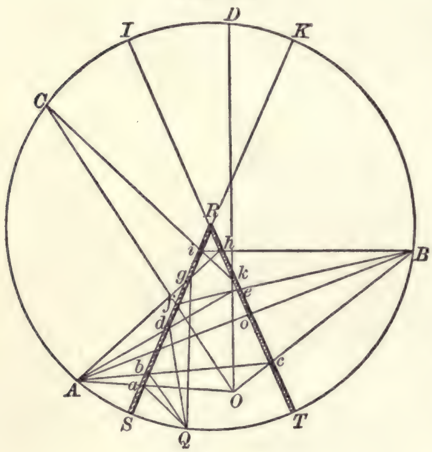
The multiplied images of a small bright object, sometimes

seen in a looking-glass, are produced by repeated reflections between the front and the silvered covering on the back side. At each internal impact on the first surface some light escapes, and shows us an image, while another portion is reflected to the back, and thence forward again. The image of a lamp viewed very obliquely in a mirror is sometimes repeated eight or ten times; and a planet, or bright star, when seen in a looking-glass, will be accompanied by three or four faint images, caused in the same way.

2. *Inclined Mirrors.* Let Q (Fig. 223) be the object, and O the position of the eye. With R as a centre and radius $R\ Q$, describe a circumference.

Suppose a chord $Q\ A$ to be drawn perpendicular to the mirror S, then A will be the image of Q. Regarding A as a *real* object, as in the case of parallel mirrors, draw a chord $A\ B$ perpendicular to the mirror T, then since $o\ B = o\ A$, B will be the image of A. Suppose a chord $B\ C$ to be drawn perpendicular to the mirror S, then C, being as far behind the mirror S as the object B, assumed as *real*, is in front of it, will be the image of B; and for like reasons D will be the image of C in mirror T. All the images formed by the inclined mirrors are thus seen to be confined to the circumference of a circle described as above stated. There can be no image of D, since it lies behind both mirrors prolonged. The image A is seen by rays which proceed from Q to a and thence to the eye at O. B is seen as though the rays came from B to O, these having been reflected at c as though they came from A, the reflection at b being direct from Q. The image C is seen by rays reflected from the points f, e, d; and D by rays reflected from k, i, h, g. The reflections occur in the order d, e, f, and g, h, i, k. Only images formed by light first reflected from S have been considered; a second series produced by light first reflected from T may be constructed in like manner.

FIG. 223.



361. The Kaleidoscope.—This instrument, when carefully constructed, beautifully exhibits the phenomenon of multiplied

reflection by inclined mirrors. It consists of a tube containing two long, narrow, metallic mirrors, inclined at a suitable angle; and is used by placing the objects (fragments of colored glass, &c.) at one end, and applying the eye to the other. In order that there may be perfect symmetry in the figure made up of the objects and their successive images, the angle of the mirrors should be of such size, that it can be exactly contained an even number of times in 360°. The best inclination is 30°; and the field of view is then composed of 12 sectors. It is also essential, that the small objects forming the picture, should lie at the least possible distance beyond the mirrors. To insert three mirrors instead of two, as is often done, only serves to confuse the picture, and mar its beauty.

362. Images by the Concave Mirror.—The concave mirror forms various images, either *real* or *apparent,* either *greater* or *less* than the object, either *erect* or *inverted,* according to the place of the object.

1. The object *between the mirror and its principal focus.* By Art. 352 (2), rays which diverge from a point between the mirror and its principal focus, continue to diverge after reflection, but in a less degree. Let C be the centre, and F the principal focus of the mirror $M\ N$ (Fig. 224), and $A\ B$ the object. Draw the axes, $C\ A$, $C\ B$, and produce them behind the mirror. The pencil from

FIG. 224.

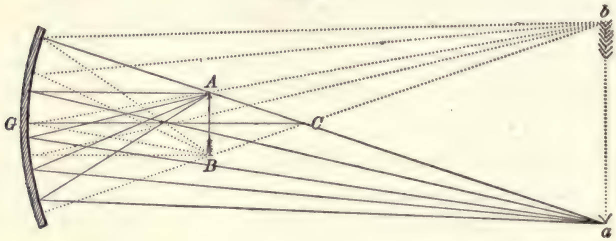
A will be reflected to the eye at H, radiating as from a, in the same axis; likewise, that from B, as from b. Therefore, the image is *apparent,* since rays do not actually flow from it; *erect,* as the axes do not cross each other between the object and image; *enlarged,* because it subtends the angle of the axes at a greater distance than the object does. As the object approaches, and finally reaches the principal focus, the reflected rays approach parallelism, and the image departs from the mirror, till it is at an infinite distance.

Other rays than those given in the figure fall upon the mirror from A, but are reflected either above or below the eye, and therefore have no part in the production of the image, and for that reason are omitted. The same is true of rays from every other point of the object.

2. Object *between the principal focus and the centre.* As soon as the object passes the principal focus, the rays of each pencil begin to converge; and each radiant of the object has its conjugate focus in the same axis beyond the centre (Art. 353).

For example the rays diverging from the point A, represented in Fig. 225 by full lines, after reflection are converged to a situated somewhere on the secondary axis $A\ C\ a$, and rays from B,

Fig. 225.



given as dotted lines, converge finally to b on the axis $B\ C\ b$. The images of intermediate points are formed in the same way.

If an observer is beyond $a\ b$, the rays, after crossing at the image, will reach him, as though they originated in $a\ b$; or if a screen is placed at $a\ b$, the light which is collected in the focal points will be thrown in all directions by radiant reflection from the screen. Hence, the image is *real;* it is also *inverted,* because the axes cross between the conjugate foci; and it is *enlarged,* since it subtends the angle of the axes at a greater distance than the object does. That $b\ C$ is greater than $B\ C$, is proved by joining $C\ G$, which bisects the angle $B\ G\ b$, and therefore divides $B\ b$ so that $B\ C : C\ b :: B\ G : G\ b$. As $G\ b$ is greater than $B\ G$, so $C\ b$ is greater than $B\ C$. When the object reaches the centre, the image is there also, but inverted in position, since rays which proceed from one side of C, are reflected to the other side of it.

3. Object *beyond the centre.* This is the reverse of (2), the conjugate foci having changed places; $a\ b$, therefore, being the object, $A\ B$ is its image, *real, inverted, diminished.* As the object removes to infinity, the image proceeds only to the principal focus F.

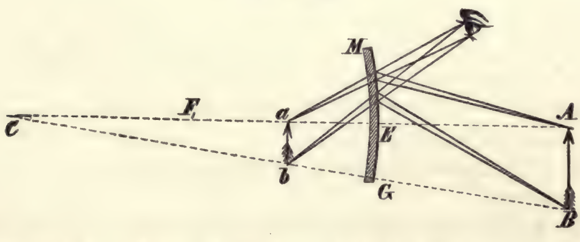
363. Illustrated by Experiment.—These cases are shown experimentally by placing a lamp close to the mirror, and then carrying it along the axis to a considerable distance away. While the lamp moves from the mirror to the principal focus, its image behind the mirror recedes from its surface to infinity; we may then regard it as being either at an infinite distance behind, or an infinite distance in front, since the rays of every pencil are par-

allel. After the lamp passes the principal focus, the image appears in the air at a great distance in front, and of great size, and they both reach the centre together, where they pass each other; and, as the lamp is carried to great distances, the image, growing less and less, approaches the principal focus, and is there reduced to its smallest size. The only part of the infinite line of the axis before and behind, in which no image can appear, is the small distance between the mirror and its principal focus.

If a person looks at *himself*, so long as he is between the mirror and the principal focus, he sees his image behind the mirror and enlarged. But when he is between the principal focus and centre, the image is *real*, and behind him; the converging rays of the pencils, however, enter his eyes, and give an indistinct view of his image as if at the mirror. When he reaches the centre, the pupil of the eye is seen covering the entire mirror, because rays from the centre are perpendicular, and return to it from all parts of the surface. Beyond the centre, he sees the real image in the air before him, distinct and inverted.

364. Images by the Convex Mirror.—The convex mirror affords no variety of cases, because diverging rays, which fall upon it, are made to diverge still more by reflection. In Fig. 226 the pencil from *A* is reflected, as if radiating from *a* in the same

FIG. 226.

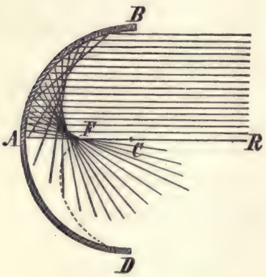


axis *A C*, and that from *B*, as from *b* in the axis *B C*; and these apparent radiants are always nearer the surface than the middle point between it and *C* (Art. 354). The image is therefore *apparent*; it is *erect*, since the axes do not cross between the object and image; and it is *diminished*, as it subtends the angle of the axes at a less distance than the object.

As in Fig. 224, rays from *A* and *B*, which after reflection pass above or below the eye, have been omitted; only that portion of the mirror from which rays are represented as being reflected has any part in the formation of the image. For eyes in other positions other rays would be used, still seeming to come from the same image *a b*.

365. Caustics by Reflection.—These are luminous curved surfaces, formed by the intersections of rays reflected from a hemispherical concave mirror. The name *caustic* is given from the circumstance that *heat*, as well as light, is concentrated in the focal points which compose it. *B A D* (Fig. 227), represents a section of the mirror, and *B F D* of the caustic; the point *F*, where all the sections of the caustic through the axis meet each other, is called the *cusp*. When the incident rays are parallel, as in the figure, the cusp is at the principal focus, that is, the middle point between *A* and *C*. The rays near the axis *R A*, after reflection meet at the cusp (Art. 352); but those a little more distant cross them, and meet the axis a little further toward *A*. And the more distant the incident ray from the axis, the further from the centre does the reflected ray meet the axis. Thus each ray intersects all the previous ones, and this series of intersections constitutes the curve, *B F*. The curve is luminous, because it consists of the foci of the successive pencils reflected from the arc *A B*.

Fig. 227.



If the incident rays, instead of being parallel, diverge from a lamp near by, the form of the caustic is a little altered, and the cusp is nearer the centre. This case may be seen on the surface of milk, the light of the lamp being reflected by the edge of the bowl which contains it.

If parallel or divergent light falls on a convex hemispherical mirror, there will be *apparent* caustics behind the mirror; that is, the light will be reflected as if it radiated from points arranged in such curves.

366. Spherical Aberration of Mirrors.—It has already been mentioned (Art. 351), that the statements in this chapter relating to focal points and images, as produced by spherical mirrors, are true only when the mirror is a very small part of the whole spherical surface. In Art. 365 we have seen the effect of using a large part of the spherical surface—viz., the rays neither converge *to*, nor diverge *from* a single point, but a series of points arranged in a curve. This general effect is called the *spherical aberration* of a mirror; since the deviation of the rays is due to the spherical curvature. The deviation, as we have seen, is quite apparent in a hemisphere, or any considerable portion of one; but it exists in some degree in any spherical mirror, unless infinitely small compared with the hemisphere.

But there are curves which will reflect without aberration. Let a concave mirror be ground to the form of a paraboloid, and rays parallel to its axis will be converged to the focus without aberration. For, at any point on such a mirror, a line parallel to the axis, and a line drawn to the focus, make equal angles with the tangent, and therefore, equal angles with the perpendicular to the surface. And rays, parallel to the axis of a convex paraboloid, will diverge as if from its focus, on the same account. Again, if a radiant is placed at the focus of a concave parabolic mirror, the reflected rays will be parallel to the axis, and will illuminate at a great distance in that direction. Such a mirror, with a lamp in its focus, is placed in front of the locomotive engine to light the track, and has been much used in light-houses. If a concave mirror is ellipsoidal, light emanating from one focus is collected without aberration to the other, because lines from the foci to any point of the curve make equal angles with the tangent at that point.

Since heat is reflected according to the same law as light, a concave mirror is a burning-glass. When it faces the sun, the light and heat are both collected in a small image of the sun at the principal focus. And, if no heat were lost by the reflection, the intensity at the focus would be to that of the direct rays, as the area of the mirror to the area of the sun's image. Burning mirrors have sometimes been constructed on a large scale, by giving a concave arrangement to a great number of plane mirrors.

CHAPTER III.

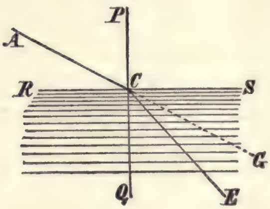
REFRACTION OF LIGHT.

367. Division of the Incident Beam.—When light falls on an *opaque* body, we have noticed that it is arrested, and a shadow formed beyond. Of the light thus arrested, a portion is reflected, and another portion lost, which is said to be absorbed by the body. When light meets a *transparent* body, a part is still reflected, and a small portion absorbed, but, in general, the greater part is transmitted. The ratio of intensities in the reflected and transmitted beams varies with the angle of incidence, but little being reflected at small angles of incidence, and almost the whole at angles near 90°.

368. Refraction.—The transmitted beam suffers important changes, one of which is a change in *direction*. This change is

called *refraction*, and takes place at the surface of a new medium. In Fig. 228, *A C*, incident upon *R S*, the surface of a different medium, is turned at *C* into another line, as *C E*, which is called the *refracted ray*. The angle *E C Q*, between the refracted ray and the perpendicular is called the *angle of refraction;* the angle *G C E*, between the directions of the incident and the refracted rays, is the *angle of deviation.*

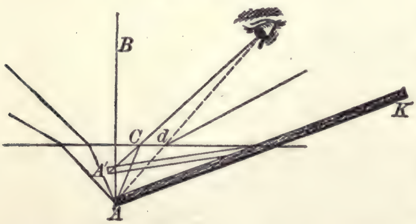
Fig. 228.



It is a general fact, to which there are but few exceptions, that a ray of light in passing out of a rarer into a denser medium is refracted *toward* the perpendicular to the surface; and in passing out of a denser into a rarer medium, it is refracted *from* the perpendicular. But the chemical constitution of bodies sometimes affects their refracting power. Some inflammable bodies, as sulphur, amber, and certain oils, have a great refracting power in comparison with other bodies; and in a given instance, a ray of light in passing out of one of these substances into another of greater density may be turned from the perpendicular instead of toward it. In the optical use of the words, therefore, *denser* is understood to mean, *of greater refractive power;* and *rarer* signifies, *of less refractive power.* In Fig. 228, the medium below *R S* is of greater refractive power than that above.

Let *A K* (Fig. 229) represent a straight rod, the lower end *A* being beneath the surface of water. The rays which diverge from the point *A* are bent from the perpendicular *A B*. The ray *A d*, which, if prolonged, would enter the eye, is by refraction bent so as to pass below, while the ray *A C* deviates at *C* and enters the eye as though coming from *A'*, thus giving to the rod the bent appearance noticed in an oar when in use.

Fig. 229.



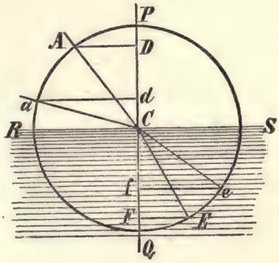
In the same manner, the bottom of a river appears elevated, and diminishes the apparent depth of the stream. Let a small object be placed in the bottom of a bowl, and let the eye be withdrawn till the object is hidden from view by the edge of the bowl. If now the bowl be filled up with water, the object is no longer concealed, for the light, as it emerges from the water, is bent away from the perpendicular, and brought low enough to enter the eye.

369. Law of Refraction.—The law which is found to hold true in all cases of common refraction is this :

The angles of incidence and refraction are on opposite sides of the perpendicular to the surface, and, for any given media, the sines of the angles have a constant ratio for all inclinations.

For example, in Fig. 230, if $A\ C$ is refracted to E, then $a\ C$ will be refracted to e, so that $A\ D : E\ F :: a\ d : e\ f$; and if the rays pass out in a contrary direction, the ratio is also constant, being the reciprocal of the former, viz., $E\ F : A\ D :: e\ f : a\ d$.

FIG. 230.



This constant ratio is called the *Index of Refraction* and is found by dividing the sine of the angle of incidence by the sine of the angle of refraction.

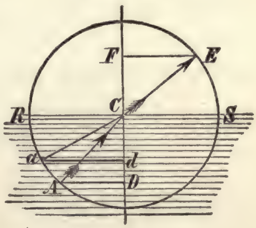
A ray perpendicular to the surface, passing in either direction, is not refracted; for, according to the law, if the sine of one angle is zero, the sine of the other must be zero also.

The following table gives the indices of refraction, the ray being supposed to pass from a vacuum into the substance ; such indices are termed *absolute indices :*

Diamond	2.450	Crown glass (mean)	1.530
Carbon disulphide	1.678	Alcohol	1.372
Oil of cassia	1.630	Water	1.336
Flint glass (mean)	1.600	Ice	1.309
Quartz	1.548	Air	1.000294
Canada Balsam	1.540		

370. Limit of Transmission from a Denser to a Rarer Medium.—As a consequence of the law of refraction, there is a limit beyond which a ray cannot escape from a denser medium. Let $A\ C$ (Fig. 231) be the ray incident upon the rarer medium $R\ E\ S$. It will be refracted from the perpendicular $D\ F$ into the direction $C\ E$, so that $A\ D$ is to $E\ F$ in a constant ratio (Art. 369). If the angle $A\ C\ D$ be increased, $F\ C\ E$ must also increase till at length its sine equals $C\ S$.

FIG. 231.



Suppose the denser medium to be water and the rarer air, then

$$\frac{\text{Sine } A\ C\ D}{\text{Sine } E\ C\ F} = \frac{1}{1.336} \text{ nearly};$$

hence, sine $E\,C\,F = 1.336 \times$ sine $A\ C\,D$. If $E\,C\,F$ be increased to $90°$, then sine $90° = 1 = 1.336 \times$ sine $A\ C\,D$, from which we find sine $A\ C\,D = .7485$, the angle corresponding to which is $48°\ 28'$. If the angle of incidence be greater than $48°\ 28'$, its sine would exceed .7485, and therefore the sine of the angle in air should exceed unity, which is impossible. Hence it follows, that whenever the angle of incidence is greater than that at which the sine of the angle of refraction becomes equal to radius, the ray cannot be refracted consistently with the constant ratio of the sines.

This is proved also by experiment; the emerging ray increases its angle of refraction till it at length ceases to pass out. Beyond that limit all the incident rays are *reflected* from the inner surface of the denser medium; and this reflection is more perfect than any external reflection, and is called *total reflection.*

The limiting angle for diamond is $24°\ 12'$, and its great brilliancy, when properly cut, is due to numerous internal total reflections which cause the light to emerge in different directions.

371. Opacity of Mixed Transparent Media.—Light in passing from a medium to a different one, is partly reflected and partly refracted; if this be often repeated in a mixed medium no light is transmitted. It is the frequency of reflection at the limiting surfaces of air and water that renders foam opaque. So also a transparent crystal, when crushed, becomes an opaque powder. If the powder be wetted with a liquid having the same refractive index as the crystal, the reflections will be prevented and transparency will result.

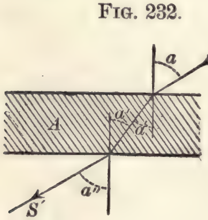
372. Transmission through Parallel Plane Surfaces.—Let S (Fig. 232) enter the medium A, and represent the emergent ray by S'. Suppose the ray to enter from a vacuum, and to emerge into a vacuum again, and call the index of refraction m. Then

$$\text{sine } a = m \times \text{sine } a', \text{ and}$$

$$\text{sine } a' = \frac{1}{m}\,\text{sine } a'';$$

multiplying these together, we have sine $a =$ sine a'', whence $a = a''$, and the emergent and incident rays are parallel. Suppose the ray S' to enter a second medium B, bounded by parallel faces, it will emerge parallel to S', and therefore parallel to S. Hence if a ray traverse any number of media with parallel faces, these media being separated by vacua the

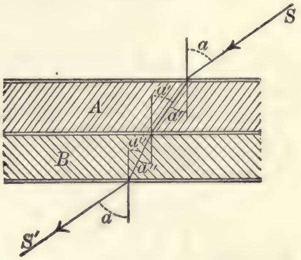
FIG. 232.



finally emergent ray will be parallel to the first incident ray S. If now the spaces between the media be diminished, the result will not be changed, and finally when the diminution reaches its limit the faces of the media will be in contact, and we shall still have the incident and emergent rays parallel.

373. Determination of Relative Indices of Refraction.—When a ray passes from a medium A into another B (Fig. 233), the absolute indices of these being known the relative index may be found. Suppose the media to be bounded by parallel plane faces. Let m be the *absolute index* of A, and n that of B. Denote the relative index, $\frac{\text{sine } a'}{\text{sine } a''}$, by i. Suppose the ray S to enter A from a vacuum, then

Fig. 233.



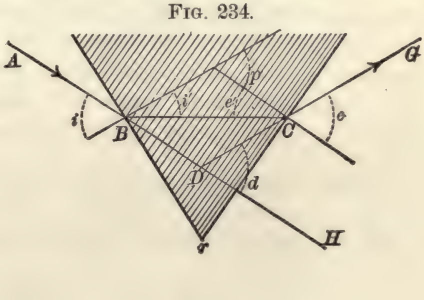
$$\text{sine } a = m \times \text{sine } a'$$
$$\text{sine } a' = i \times \text{sine } a''$$
$$\text{sine } a'' = \frac{1}{n} \text{sine } a$$ since the emergent ray S' is parallel to the incident ray S (Art. 372). By multiplying these equations together, we find $i = \frac{n}{m}$; hence, to find the relative index of refraction *when a ray passes from medium A into medium B, divide the absolute index of B by that of A.*

Suppose a ray to pass from air into carbon disulphide, then $\frac{1.678}{1.0003} = 1.6774$, knowing which the deviation of the ray for any given angle of incidence can be found.

The same principle may be applied to find the relative index of two substances whose relative indices with respect to a third are known.

374. Transmission through a Medium Bounded by Inclined Planes.—A medium bounded by *inclined* planes is called a *prism*. The angle included by the planes through which the light passes is called the *refracting angle* of the prism, and the planes are *deviating planes*.

Fig. 234.



Let A B (Fig. 234) be the incident ray, and C G

the emergent ray. The total deviation will be $G\,D\,H = d$. Adopting the notation of the figure, we have $G\,D\,H = D\,B\,C + D\,C\,B$ or $d = (i - i') + (e - e') = i + e - (i' + e')$. Because of the perpendiculars through B and C we have $r = p$, but $p = i' + e' = r$; hence, $d = i + e - r$; that is to say, *the total deviation is equal to the sum of the angles of incidence and emergence diminished by the refracting angle of the prism.*

375. Prism Used for Measuring Refractive Power.— For any given prism the deviation will depend upon the angles of incidence and emergence.

If a prism rotate about an axis parallel to its refracting edge, a position of minimum deviation will be found such that any rotation either to right or left will increase the deviation of the ray; if now the angles of incidence and emergence be measured, they will be found equal.

From the equations

$$r = i' + e'$$

$$d = i + e - r,$$ by making $i = e$, and consequently $i' = e'$, we obtain

$$i = \tfrac{1}{2}(r + d), \text{ and } i' = \tfrac{1}{2}r;$$

from which we find the relative index of refraction

$$m = \frac{\text{sine } i}{\text{sine } i'} = \frac{\text{sine } \frac{1}{2}(r + d)}{\text{sine } \frac{1}{2} r}.$$

Thus having measured the refracting angle of the prism and the minimum deviation of the ray we can at once determine the index of refraction of the substance of which the prism is formed.

If the angle r be very small, d will also be small, and the ratio of the angles may be used instead of the ratio of their sines, and the formula then becomes $m = \dfrac{r + d}{r} = 1 + \dfrac{d}{r}$.

This is one of the best methods by which to determine the index of refraction of a solid, transparent substance.

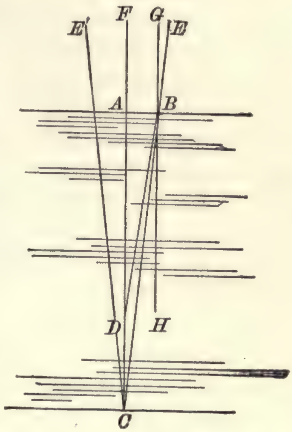
The final deviation of the ray being unaffected by its passage through glass plates with parallel faces, hollow prisms formed of such plates may be filled with a liquid whose index of refraction is to be determined. A tube whose end sections are glass planes equally inclined to the axis of the tube, may be used to determine the relative indices of gases and air, and by exhausting the tube to form a vacuum, the absolute indices may be found.

376. Light through One Surface.—

1. *Plane Surface.* When *parallel* rays pass into another medium through a plane surface, they remain parallel. For the per-

pendiculars being parallel, the angles of incidence are equal, and therefore the angles of refraction are equal also, and the refracted rays parallel. But a pencil of *diverging* rays is made to diverge *less*, when it *enters a denser medium.* For the outer rays make the largest angles of incidence, and are therefore most refracted toward the perpendiculars, and thus toward parallelism with each other. And when *diverging* rays *enter a rarer medium,* they diverge *more ;* because the outside rays make the largest angles of incidence, and therefore the largest angles of refraction, by which means they spread more from each other.

FIG. 235.

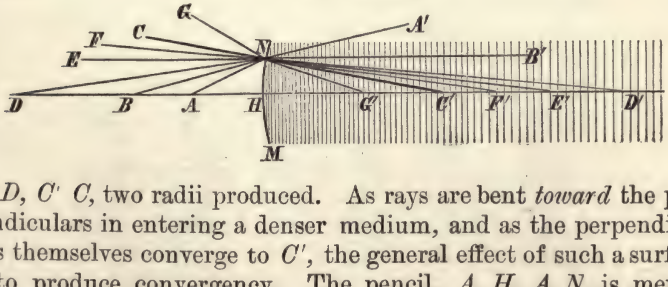


The last case is illustrated when we look perpendicularly into water, and see its depth apparently diminished by about one-fourth of the whole. Let $A\ B$ (Fig. 235) be the surface, and C a point at the bottom, from which pencils come to the eyes at E, E'. Let $C\,F$ be perpendicular to the surface $A\ B$, and $C\ B\ E$ the axis of an oblique pencil to the eye at E. As the distance between the pupils of the eyes is less than $2\frac{1}{2}$ inches, the obliquity of the pencil $C\ B\ E$ will be very slight.

The angle $C = C\,B\,H =$ angle of incidence ; and $A\,D\,B = G\,B\,E =$ angle of refraction. Now, in the triangle $B\ D\ C$, $B\ C$: $B\ D$ (: : $A\ C$: $A\ D$ nearly) : : sin D : sin C : : sine of refraction : sine of incidence : : 1.34 : 1. Hence the apparent depth is one fourth less than the real depth. The apparent depth of water may be diminished much more than this by looking into it obliquely.

2. *Convex surface of the denser. A convex surface tends to converge rays.* Let C' (Fig. 236) be the centre of convexity, and

FIG. 236.



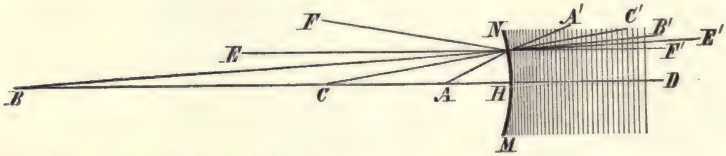
$C'\ D$, $C'\ C$, two radii produced. As rays are bent *toward* the perpendiculars in entering a denser medium, and as the perpendiculars themselves converge to C', the general effect of such a surface is to produce convergency. The pencil, $A\ H$, $A\ N$, is merely made less divergent, $H\ D'\ N\ A'$; $B\ H$, $B\ N$ become parallel,

$H D'$, $N B'$; $D H$, $D N$, convergent to D'; the parallel rays, $D H$, $E N$, convergent to E'; the convergent pencil, $D H$, $F N$, more convergent to F'; but $D H$, $C N$, which converge equally with the radii, are not changed; and $D H$, $G N$, which converge more than the radii, converge less than before, to G'. The two last cases, which are exceptions to the general effect, rarely occur in the practical use of lenses.

If we trace in the opposite direction the rays, A', B', D', &c., comparing each with $D' D$, we find, in this case also, that the convex surface tends to converge the rays, by bending them *from* their respective perpendiculars.

3. *Concave surface of the denser. A concave surface tends to diverge rays.* Let $C C'$, $C D$ (Fig. 237), be the radii of concavity produced. As the radii *diverge* in the direction in which the light

FIG. 237.

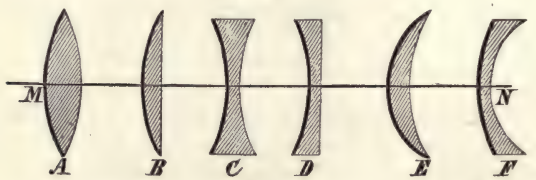


moves, the rays, being bent *toward* them, will generally be made to diverge also. Hence, parallel rays, $B H$, $E N$, are diverged, $H D$, $N E'$; and diverging rays, $B H$, $B N$, are diverged more, $H D$, $N B'$. If, however, rays diverge as much as the radii, or more, they proceed in the same direction, or diverge less, a case which rarely occurs.

If the rays are traced in the opposite direction, the tendency in general to produce divergency appears from the fact that the perpendiculars are now *converging* lines, and the rays are refracted *from* them.

377. Lenses.—A *lens* is a transparent medium bounded by curved surfaces whose céntres of curvature lie upon a normal common to the two surfaces. If the radius of curvature is made

FIG. 238.



infinite, the corresponding surface becomes a plane. The usual varieties are shown in Fig. 238.

A *double convex lens* (A) consists of two spherical segments, either equally or unequally convex, having a common base.

A *plano-convex lens* (B) is a lens having one of its sides convex and the other plane, being simply a segment of a sphere.

A *double concave lens* (C) is a solid bounded by two concave spherical surfaces, which may be either equally or unequally concave.

A *plano-concave lens* (D) is a lens one of whose surfaces is plane and the other concave.

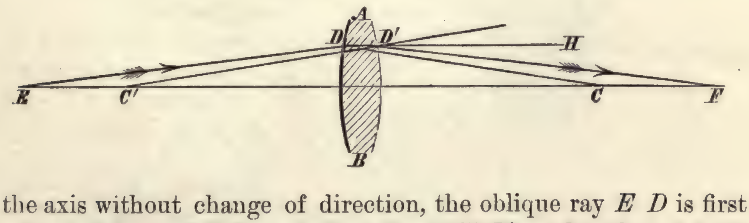
A *meniscus* (E) is a lens one of whose surfaces is convex and the other concave, but the concavity being less than the convexity, it takes the form of a crescent, and has the effect of a convex lens whose convexity is equal to the difference between the sphericities of the two sides.

A *concavo-convex lens* (F) is a lens one of whose surfaces is convex and the other concave, the concavity exceeding the convexity, and the lens being therefore equivalent to a concave lens whose concavity is equal to the difference between the sphericities of the two sides.

A line ($M\ N$) passing through a lens, perpendicular to its opposite surfaces, is called the *axis*. The axis usually, though not necessarily, passes through the centre of the figure.

378. General Effect of the Convex Lens.—Whether double-convex or plano-convex, its general effect is to converge light. It has been shown (Art. 376) that the convex surface of a denser medium tends to converge rays, whichever way they pass through it. Therefore, if E (Fig. 239) is a radiant, while $E\ C'\ C$ follows

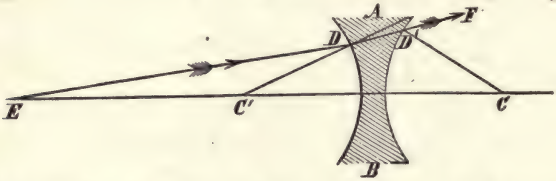
FIG. 239.



the axis without change of direction, the oblique ray $E\ D$ is first refracted *toward* $D\ C$, and then *from* $C'\ D'$ produced, and both actions conspire to converge it to the axis. The rays are represented as meeting in the focus F. Whether the rays are *actually* converged, depends on their previous relation to each other. If the lens is *plano-convex*, the plane surface has usually but little effect in converging the light; but by Art. 376 it may be shown that its action will usually conspire with that of the convex surface.

379. General Effect of the Concave Lens.—This lens, whether double-concave or plano-concave, tends to produce *divergency*. This is evident from what has been shown in Art. 376. The ray $E\,D$ (Fig. 240), in entering the denser medium, is first

FIG. 240.



refracted *toward* $C'\,D$ produced, and on leaving the medium at D', is refracted *from* $D'\,C$; and is thus twice refracted from the ray $E\,C$, which being in the axis, is not refracted at all. If the lens is *plano-concave*, the effect of the plane surface may, or may not, conspire with that of the concave surface.

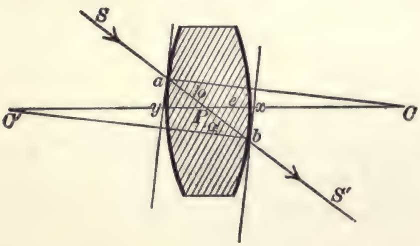
380. The Optic Centre of a Lens.—The incident and emergent portions of a ray which enters and leaves a lens at the points of contact of parallel tangent planes will be parallel according to Art. 372.

The point where the part of such ray included between the bounding surfaces cuts the axis of the lens, or would cut it if produced, is called the optic centre.

In Fig. 241 let a and b be points of contact of parallel tangent planes, then the radii $C\,a$ and $C'\,b$ being perpendicular to these parallel planes are themselves parallel, hence the angles o and o are equal; the angles at P are also equal, and hence the triangles $C\ a\ P$ and $C'\ b\ P$ are similar, and

$$C'\,P : C\,P :: C'\,b : C\,a.$$

FIG. 241.



Represent the thickness of the lens $x\,y$, measured on the axis, by t, and the distance from P, the optic centre, to the surface x by e; also make the radius $C\,a = r$ and $C'\,b = r'$. Substituting these values above we have

$$r' - e : r - (t - e) :: r' : r,$$

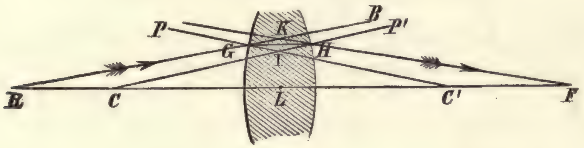
from which we obtain

$$e = \frac{r'\,t}{r' + r} = \frac{r'}{\frac{r'}{t} + \frac{r}{t}}.$$

But this value of e is constant since r, r' and t are constant; therefore all rays which suffer no deviation in passing through the lens must pass through a common point P, called the optic centre. The optic centre is within the lens in the cases of double concave and double convex lenses, but without in the meniscus and concavo-convex. If $r = r'$ the optic centre is midway between the faces.

381. Conjugate Foci.—If the rays from R (Fig. 242) are collected at F, then rays emanating from F will be returned to R; and the two points are called *conjugate foci*. Their relative distances from the lens may be determined when the radii of the

FIG. 242.



surfaces and the index of refraction are known. Let n be the index of refraction, and assume, what is practically true, that the angles of incidence and refraction are so small that their ratio is the same as the ratio of their sines. Then

$$R\,G\,P\,(=K\,G\,I) : I\,G\,H :: n : 1;$$

$$\therefore \quad K\,G\,H : I\,G\,H :: n-1 : 1;$$

in like manner $\quad K\,H\,G : I\,H\,G :: n-1 : 1;$

$$\therefore K\,G\,H + K\,H\,G : I\,G\,H + I\,H\,G :: n-1 : 1.$$

But $\quad K\,G\,H + K\,H\,G = B\,K\,F = R + F;$

and $\quad I\,G\,H + I\,H\,G = G\,I\,C = C + C';$

naming the acute angles at $R, C, C'\ F$, by those letters respectively,

$$\therefore R + F : C + C' :: n-1 : 1.$$

Now, the lens being thin, and the angles R, C, C', and F very small, the same perpendicular to the axis, at L, the centre of the lens, may be considered as subtending all those angles. Hence, each angle is as the reciprocal of its distance from L. Let $R\,L = p$; $F\,L = q$; $C\,L = r$; and $C'\,L = r'$. Then the equation above becomes,

$$\frac{1}{p} + \frac{1}{q} : \frac{1}{r} + \frac{1}{r'} :: n-1 : 1;$$

which expresses in general the relation of the conjugate foci.

382. To Find the Principal Focus.—The radiant from which parallel rays come is at an infinite distance. Therefore,

making $p = \infty$, and the distance of the principal focus $= F$, we have $\frac{1}{p} = 0$, and

$$\frac{1}{F} : \frac{1}{r} + \frac{1}{r'} :: n - 1 : 1.$$

If the curvatures are equal, for crown-glass, for which $n = \frac{3}{2}$, F reduces to r; that is, the principal focus of a double convex lens of crown-glass, having equal curvatures, is at the centre of convexity.

The foregoing formulæ are readily adapted to the other forms of lens. When a surface is plane, its radius is infinite, and $\frac{1}{r}$, or $\frac{1}{r'} = 0$. When concave, its centre is thrown upon the same side as the surface, and its radius is to be called negative. And if the focal distance, as given by the formula, becomes negative, it is understood to be on the same side as the radiant; that is, the focus is a virtual radiant.

383. Powers of Lenses Practically Determined.—The reciprocal of the principal focal length of a lens $\frac{1}{F}$, is called the *power of a lens.* From Art. 381 we find

$$\frac{1}{p} + \frac{1}{q} = (n - 1) \left(\frac{1}{r} + \frac{1}{r'}\right),$$

and from Art. 382

$$\frac{1}{F} = (n - 1)\left(\frac{1}{r} + \frac{1}{r'}\right);$$

whence we have

$$\frac{1}{F} = \frac{1}{p} + \frac{1}{q}.$$

As the index of refraction and the radii of curvature are not generally known in respect to any particular lens which we may happen to be using, some practical method by which to determine F will enable us to calculate readily either p or q, the other being given.

(1.) *To find F for a convex lens.*—Form an image of the sun upon a plate of ground glass, and measure the distance of the image from the lens. Or, place a light on one side of the lens and find its sharp image upon a screen on the other side. These distances measured, give p and q, whence $F = \frac{p\,q}{p + q}$.

(2.) These two methods assume the thickness of the lens to be small compared with the focal length. The focal length of a

thick lens, or system of lenses, may be found thus: On one side, at a distance a little greater than F, place a scale strongly illuminated by transmitted light, and receive the sharp and greatly magnified image of one of its divisions upon a screen upon the other side of the lens or lenses. Then let l = length of one division, L = length of its image, p = distance of the screen from the lens (very great compared with its thickness), and we find, from similar right-angled triangles, $L : l :: p : \frac{p\,l}{L} = q$, and these values of p and q give

$$F = \frac{p\,q}{p+q} = \frac{p\,\frac{p\,l}{L}}{p + \frac{p\,l}{L}} = p\,\frac{l}{L+l}.$$

The focal length is strictly the distance from F to the intersection of the axis by the *principal plane* of the lens or combination of lenses.

The *principal plane* passes through the point of intersection of an incident ray, parallel to the axis and its emergent ray, both produced if necessary, and is at right angles to the axis.

(3.) To find F for a concave lens. Use in contact with the concave lens a stronger convex, of known value for F, and proceed according to the preceding methods. Then if f = focal length of combination and f' = focal length of convex alone, and F = that of concave lens sought, we shall find

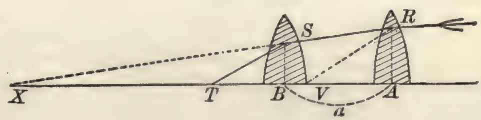
$\frac{1}{F} = \frac{1}{f} - \frac{1}{f'}$, as will be proved hereafter; or when the lens is deep and not very small, take for the focal length that distance from a screen at which the circle of light from the sun is twice the diameter of the lens.

384. Equivalent Combinations.—

To find the focal length of a lens which shall be equal to a combination of two lenses.

Suppose the lenses (Fig. 243) to be of such thickness as may be neglected. Let a ray parallel to the common axis be incident at R. If $R\ V$ be drawn parallel to $S\ T$, the emergent ray, $A\ V$ will represent the focal length, F, of a lens which would produce the same deviation as this combination. Let $A\ X = f$ = focal length of A, then $B\ X = f - a$, a being the distance between B and A.

FIG. 243.



Now if we regard T as a radiant, and $T\ S\ R$ as the path of the ray, then X is the virtual conjugate focus of the lens B corresponding to T, and calling f' the focal length of B, we have

$$\text{Art. 383, } \frac{1}{f'} = \frac{1}{B\ T} - \frac{1}{B\ X}.$$

Substituting the value of $B\ X$ above, we have

$$\frac{1}{B\ T} = \frac{1}{f'} + \frac{1}{f-a} = \frac{f'+f-a}{f'(f-a)}.$$

By similar triangles $A\ V\ R$, $B\ T\ S$ and $X\ A\ R$, $X\ B\ S$

$$\frac{B\ T}{A\ V\ (=F)} = \frac{B\ S}{A\ R} = \frac{B\ X}{A\ X}, \text{ whence}$$

$$\frac{1}{F} = \frac{f-a}{f} \times \frac{f'+f-a}{f'\ (f-a)} = \frac{f'+f-a}{f\ f'}$$

$$\frac{1}{F} = \frac{1}{f} + \frac{1}{f'} - \frac{a}{f\ f'}.$$

When the lenses are in contact the distance $a = o$, and we have $\frac{1}{F} = \frac{1}{f} + \frac{1}{f'}$; that is to say,

The power of a combination of two lenses in contact is equal to the sum of their respective powers.

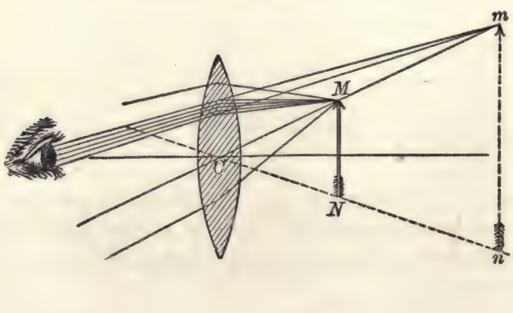
Due attention must be paid to the signs of the powers, those of concave lenses being negative.

The above rule is general, and is not confined to two lenses only.

385. Images by the Convex Lens.—The *convex* lens forms a variety of images, whose character and position depend on the place of the object. If it is at the *principal focus*, the rays of every pencil pass out parallel, and seem to come from an infinite distance. If the object is *nearer* than the principal focus, the emergent rays of each pencil diverge less than the incident rays, and therefore seem to radiate from points further back ; the image is therefore *apparent*.

Let $M\ N$ be an object (Fig. 244) nearer than the principal

FIG. 244.



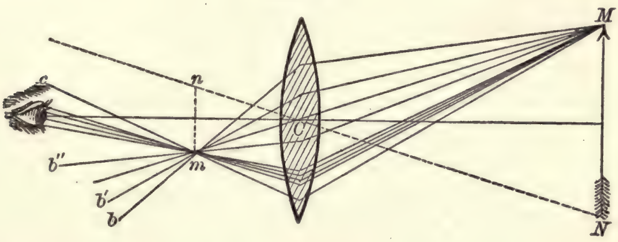
focus. Rays from the point M will diverge; and falling upon the whole surface of the lens, will be refracted; some passing above the eye and some below; only a small part of them will enter the eye, as though they came from m, situated on the secondary axis $C\ M$ produced. In like manner each point of $M\ N$ has its corresponding point in the image $m\ n$.

The image is erect, because the axes of the pencils do not cross between the object and image; and it is *enlarged*, because it subtends the angle $M\ C\ N$ at a greater distance than the object.

But if the object is *further* from the lens than the principal focus, the rays of each pencil converge to a point in the axis of that pencil produced through the lens; and thus light is collected in focal points, which consequently become actual radiants.

Let $M\ N$ (Fig. 245) be the object. A cone of rays from the point M covers the lens, and converges to the conjugate focus

FIG. 245.



m, on the axis $M\ C$ produced, whence the rays again diverge as from a real radiant. Some of these, as $m\ c$, pass above the eye, while others, $m\ b$, $m\ b'$, $m\ b''$, pass below, only a small part of them entering the eye and rendering visible the image $m\ n$.

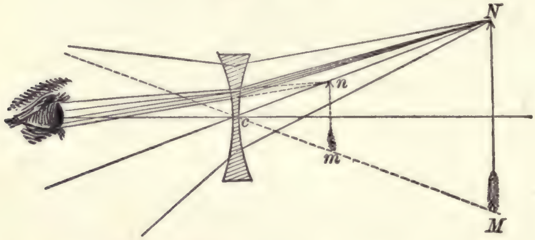
Instead of viewing the image as above, since it is a collection of *real* foci a white screen may be placed at $m\ n$, which will reflect the light of each focal point in all directions, and thus render the image visible to a large audience. Though the rays of every radiant *converge* from the lens to the conjugate focus of that radiant, yet the axes of the pencils *diverge* from each other, having all crossed at the optic centre. The image is therefore *inverted*, as are all real images, in whatever way produced.

The formula for conjugate foci shows that if p is increased, q is diminished; therefore the further $M\ N$ is removed from the lens, the nearer $m\ n$ approaches to it; but the nearest position is the principal focus, which it reaches when the object is at an infinite distance. As the object and image subtend equal angles at the optic centre, and are parallel, or nearly parallel with each other, their *diameters* are proportional to their *distances* from the

lens. But the area of the lens has no effect on the size of the image, since change of area does not alter the relation of the axes, but only the size of the luminous cones, and thus the quantity of light in each pencil.

386. Images by the Concave Lens.—As the rays of each pencil are diverged more after passing through the lens than before, the image is *apparent*, and is situated between the lens and the object. Let *M N* (Fig. 246) be the object; the cone of rays from *N* will, after refraction, diverge more, as from *n*, in the same axis

Fig. 246.

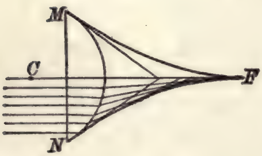


c N; and all other pencils will be affected in a similar manner, and form an apparent image *m n*. It will be *erect*, since the axes do not cross between, and *diminished*, being nearer the angle *c*, which is subtended by both object and image.

It is noticeable that the *concave mirror* and the *convex lens* are analogous in their effects, forming images on both sides, both real and apparent, both erect and inverted, both larger and smaller than the object; while the *convex mirror* and the *concave lens* also resemble each other, producing images always on one side, always apparent, always erect, always smaller than the object.

387. Caustics by Refraction.—If the convex surface of a lens is a considerable part of a hemisphere, the rays more distant from the axis will be so much more refracted than others, as to cross them and meet the axis at nearer points, thus forming caustics by refraction. Fig. 247 shows this effect in the case of parallel rays; those near the axis intersecting it at the principal focus *F*, and the intersections of remoter rays being nearer and nearer to the lens, so that the whole converging pencil assumes a form resembling a cone with concave sides.

Fig. 247.



The grating (Fig. 248), viewed through such a lens, would

appear distorted, as in Fig. 249, and if viewed through a concave lens the opposite effect would result, as in Fig. 250.

FIG. 248.

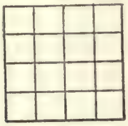


FIG. 249.

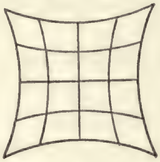
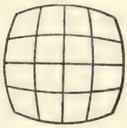
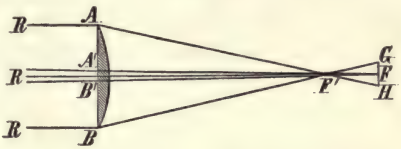


FIG. 250.



388. Spherical Aberration of a Lens.—The production of caustics is an extreme case of what is called spherical aberration. Unless the lens is of small angular breadth, not more than 10°, a pencil whose rays originated in one point of an object is not converged accurately to one point of the image, but the outer rays are refracted too much, and make their focus nearer the lens than that of the central rays, as represented in Fig. 251. If F is the focus of the central rays, and F' of the extreme ones, other rays of the same beam are collected in intermediate points, and $F F'$ is called the *longitudinal spherical aberration;* and $G H$, the breadth covered by the pencil at the focus of central rays, is called the *lateral spherical aberration.*

FIG. 251.



Such a lens cannot form a distinct image of any object; because perfect distinctness requires that all rays from any one point of the object should be collected to one point in the image. If, for example, the beam whose outside rays are $R A$, $R B$, comes from a point of the moon's disc, that point will not be perfectly represented by F, because a part of its light covers the circle, whose diameter is $G H$, thus overlapping the space representing adjacent points of the moon. And if that point had been on the edge of the moon's disc, F could not be a point of a well-defined edge of the image, since a part of the light would be spread over the distance $F G$ outside of it, and destroy the distinctness of its outline.

389. Remedy for Spherical Aberration.—As spherical lenses refract too much those rays which pass through the outer parts, it is obvious that, to destroy aberration, a lens is required whose curvature diminishes toward the edges. Accordingly, forms for *ellipsoidal* lenses have been calculated, which in theory will completely remove this species of aberration. But no curved

solids can be so accurately ground as those whose curvature is uniform in all planes, that is, the spherical. Hence, in practice it is found better to *reduce* the aberration as much as possible by spherical lenses, than to attempt an entire *removal* of it by other forms which cannot be well made.

Lenses, or combinations which are free from spherical aberration, are said to be *Aplanatic.* By lessening the aperture of a lens by a suitable diaphragm, the aberration may be much diminished.

In a plano-convex lens, whose plane surface is toward the object, the spherical aberration is 4.5 ; that is (Fig. 251), $F F' = 4.5$ times the thickness of the lens. But the same lens, with its convex side toward the object, is far better, its aberration being only 1.17. In a double convex lens of equal curvatures, the aberration is 1.67 ; if the radii of curvature are as 1 : 6, and the most convex side is toward the object, the aberration is only 1.07. By placing two plano-convex lenses near each other, the aberration may be still more reduced.

390. Atmospheric Refraction.—The atmosphere may be regarded as a transparent spherical shell, whose density increases from its upper surface to the earth. The radii of the earth produced are the perpendiculars of all the laminæ of the air ; and rays of light coming from the vacuum beyond, if oblique, are bent *gradually* toward these perpendiculars ; and therefore heavenly bodies appear more elevated than they really are. The greatest elevation by refraction takes place at the horizon, where it is about half a degree.

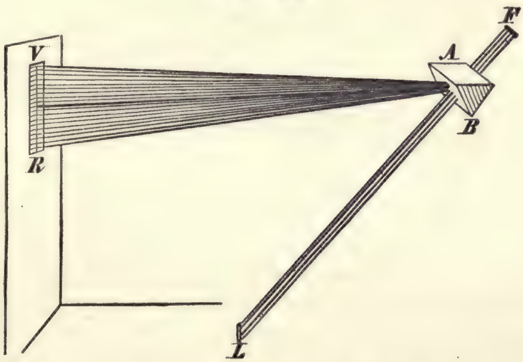
391. Mirage.—This phenomenon, called also *looming,* consists of the formation of one or more images of a distant object, caused by horizontal strata of air of very different densities. Ships at sea are sometimes seen when beyond the horizon, and their images occasionally assume distorted forms, contracted or elongated in a vertical direction. These effects are generally ascribed to *extraordinary refraction* in horizontal strata, whose difference of density is unusually great. But many cases of mirage seem to be instances of *total reflection* from a highly rarefied stratum resting on the earth. These occur frequently on extended sandy plains, as those of Egypt. When the surface becomes heated, distant villages, on more elevated ground, are seen accompanied by their images inverted below them, as in water. As the traveler advances, what appeared to be an expanse of water retires before him. By placing alcohol upon water in a glass vessel, and allowing them time to mingle a little at their common surface, the phenomena of mirage may be artificially represented.

CHAPTER IV.

DECOMPOSITION AND DISPERSION OF LIGHT.

392. The Prismatic Spectrum.—Another change which light suffers in passing into a new medium, is called *decomposition,* or the separation of light into colors. For this purpose, the glass prism is generally employed. It is so mounted on a jointed stand, that it can be placed in any desired position across the beam from the heliostat. The beam, as already noticed, is bent away from the refracting angle, both in entering and leaving the prism, and deviates several degrees from its former direction. If the light is admitted through a narrow aperture, *F* (Fig. 252), and

FIG. 252.



the axis of the prism is placed parallel to the length of the aperture, the light no longer falls, as before, in a narrow line, *L,* but is extended into a band of colors, *R V,* whose length is in a plane at right angles to the axis of the prism. This is called the prismatic spectrum. Its colors are usually regarded as seven in number—*red, orange, yellow, green, blue, indigo, violet.* The red is invariably nearest to the original direction of the beam, and the violet the most remote; and it is because the elements of white light are unequally refrangible, that they become separated, by transmission through a refracting body. The spectrum is properly regarded as consisting of innumerable shades of color. Instead of Newton's division into *seven* colors, many choose to consider all the varieties of tint as caused by the combination of *three* primitive colors, *red, yellow,* and *blue,* varying in their pro-

portions throughout the entire spectrum. The number *seven*, as perhaps any other particular number, must be regarded as arbitrary.

The spectrum contains rays of other wave lengths than those which affect the eye. The rays of longest wave length are crowded together at and beyond the red, and here the greatest heat is found upon testing with a thermometer.

The chemical or actinic rays of shortest wave lengths, are found at and beyond the violet. These invisible rays differ from those which are visible only in wave length.

Light from other sources is also susceptible of decomposition by the prism; but the spectrum, though resembling that of the sun, usually differs in the proportion of the colors.

393. The Individual Colors of the Spectrum cannot be Decomposed by Refraction.—If the spectrum formed by the prism *A* be allowed to fall on the screen *E D* (Fig. 253), and one color of it, green for example, be let through the screen, and

FIG. 253.

received on a second prism, *B*, it is still refracted as before, but all its rays remain together and of the same color. The same is true of every color of the spectrum. Therefore, so far as refrangibility is concerned, all the colors of the spectrum are alike simple.

394. Colors of the Spectrum Recombined.—It may be shown, in several ways, that if all the colors of the spectrum be combined, they will *reproduce white light.* One method is by transmitting the beam successively through two prisms whose refracting angles are on opposite sides. By the first prism, the colors are separated at a certain angle of deviation, and then fall on the second, which tends to produce the same deviation in the opposite direction, by which means all the colors are brought upon the same ground, and the illuminated spot is white as if no prism had been interposed. Or the colors may be received on a series of small plane mirrors, which admit of such adjustment as to reflect all the beams upon one spot. Or finally, the several colors can, by different methods, be passed so rapidly before the eye that their

visual impressions shall be united in one; in which case the illuminated surface appears white.

395. Complementary Colors.—If certain colors of the spectrum are combined in a compound color, and the others in another, these two are called *complementary colors,* because, when united, they will produce white. For example, if *green, blue,* and *yellow* are combined, they will produce green, differing slightly from that of the spectrum; the remaining colors, *red, orange, indigo,* and *violet,* compose a kind of purple, unlike any color of the spectrum. But these particular shades of *green* and *purple,* if mingled, will make perfectly white light, and are therefore complementary colors.

Tyndal gives these as complementary: Red and greenish blue, orange and cyanogen blue, yellow and indigo blue, greenish yellow and violet.

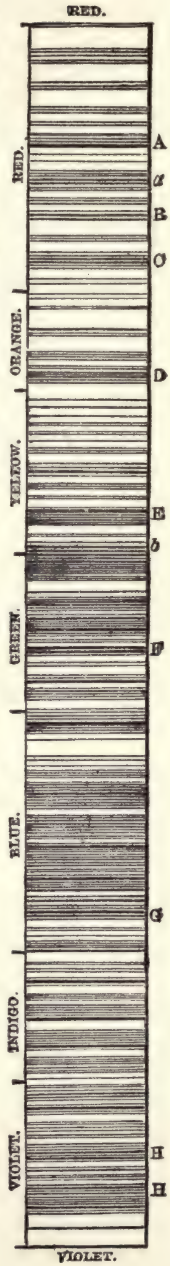
396. Natural Colors of Bodies.—The colors which bodies exhibit, when seen in ordinary white light, are owing to the fact that they decompose light by absorbing or transmitting some colors and reflecting the others. We say that a body *has* a certain color, whereas it only *reflects* that color; a flower is called red, because it reflects only or principally red light; another yellow, because it reflects yellow light, &c. A white surface is one which reflects all colors in their due proportion; and such a surface, placed in the spectrum, assumes each color perfectly, since it is capable of reflecting all. A substance which reflects no light, or but very little, is black. What peculiarity of constitution that is which causes a substance to reflect a certain color, and to absorb others is unknown.

Very few objects have a color which exactly corresponds to any color of the spectrum. This is found to result from the fact that most bodies, while they reflect some one color chiefly, reflect the others in some degree. A red flower reflects the red light abundantly, and perhaps some rays of all the other colors with the red. Hence there may be as many shades of red as there can be different proportions of other colors intermingled with it. The same is true of each color of the spectrum. Thus there is an infinite variety of tints in natural objects. These facts are readily established by using the prism to decompose the light which bodies reflect.

397. Fixed Dark Lines of the Spectrum.—Let the aperture through which the sunbeam enters be made exceedingly narrow, and let the prism be of uniform density, and then let the refracted pencil pass immediately through a small telescope, and thence into the eye, and there appears a phenomenon of great

interest—the *dark lines*, or the *Fraunhofer lines*, as they are often called from the name of their discoverer. These lines, an imperfect view of which is presented in Fig. 254, are unequal in breadth, in darkness, and in distance from each other, and so fine and crowded in many parts that the whole number cannot be counted. Fraunhofer himself described between 500 and 600, among which a few of the most prominent are marked by letters, and used in measuring refractive power. At least as many as six thousand are now known and mapped, so that any one of them may be identified. They are parallel to each other, and perpendicular to the length of the spectrum. When the pencil passes through a succession of prisms, all bending it the same way, the spectrum becomes more dilated, and more lines are seen. The instrument fitted up as above described, either with one prism or a series of prisms, is called a *spectroscope*.

Fig. 254.



398. Bright Lines in the Spectrum of Flame.—If the spectroscope be used for the examination of the flame of different substances in combustion, the spectrum is found to consist of certain detached bright bands, differing in color and number, according to the substance under examination. Thus, the spectrum of sodium flame, besides showing other fainter lines, consists mainly of two conspicuous *yellow* lines, very close together, so as ordinarily to appear as one. The flame of carbon shows two distinct lines, one of which is green, the other indigo. In this respect every substance differs from every other, and each may be as readily distinguished by the lines which compose its spectrum as by any other property. The lines of some substances are very numerous; as, for example, iron, whose spectrum lines amount to four or five hundred.

But a *solid* or *liquid* substance, when raised to a red or white heat, without passing into the gaseous state and producing flame, forms a continuous spectrum, similar to that of the sun, having neither isolated bright bands nor dark lines.

399. The Spectrum of a Heated Solid or Liquid Shining through Flame.—The condition of a spectrum is entirely changed when the light from a heated solid or liquid substance shines through the flame of a burning gas. The *bright* lines peculiar to that gas instantly become *dark* lines. The flame seems to absorb just those rays, and only those, which are like the rays emitted by itself. As an example, the spectrum of sodium flame consists of a bright double yellow line, and a few fine luminous lines of other colors. If now iron at an intense white heat shines through this flame, the whole spectrum becomes luminous, except the very lines which were before bright; these are now dark.

400. Composition of the Sun's Surface.—A great number of the dark lines of the solar spectrum are identical in position with lines in the spectrum of terrestrial substances. The spectroscope can be attached to the eye-piece of a telescope, so as to bring half the breadth of the solar spectrum side by side with half the breadth of the spectrum of the flame of some substance; and their lines can thus be compared with each other on the divisions of the same scale. When this is done, there is found, with regard to several substances, an identity of position and relative breadth and intensity so exact that it is impossible to regard the agreement as accidental. The double line D of the sunbeam, is the prominent line of sodium. So all the numerous lines of potassium, iron, and several other simple substances, exactly coincide with the dark lines of the spectrum of sunlight.

The foregoing facts seem to indicate that the photosphere of the sun consists of the flame of many substances, among which are some such as belong to the earth, namely, sodium, potassium, iron, &c.; and that the luminous liquid matter beneath the photosphere shines through it, and changes all the bright lines to dark ones.

401. Dispersion of Light.—Decomposition of light refers to the *fact* of a separation of colors; *dispersion*, rather to the *measure* or *degree* of that separation. The *dispersive power* of a medium indicates the amount of separation which it produces, compared with the amount of refraction.

The deviation of the line E is usually taken as the deviation of the beam regarded as a whole. The difference of the deviations of the lines A and H is the dispersion.

For example, if a substance, in refracting a beam of light $1° \ 51'$ from its course, separates the violet from the red by $4'$, then its dispersive power is $\frac{4}{111} = .036$. The following table

gives the dispersive power of a few substances much used in optics:

	Dispersive power.		Dispersive power.
Oil of cassia	0.139	Plate-glass	0.032
Sulphuret of carbon	0.130	Sulphuric acid	0.031
Oil of bitter almonds	0.079	Alcohol	0.029
Flint-glass	0.052	Rock-crystal	0.026
Muriatic acid	0.043	Blue sapphire	0.026
Diamond	0.038	Fluor-spar	0.022
Crown-glass	0.036		

The discovery that different substances produce different degrees of dispersion, is due to Dollond, who soon applied it to the removal of a serious difficulty in the construction of optical instruments.

402. Chromatic Aberration of Lenses.—This is a deviation of light from a focal point, occasioned by the different refrangibility of the colors. If the surface of a lens be covered, except a narrow ring near the edge, and a sunbeam be transmitted through the ring, the chromatic aberration becomes very apparent; for the most refrangible color, violet, comes to its focus nearest, and then the other colors in order, the focus of red being most remote. Since the distinctness of an image depends on the accurate meeting of rays of the same pencil in one point, it is clear that discoloration and indistinctness are caused by the separation of colors.

403. Achromatism.—In order to refract light, and still keep the colors united, it is necessary that, after the beam has been refracted, and thus separated, a substance of greater dispersive power should be used, which may bring the colors together again, by refracting the beam only a part of the distance back to its original direction. For instance, suppose two prisms, one of crown-glass and one of flint-glass, each ground to such a refracting angle as to separate the violet from the red ray by 4′. In order for this, the crown-glass, whose dispersive power is .036, must refract the beam 1° 51′; for $\frac{4'}{1^\circ\ 51'} = .036$; and the flint-glass, whose dispersive power is .052, must refract only 1° 17′; for $\frac{4'}{1^\circ\ 17'} = .052$. Place these two prisms together, base to edge, as in Fig. 255, *C* being the crown-glass and *F* the flint-glass. Then *C* will refract the beam *b b*, downward 1° 51′, and the violet, *v*, 4′ more than the red, *r*; *F* will refract this decomposed beam

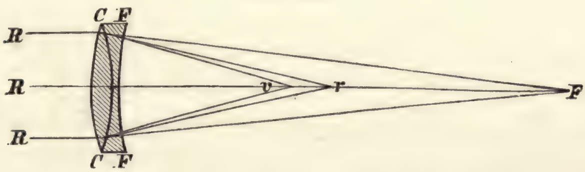
upward 1° 17′, and the violet 4′ more than the red, which will just bring them together at $v\ r$. Thus the colors are united again, and

FIG. 255.

yet the beam is refracted downward 1° 51′ — 1° 17′ = 34′, from its original direction.

404. Achromatic Lens.—If two prisms can thus produce achromatism, the same may be effected by lenses; for a convex lens of crown-glass may converge the rays of a pencil, and then a concave lens of flint-glass may diminish that convergency sufficiently to unite the colors. A lens thus constructed of two lenses of different materials and opposite curvatures, so adapted as to produce an image free from chromatic aberration, is called an *achromatic lens.* Fig. 256 shows such a combination. The con-

FIG. 256.



vex lens of crown-glass alone would gather the rays into a series of colored foci from v to r; the concave flint-glass lens refracts them partly back again, and collects all the colors at one point, F.

405. Colors not Dispersed Proportionally.—It is assumed in the foregoing discussion, that when the red and violet are united, all the intermediate colors will be united also. It is found that this is not strictly true, but that different substances separate two given colors of the spectrum by intervals which have different ratios to the whole length of the spectrum. This departure from a constant ratio in the distances of the several colors, as dispersed by different media, is called the *irrationality of dispersion.* In consequence of it there will exist some slight discoloration in the image, after uniting the extreme colors. It is found better in practice to fit the curvatures of the lenses, for uniting those rays which most powerfully affect the eye.

In a well-corrected telescope, when pointed at a bright object, such as Jupiter or the moon, a purple color will be seen when the eye-piece is pushed inwards from its position of adjustment, and a greenish color will show when the eye-piece is pulled out too far.

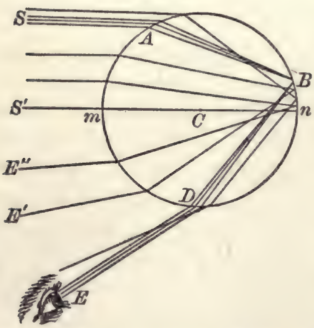
CHAPTER V.

RAINBOW AND HALO.

406. The Rainbow.—This phenomenon, when exhibited most perfectly, consists of two colored circular arches, projected on falling rain, on which the sun is shining from the opposite part of the heavens. They are called the *inner* or *primary* bow, and the *outer* or *secondary* bow. Each contains all the colors of the spectrum, arranged in contrary order; in the primary, red is outermost; in the secondary, violet is outermost. The primary bow is narrower and brighter than the secondary, and when of unusual brightness, is accompanied by *supernumerary* bows, as they are called; that is, narrow red arches just within it, or overlapping the violet; sometimes three or four supernumeraries can be traced for a short distance. The common centre of the bows is in a line drawn from the sun through the eye of the spectator.

407. Action of a Transparent Sphere on Light.—Let a hollow sphere of glass be filled with water, and cause a beam of parallel rays of homogeneous yellow light to fall upon it. To prevent confusion in Fig. 257, we will consider only those rays which fall upon the upper half of the section of the sphere, and will trace them as they emerge at the lower half. Those rays which enter near the axis $S'\,m$ will be refracted to points near n. Rays still farther from $S'\,m$ will be refracted to points still farther from n. Rays at about 59° from m, at A, will be refracted to B, and no ray, no matter where it may enter the sphere, can be refracted to a point higher than B. Now as B is the limit of the arc $n\,B$, it follows that rays close to the middle *ray* of the pencil

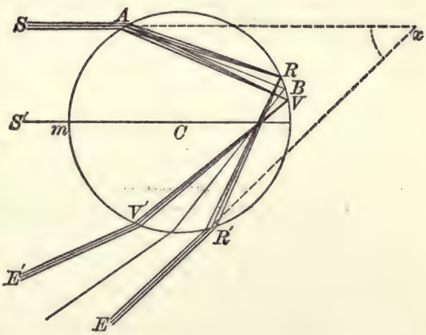
FIG. 257.



$S\ A$, both above it and below it, will be refracted to B, crowded together as it were, and after reflection a large portion will emerge at D. As on passing into the sphere at A from air, these converged to B, so on emerging the reverse action takes place at D, and we have a compact pencil of parallel rays. An eye at E would receive an impression of bright light in the direction $E\ D$; an eye below $E\ D$ would receive no light at all, and at E' or E'', while some light would be received from the diverging rays, the impression would be much less vivid than at E. We have been considering only a section of the sphere through the axis; if now we conceive this section to revolve about the axis $S'\ C$, our beam $S\ A$ becomes a hollow cylinder of light, and the emergent beam $E\ D$ becomes an emergent hollow frustum of a cone, and if the eye be placed at any element of this cone the effect will be as described for the element E.

408. The Primary Bow.—In the preceding article, homogeneous yellow light was considered. Let us examine the results when white light from the sun falls upon a rain-drop. Suppose $S\ A$ (Fig. 258) to be a beam of parallel rays from the sun incident at 59° from m. As B was the point at which yellow rays of the beam were concentrated, the red rays which are less refrangible will all concentrate at R, the distance $R\ B$ being very greatly exaggerated for the sake of clearness in the diagram. After reflection these red rays will emerge as a beam of *parallel red* rays at R'. The violet rays of the beam $S\ A$ being most refrangible will all meet at V, below B, and will emerge at V' as a beam of parallel violet rays. Between these will be beams of the intermediate colors of the spectrum. These are beams of parallel *rays*, but are not parallel *beams*, as is shown in the figure. The angle x included between the incident beam $S\ A$ and the emergent red beam produced backward to x is found by calculation to be 42° 2′, and the like angle for the violet beam is 40° 17′.

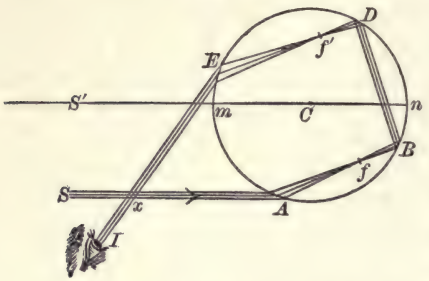
FIG. 258.



409. Course of Rays in Secondary Bow.—If we examine the conditions of two internal reflections (Fig. 259), we find that a beam of monochromatic light entering at a certain

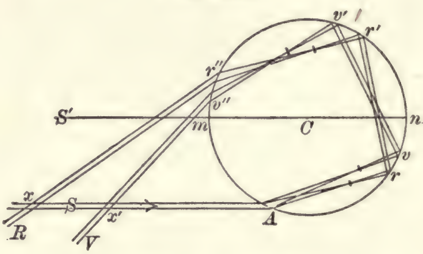
distance from the axis $S'\,n$, about 71° 42′, suffers the least deviation possible after two reflections, that is to say, the angle $S\,x\,I$ will be a minimum. Rays near this ray $S\,A$ of minimum deviation both on the side towards the axis $S'\,n$ and on the side away from it will tend to meet in a focus at f about $\frac{3}{4}$ of the distance $A\,B$, and will then be reflected parallel to D, again being reflected to a focus at f' ($E\,f' = \frac{3}{4}$ $E\,D$), and finally emerging at E, a parallel beam as on entering. An observer at I would receive an intense beam of light of the particular color used.

FIG. 259.



Now substitute for the monochromatic light, light from the sun, and the results will be as illustrated in Fig. 260, in which the difference in direction between the red and the violet rays has been greatly exaggerated. At A the red rays, following the course given in Fig. 260 are converged, cross at the focus, and at r are reflected as a parallel beam to r'; here they are again reflected to a focus, and again diverging pass on to r'', where they emerge as a parallel beam $r''\,R$. The violet rays are separated from the red at A, and being more refrangible, take the path indicated in the figure. The angle $A\,x\,r'' = 50°\ 59'$ and $A\,x'\,v'' = 54°\ 9'$. In order that the emergent pencil may enter the eye the incident ray must enter on the side of the drop nearest the observer. The rays just considered are the only ones which, after *two* reflections, emerge compact and parallel, and give bright color at a great distance.

FIG. 260.



The explanation just concluded gives a general and sufficiently exact conception of the phenomena of the primary and secondary bows.

A rigid mathematical analysis would take note of the caustics by reflection and refraction, and would vary slightly the places of maximum illumination. To such analysis, and to the principle of interference, the student must refer for an account of the super-

numerary bows accompanying the primary, which are sometimes observed.

410. Axis of the Bows.—Let *A B D G I* (Fig. 261) represent the path of the pencil of red light in the primary bow. If *A B* and *I G* are produced to meet in *K*, the angle *K* is the deviation, 42° 2′, of the incident and emergent red rays. Suppose the spectator at *I*, and let a line from the sun be drawn through his position to *T*; it is sensibly parallel to *A B*, and therefore the angles *I* and *K* are equal. As *T* is opposite to the sun, the red

Fig. 261.

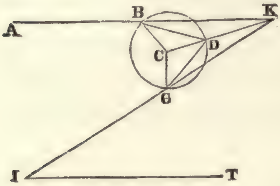
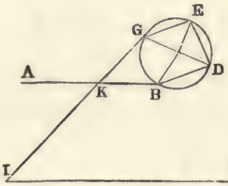


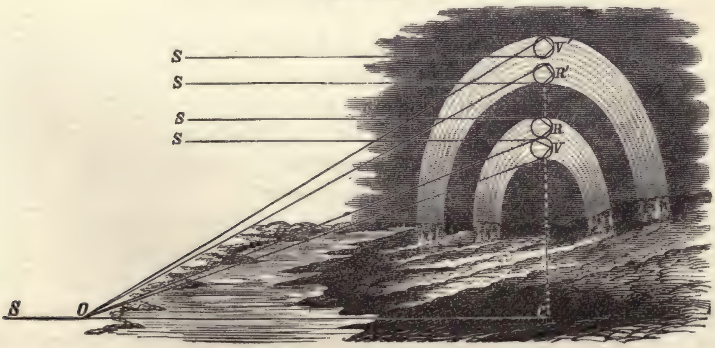
Fig. 262.



color is seen at the distance of 42° 2′, on the sky, from the point *T*; and so the angular distance of each color from *T* equals the angle which the ray of that color makes with the incident ray. In like manner, in the secondary bow, if *I T* (Fig. 262) be drawn through the sun and the eye of the observer, it is parallel to *A B*, and the angular distance of the colored ray from *T* is equal to *K*, the deviation of the incident and emergent rays. *I T* is called the *axis* of the bows, for a reason which is explained in the next article.

411. Circular Form of the Bows.—Let *S O C* (Fig. 263) be a straight line passing from the sun, through the observer's

Fig. 263.



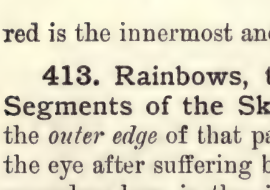
place at *O*, to the opposite point of the sky; and let *V O*, *R O* be the extreme rays, which after *one* reflection bring colors to the eye

at O, and $R'\ O$, $V'\ O$, those which exhibit colors after *two* reflections; then (according to Arts. 408, 409), $V\ O\ C = 40°\ 17'$, $R\ O\ C = 42°\ 2'$, $R'\ O\ C = 50°\ 59'$, $V'\ O\ C = 54°\ 9'$. Now, if we suppose the whole system of lines, $S\ V'\ O$, $S\ V\ O$, to revolve about $S\ O\ C$, as an axis, the relations of the rays to the drops, and to each other, will not be at all changed; and the same colors will describe the same lines, whatever positions those lines may occupy in the revolution. The emergent rays, therefore, all describe the surfaces of cones, whose common vertex is in the eye at O; and the colors, as seen on the cloud, are the circumferences of their bases.

In a given position of the observer, the extent of the arches depends on the elevation of the sun. When on the horizon, the bows are semicircles; but less as the sun is higher, because their centre is depressed as much below the horizon as the sun is elevated above it. From the top of a mountain, the bows have been seen as almost entire circles.

412. Colors of the Two Bows in Reversed Order.—Suppose the eye to receive a red ray from a drop a (Fig. 264); rays of all other colors being more refrangible than the red would pass above the eye, as does v'. In order that a violet ray may enter the eye it must proceed from a lower drop, as b, and the less refrangible rays from this drop will pass below the eye, as at R'. Hence in the primary bow the drops which send violet to the eye are nearer to the axis of the bow than those which send red, red being therefore the outermost color. A like examination of the secondary bow shows that red is the innermost and violet the outermost color.

FIG. 264.



413. Rainbows, the Colored Borders of Illuminated Segments of the Sky.—The primary bow is to be regarded as the *outer edge* of that part of the sky from which rays can come to the eye after suffering but *one* reflection in drops of rain; and the secondary bow is the *inner edge* of that part from which light, after being *twice* reflected, can reach the eye.

It is found by calculation, that in case of one reflection, the incident and emergent rays can make no inclinations with each other greater than 42° 2′ for red light, and 40° 17′ for violet; but the inclinations may be less in any degree down to 0°. Therefore, all light, once reflected, comes to the eye from *within* the primary bow.

But the angles, 50° 59′ and 54° 9′, are, by calculation, the *least*

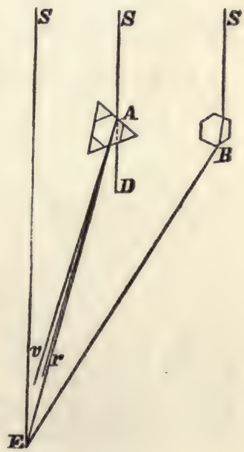
deviations of red and violet light from the incident rays after *two* reflections. But the deviations may be greater than these limits up to 180°. Therefore rays twice reflected can come to the eye from any part of the sky, except between the secondary bow and its centre.

It appears, then, that from the zone lying between the two bows, no light, reflected by drops internally, either once or twice, can possibly reach the eye. Observation confirms these statements; when the bows are bright, the rain within the primary is more luminous than elsewhere; and outside of the secondary bow, there is more illumination than between the two bows, where the cloud is perceptibly darkest.

414. The Common Halo.—This, as usually seen, is a white or colored circle of about 22° radius, formed around the sun or moon. It might, without impropriety, be termed the *frost-bow*, since it is known to be formed by light refracted by crystals of ice suspended in the air. It is formed when the sun or moon shines through an atmosphere somewhat hazy. About the sun it is a white ring, with its inner edge red, and somewhat sharply defined, while its outer edge is colorless, and gradually shades off into the light of the sky. Around the moon it differs only in showing little or no color on the inner edge.

415. How Caused.—The phenomenon is produced by light passing through crystals of ice, having sides inclined to each other at an angle of 60°. Let the eye be at E (Fig. 265), and the sun in the direction $E\ S$. Let $S\ A$, $S\ B$, &c., be rays striking upon such crystals as may happen to lie in a position to refract the light toward $S\ E$ as an axis. Each crystal turns the ray from the refracting edge on entering; and again, on leaving, it is bent still more, and the emergent pencil is decomposed. The color, which comes from each one to the eye E, depends on its angular distance from $E\ S$, and the position of its refracting angle. The angle of deviation for A is $E\ A\ D = S\ E\ A$; for B, it is $S\ E\ B$, and so on. It is found by calculation, that the least deviation for red light is 21° 45′; the least for orange must be a little greater, because it is a little more refrangible, and so on for the colors in order. The greatest deviation for the rays generally is about 43° 13′. All light, therefore, which can be transmitted

FIG. 265.



by such crystals must come to the observer from points somewhere between these two limits, 21° 45′ and 43° 13′ from the sun. But by far the greater part of it, as ascertained by calculation, passes through near the least limit.

416. Its Circular Form.—What takes place on one side of $E\,S$ may occur on every side; or, in other words, we may suppose the figure revolved about $E\,S$ as an axis, and then the transmitted light will appear in a ring about the sun S. The inner edge of the ring is red, since that color deviates least; just outside of the red the orange mingles with it; beyond that are the red, orange, and yellow combined; and so on, till, at the minimum angle for violet, all the colors will exist (though not in equal proportions), and the violet will be scarcely distinguishable from white. Beyond this narrow colored band the halo is white, growing more and more faint, so that its outer limit is not discernible at all.

417. The Halo, a Bright Border of an Illuminated Zone.—As in the rainbow, so in the halo, the visible band of colors is only the border of a large illuminated space on the sky. The ordinary halo, therefore, is the bright inner border of a zone, which is more than 20° wide. The whole zone, except the inner edge, is too faint to be generally noticed, though it is perceptibly more luminous than the space between the halo and the luminary.

418. Frequency of the Halo.—The halo is less brilliant and beautiful, but far more frequent, than the rainbow. Scarcely a week passes during the whole year in which the phenomenon does not occur. In summer the crystals are three or four miles high, above the limit of perpetual frost. As the rainbow is sometimes seen in dew-drops on the ground, so the frost-bow, just after sunrise, has been noticed in the crystals which fringe the grass.

419. The Mock Sun.—The mock sun, or sun-dog, is a short arc of the halo, occasionally seen at 22° distance, on the right and left of the sun, when near the horizon. The crystals, which are concerned in producing the mock sun, are supposed to have the form of *spiculæ*, or six-sided *needles*, whose alternate sides are inclined to each other at an angle of 60°; these falling through the air with their axes vertical, refract the light only in directions nearly horizontal, and therefore present only the right and left sides of the halo.

In high latitudes, other and complex forms of halo are frequent, depending for their formation on the prevalence of crystals of other angles than 60°. [See Appendix for calculations of the angular radius of rainbows and halo.]

CHAPTER VI.

NATURE OF LIGHT.—WAVE THEORY.

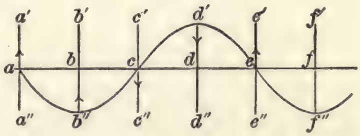
420. The Wave Theory.—Light has sometimes been regarded as consisting of *material particles* emanating from luminous bodies. But this, called the *corpuscular* or *emission theory,* has mostly yielded to the *undulatory* or *wave theory,* which supposes light to consist of vibrations in a medium. This medium, called the *luminiferous ether,* is imagined to exist throughout all space, and to be of such rarity as to pervade all other matter. It is supposed also to be elastic in a very high degree, so that undulations excited in it are transmitted with great velocity.

There is no independent evidence of the existence of this theoretical ether.

Radiant heat consists of undulations of the same ether, which differ from those of light only in being slower. For it is a familiar fact, that when the heat of a body is increased to about 500° or 600° C. the body becomes luminous, and the brightness increases as the temperature is raised.

421. Nature of the Wave.—Suppose a number of ether molecules, as *a, b, c,* &c. (Fig. 266), to be equidistant upon the straight line *a f.* Now conceive that *a* moves to *a'*, then back to *a''*, thence to *a* again, occupying four equal intervals of time in the circuit. Suppose *b* to start on the same round, at a time one interval later; when *a* reaches its original position, and is just beginning its upward motion, as in the figure, *b* will be at *b''* moving towards *b*. In like manner, starting *c* and *d* at intervals later by one, we shall find their positions and directions of motion, when *a* begins its second circuit, to be as given in the figure. The motion will have been transmitted to *e,* which will begin its first circuit at the instant that *a* starts upon its second. Molecules which like *a* and *e* are in the same condition as to *place* and *direction of motion,* are said to be in the same phase. A wave length is the distance between two *consecutive* like phases. The amplitude of vibration is the distance between the two limits of the excursion of the particle. It is evident from the figure that the motion is communicated

FIG. 266.



along the axis *a f* a distance *a e*, or one wave length, during the time of one vibration.

422. Postulates of the Wave Theory.—

1. *The waves are propagated through the ether at the rate of 186,300 miles per second.*

As this is the known velocity of light, it must be the rate at which the waves are transmitted.

2. *The atoms of the ether vibrate at right angles to the line of the ray in all possible directions.*

It was at first assumed that the luminous vibrations, like the vibrations of sound, are *longitudinal*, that is, back and forth in the line of the ray; but the discoveries in polarization require that the vibrations of light should be assumed to be *transverse*, that is, in a plane perpendicular to the line of the ray; and, moreover, that in that plane the vibrations are in every possible direction within an inconceivably short space of time. Thus, if a person is looking at a star in the zenith, we must consider each atom of the ether between the star and his eye as vibrating across the vertical in all horizontal directions, north and south, east and west, and in innumerable lines between these.

3. *Different colors are caused by different rates of vibration.*

Red is caused by the *slowest* vibrations, and *violet* by the *quickest*, and other colors by intermediate rates. White light is to the eye what harmony is to the ear, the resultant effect of several rates of vibration combined. There are slower vibrations of the ether than those of red light, and quicker ones than those of violet light, but they are not adapted to affect the vision. The former affect the sense of feeling as *heat*, the latter produce chemical effects, and are called *actinic* rays.

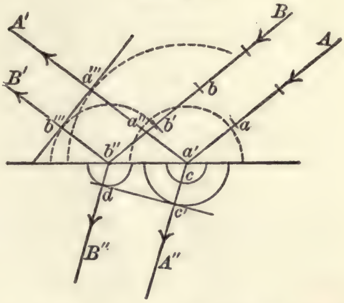
4. *The ether within bodies is less elastic than in free space.*

This is inferred from the fact that light moves with less velocity in passing through bodies than in free space; the greater the refractive power of a body, the slower does light move within it. And in some bodies of crystalline structure, it happens that the velocity is different in different directions, so that the elasticity of the ether within them must be regarded as varying with the direction.

423. Reflection and Refraction according to the Wave Theory.—The vibrations of the ether are transmitted from the source of motion as spherical waves. In a luminous body are an infinite number of radiants, each the centre of a succession of spherical waves. A beam of parallel rays is a collection of parallel radii of spherical waves, having different centres of disturbance, and the *wave front* of such a beam is the *tangent plane* common to all the spheres.

Suppose A and B to be two parallel rays of a beam of light. Let a and b (Fig. 267) represent two like wave fronts, and $a\ a' = b\ b'$ be the distance light moves in any small unit of time. When the wave a reaches a', b will have reached b'. While b' moves to b'', a' regarded as a centre of disturbance will have sent out a spherical wave to a''. While b'' is transmitting a spherical wave to b''', a'' will have extended to a''', and the common tangent plane $b'''\ a'''$ will be the wave front, and $A'\ B'$, the reflected rays, represent the beam. All rays from a' and b'' which move obliquely with respect to each other are destroyed by their mutual reactions, only those remaining which move in parallel directions, as $A'\ B'$.

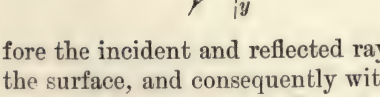
FIG. 267.



While b' moves to b'', a' is sending a wave into the medium at a slower rate, suppose with only half the velocity, which has advanced to c by the time b' reaches b''. While b'' is moving into the medium to d, c has moved to c', and a common tangent $d\ c'$ is the front of the refracted beam, of which $A''\ B''$ are rays.

424. Relation of Angles of Incidence, Reflection, and Refraction.—The velocity of light in any medium being uniform, retaining the notation of Fig. 267 we have in the triangles $a'\ b'\ b''$ and $a'\ a''\ b''$ (Fig. 268) $a'\ a'' = b'\ b''$, $a'\ b''$ common, and the angles at a'' and b' equal, being right angles formed by the radii and tangents; hence the angles $a''\ a'\ b''$ and $b'\ b''\ a'$ are equal; therefore the incident and reflected rays *must* make equal angles with the surface, and consequently with the normal.

FIG. 268.



In the triangle $a'\ b'\ b''$ we have $b'\ b'' : a'\ b'' :: \text{sine } b'\ a'\ b'' : \text{sine } 90°$; and from $a'\ b''\ c$ we have

$$a'\ c : a'\ b'' :: \text{sine } a'\ b''\ c : \text{sine } 90°;$$

combining these, $b'\ b'' : a'\ c :: \text{sine } b'\ a'\ b'' : \text{sine } a'\ b''\ c$; or,

$$\frac{b'\ b''}{a'\ c} = \frac{\text{sine } b'\ a'\ b''}{\text{sine } a'\ b''\ c} = \frac{\text{sine } x\ a'\ a}{\text{sine } c\ a'\ y} = \frac{\text{sine ang. Inc.}}{\text{sine ang. Refrac.}}.$$

But $b'\ b''$ and $a'\ c$ are spaces through which the wave is propa-

gated in the same interval of time, and as the velocities are constant in each medium, the ratio $\frac{b'\ b''}{a'\ c}$ must be a constant ratio; therefore its equal ratio $\frac{\text{sine ang. Inc.}}{\text{sine ang. Refrac.}}$ must also be a constant ratio.

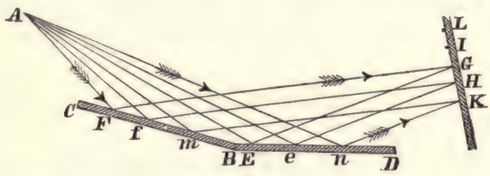
In the above it has been assumed that the velocity of transmission in the denser medium is less than in the rarer; this assumption is verified by direct experiment.

425. Interference.—As two systems of water-waves may increase or diminish their height by being combined, and as sounds, when blended, may produce various results, and even destroy each other, so may two pencils of light either augment or diminish each other's brightness, and even produce darkness.

If unlike phases coincide, the vibrations are destroyed and darkness follows, while if like phases meet, increase of brightness results.

To illustrate this, let two plane reflectors, inclined at a very obtuse angle, nearly 180°, receive light from a minute radiant, and reflect it to one spot on a screen; the reflected pencils will interfere, and produce bright and dark lines. Suppose light of one color, as violet, flows from a radiant point A (Fig. 269); let mirrors $B\ C$ and $B\ D$ reflect it to the screen $K\ L$. F and E may be so selected that the ray $A\ F + F\ G$ equals the ray $A\ E + E\ G$.

Fig. 269.



Then G will be luminous, because the two paths being equal, the same phase of wave in each ray will occur at the point G. But if H be so situated that $A\ f + f\ H$ differs *half a violet wave* from $A\ e + e\ H$, then H will be a dark point, because opposite phases meet there. A similar point, I, will lie on the other side of G. Again, there are two points, K and L, one on each side of G, to each of which the whole path of light by one mirror will exceed the whole by the other by just *one* violet wave; those points are bright.

If the paths differ by any odd multiple of $\frac{1}{2}$ wave length, light is destroyed and a dark band is seen; but if these paths differ by

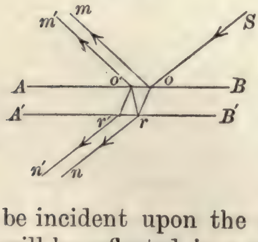
any multiple of a whole wave length, the light is intensified and bright bands are seen.

Thus, there is a series of bright and dark points on the screen; or rather a series of bright and dark hyperbolic *lines*, of which these points are sections. Other colors will give bands separated a little further, indicating longer waves. And white light, producing all these results at once, will give a repetition of the prismatic series.

426. Striated Surfaces.—If the surface of any substance is ruled with fine parallel grooves, 2000 or more to the inch, these grooves will act like the inclined mirrors of the last paragraph, and it will reflect bright colors when placed in the sunbeam. *Mother-of-pearl* and many kinds of sea-shell exhibit colors on account of delicate striæ on their surface. It may be known that the color arises from such a cause, if, when the substance is impressed on fine cement, its colors are communicated to the cement. Indeed, it was in this way that Dr. Wollaston accidentally discovered the true cause of such colors. The changeable hues in the plumage of some birds, and the wings of some insects, are owing to a striated structure of their surfaces. But the metals can be made to furnish the most brilliant spectra, by stamping them with steel dies, which have been first ruled by a diamond with lines from 2000 to 10,000 per inch, and then hardened. Gilt buttons and other articles for dress are sometimes prepared in this manner, and are called *iris ornaments*. The color in a given case depends on the distance between the grooves, and the obliquity of the beam of light. Hence, the same surface, uniformly striated, may reflect all the colors, and every color many times, by a mere change in its inclination to the beam of light.

427. Thin Laminæ.—Any transparent substance, when reduced in thickness to a few millionths of an inch, reflects brilliant colors, which vary with every change of thickness. Examples are seen in the thin laminæ of air occupying cracks in glass and ice, and the interstices between plates of mica, also in thin films of oil on water, and alcohol on glass, but most remarkably in soapy water blown into very thin bubbles.

Fig. 270.



Let a beam $S\,o$ of red light (Fig. 270) be incident upon the first surface $A\ B$ of the thin plate; a part will be reflected in $o\ m$ while another part will be refracted to r in the second surface $A'\ B'$; this refracted beam will again divide, a portion being reflected to o' and then refracted in $o'\ m'$; $o\ m$

and $o' m'$ are parts of the same beam and will reinforce each other, or will interfere and destroy each other, according to the phases which are superimposed. At each internal reflection, owing to the change in density and elasticity at the surfaces of contact of the medium and air, there is a loss or retardation equal to one half a wave length.

Calling the part of the path $o\ r\ o'$, included between the surfaces, $2\ t$, then whenever $2\ t$ is a multiple, $x\ l$, of a whole wave length, the difference of the paths $S\ o\ m$ and $S\ o\ r\ o'\ m'$ will be $x\ l + \frac{l}{2}$, $\frac{l}{2}$ being the retardation spoken of above, and the interference results in a dark band. When $2\ t$ is an odd multiple of one-half a wave length, suppose $\frac{3\ l}{2}$, then $\frac{3\ l}{2} + \frac{l}{2} = 2\ l$, and therefore $o\ m$ and $o'\ m'$ will reinforce each other, producing a light band.

If we examine the transmitted rays $S\ n$ and $S\ r\ o'\ n'$, the latter having been twice reflected internally, we find the difference of path to be $2\ t$, and as two half-wave lengths must be added to this because of the two internal reflections we have for the retardation of $r'\ n'$, as compared with $r\ n$, $2\ t + l$. When $2\ t$ is a multiple of a whole wave length, there will be no interference, and when $2\ t$ is an odd multiple of one-half a wave length there will be interference, results the opposite of those found for reflected light; hence when a thin plate shows no light by reflection it shows bright light by transmission.

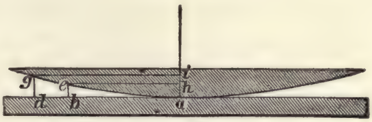
If we use a film of air of varying thickness, and the light of the sun, various colors will be produced.

If a lens of slight convexity is laid on a plane lens, and the two are pressed together by a screw, and viewed by reflected light, rings of color are seen arranged around the point of contact. The rings of least diameter are broadest and most brilliant, and each one contains the colors of the spectrum in their order, from violet on the inner edge to red on the outer. But the larger rings not only become narrower and paler, but contain fewer colors; yet the succession is always in the same order as above. Increased pressure causes the rings to dilate, while new ones start up at the centre, and enlarge also, until the centre becomes black, after which no new rings are formed. These are commonly called Newton's rings, because Sir Isaac Newton first investigated their phenomena.

428. Ratio of Thicknesses for Successive Rings.— A given color appears in a circle around the point of contact, because equal thicknesses are thus arranged. If the diameters of the suc-

cessive rings of any one color be carefully measured, their squares are found to be as the odd numbers, 1, 3, 5, 7; and hence the thicknesses of the laminæ of air at the repetitions of the same color are as the same numbers. For, let Fig. 271 represent a section of the spherical and plane surfaces in contact at *a*. Let *a b*, *a d*, be the radii of two rings at their brightest points. Suppose *a i*, perpendicular to *m n*, to be produced till it meets the opposite point of the circle of which *a g* is an arc, and call that point *f*; then *a f* is the diameter of the sphere of which the lens is a segment. Let *b e*, *d g*, be parallel to *a i*, and *e h*, *g i*, to *m n*, then we have

FIG. 271.



$$(e\,h)^2 : (g\,i)^2 :: a\,h \times h\,f : a\,i \times i\,f.$$

But the distances between the two lenses being exceedingly small in comparison with the diameter of the sphere, *h f* and *i f* may be taken as equal to *a f*; whence, by substitution,

$$(e\,h)^2 : (g\,i)^2 :: a\,h \times a\,f : a\,i \times a\,f :: a\,h : a\,i :: b\,e : d\,g.$$

Therefore the thicknesses of successive rings are as the odd numbers.

429. Thickness of Laminæ for Newton's Rings.—The absolute thickness, *b e*, *d g*, &c., can also be obtained, *a f* being known, since

$$a\,f : a\,e :: a\,e : a\,h \text{ or } b\,e;$$

for in so short arcs the chord may be considered equal to the sine, that is, the radius of the ring.

With air between the lenses Newton found the thickness of the first bright ring of orange-yellow light, to be $\frac{1}{178000}$ of an inch. Twice this, plus $\frac{1}{2}$ a wave length, must equal one whole wave length, or

$$\frac{2}{178000} + \frac{l}{2} = l, \text{ whence}$$

$$l = \frac{4}{178000} = .000022.$$

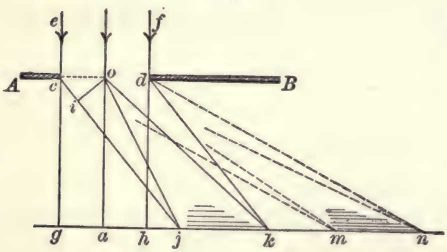
When air is between the lenses, all the rings range between the thickness of *half a millionth* of an inch and 72 *millionths*; if water is used, the limits are $\frac{3}{8}$ *of a millionth* and 58 *millionths*. Below the smaller limit the medium appears black, or no color is reflected; above the highest limit the medium appears white, all colors being reflected together. When water is substituted for air, all the rings contract in diameter, indicating that a particular order of color requires less thickness of water than of air; the thicknesses for different media are found to be in the inverse ratio of the indices of refraction.

430. Relation of Rings by Reflection and by Transmission.—If the eye is placed beyond the lenses, the transmitted light also is seen to be arranged in very faint rings, the brightest portions being at the same thicknesses as the darkest ones by reflection; and these thicknesses are as the even numbers, 2, 4, 6, &c. The centre, when black by reflection, is white by transmission, and where red appears on one side, blue is seen on the other; and, in like manner, each color by reflection answers to its *complementary color* by transmission, according to Article 427.

431. Newton's Rings by a Monochromatic Lamp.—The number of reflected rings seen in common light is not usually greater than from *five* to *ten*. The number is thus small, because as the outer rings grow narrower by a more rapid separation of the surfaces, the different colors overlap each other, and produce whiteness. But if a light of only one color falls on the lenses, the number may be multiplied to several hundreds; the rings are alternately of that color and black, growing more and more narrow at greater distances, till they can be traced only by a microscope. A good light for such a purpose is the flame of an alcohol lamp, whose wick has been soaked in strong brine, and dried.

432. Diffraction or Inflection.—One of the forms of inflection is explained as follows: Through an opaque screen, *A B* (Fig. 272), let there be a very narrow aperture, *c d*, by which is admitted the beam of red light, *e f g h*, emanating from a single point.

FIG. 272.



All points of the wave front *c d* are radiants, from which emanate waves in all directions. The original wave will move on through the aperture forming the band of light *g a h*. A ray from *c* as a radiant will interfere with that ray from *o*, which is one-half wave length in advance of it, and will give darkness at some point *j*; all rays parallel to *c j* emanating from points between *c* and *o*, will interfere with corresponding rays from points between *o* and *d*, parallel to *o j*, and the total result will be the dark band *j k*.

In like manner rays from those points which differ by $\frac{3}{2}$ wave lengths will interfere and produce the second dark band *m n*. Between these will be a bright band, *k m*, due to waves which differ by one whole wave length. The obliquity of the rays in the

figure has been very greatly exaggerated, that the lines may not be confused. The general plan of the determination of the wave length of the color used may be made plain by reference to the figure. Let $o\ i$ be drawn perpendicular to $c\ j$. Then in the triangle $c\ o\ i$ we have $c\ i = c\ o \times \cos o\ c\ i = \frac{1}{2}\ c\ d \times \text{sine}\ j\ c\ g$.

Now, because the screen is at a great distance from the slit as compared with $c\ d$, which is about $\frac{1}{25}$ of an inch, and as $g\ c\ j$ is very small indeed, about 1.5′, we have $j\ o = j\ i$, and hence $c\ j - o\ j = c\ j - i\ j = c\ i$; therefore, in order to find $c\ i$ ($= \frac{1}{2}$ a wave length) we must measure $c\ d$ and the angular deviation $g\ c\ j$. Instead of a single aperture, $c\ d$, a great number of very fine parallel lines ruled on glass are used, and details of measurement are adopted, a description of which is beyond our limits. With white light, prismatic fringes would be produced.

433. Inflection by One Edge of an Opaque Body.—If one side of the aperture $g\ h$ in the last paragraph be removed, the effect, while due to the same cause, will be somewhat modified. Let a convex lens converge sunlight to a focus from which it again diverges, the room being dark. If we introduce into the divergent pencil any opaque body, as a knife-blade, for example, and observe the shadow which it casts on a white screen, we shall observe on both sides of the shadow *fringes of colored light*, the different colors succeeding each other in the order of the spectrum, from violet to red. Three or four series can usually be discerned, the one nearest to the shadow being the most complete and distinct, and the remoter ones having fewer and fainter colors. The phenomenon is independent of the density or thickness of the body which casts the shadow. The light, in passing by the edge or back of a knife, by a block of marble or a bubble of air in glass, is in each case affected in the same way. But if the body is very narrow, as, for example, a fine wire, a modification arises from the light which passes the opposite side; for now fringes appear *within* the shadow, and at a certain distance of the screen the central line of the shadow is the most luminous part of it.

If light of one color be used, and the distance of the color from the edge of the shadow be measured when the screen is placed at different distances from the body, it will be found that the distances from the shadow are not proportional to the distances of the screen from the body; which proves that the color is not propagated in a straight line, but in a curve. These curves are found to be *hyperbolas*, having their concavity on the side next the shadow, and are in fact a species of caustics.

434. Light through Small Apertures.—The phenomena of inflection are exhibited in a more interesting manner when we

view with a magnifying glass a pencil of light after it has passed through a small aperture. For instance, in the cone already described as radiating from the focus of a lens in a dark room, let a plate of lead be interposed, having a pin-hole pierced through it, and let the slender pencil of light which passes through the pin-hole fall on the magnifier. The aperture will be seen as a luminous circle surrounded by several rings, each consisting of a prismatic series. These are, in truth, the fringes formed by the edge of the circular puncture, but they are modified by the circumstance that the opposite edges are so near to each other. If, now, the plate be removed, and another interposed having *two* pin-holes, within one-eighth of an inch of each other, besides the colored rings round each, there is the additional phenomenon of long lines crossing the space between the apertures; the lines are nearly straight, and alternately luminous and dark, and varying in color, according to their distance from the central one. These lines are wholly due to the overlapping of two pencils of light, for on covering one of the apertures they entirely disappear. By combining circular apertures and narrow slits in various patterns in the screen of lead, very brilliant and beautiful effects are produced.

435. Why Inflection is not Always Noticed in Looking by the Edges of Bodies.—It must be understood that light is *always* inflected when it passes by the edges of bodies; but that it is rarely observed, because, as light comes from various sources at once, the colors of each pencil are overlapped and reduced to whiteness by those of all the others. By using care to admit into the eye only isolated pencils of light, some cases of inflection may be observed which require no apparatus. If a person standing at some distance from a window holds close to his eye a book or other object having a straight edge, and passes it along so as to come into apparent coincidence with the sash-bars of the window, he will notice, when the edge of the book and the bar are very nearly in a range, that the latter is bordered with colors, the violet extremity of the spectrum being on the side of the bar nearest to the book, and the red extremity on the other side. Again, the effect produced when light passes through a narrow aperture may be seen by looking at a distant lamp through the space between the bars of a pocket-rule, or between any two straight edges brought almost into contact. On each side of the lamp are seen several images of it, growing fainter with increased distance, and finely colored. An experiment still more interesting is to look at a distant lamp through the net-work of a bird's feather. There are several series of colored images, having a fixed arrange-

ment in relation to the disposition of the minute apertures in the feather; for the system of images revolves just as the feather itself is revolved.

436. Length and Number of Luminous Waves.—The careful measurements which have been made in cases of interference have led, by many independent methods, to the accurate determination of the length of a wave of each color. When the length of a wave of any color is known, the number of vibrations per second is readily obtained by dividing the velocity of light by the length of the wave, for light moves a distance equal to one wave length during one vibration (Art. 421); therefore if we divide the distance per second, 186,300 miles, by the length of one wave, we have the number of vibrations per second as above.

The results of these investigations give for the

	Wave Length in Inches.	Number of Vibrations per Second.
Red...........................	.0000256	461,000,000,000,000
Violet........................	.0000174	678,000,000,000,000
Mean	.0000225	525,000,000,000,000

437. Calorescence and Fluorescence. — Rays of less refrangibility than the extreme red are due to vibrations too slow to effect vision, but by their great number they possess great heating power. If a beam of light be allowed to fall upon a thin layer of a solution of iodine contained in a suitable cell, all light rays will be absorbed, and nearly all the rays of slow vibration will pass through. These, if brought to a focus, will communicate to refractory substances vibrations sufficiently rapid to be recognized by the organ of vision, the substance becoming heated to whiteness.

Other rays, of vibrations too rapid to be recognized as light, are also found in the spectrum far beyond the violet. If these are allowed to fall upon a solution of sulphate of quinine, or upon paper impregnated with æsculine, or upon other substances capable of reducing the rate of vibration, these substances become visible, glowing with a color peculiar to each solution, and determined by the rate of vibration which has resulted. This property of substances by which the ultra violet rays are made visible is called Fluorescence.

438. Phosphorescence.—Very many substances, if they are exposed to a strong light and then are transferred to a dark chamber, continue to emit light for longer or shorter periods, depending upon the substance used. The sulphides of calcium

and strontium remain luminous for hours after exposure to sunlight.

Many other substances possess the property of phosphorescence in so slight a degree, that they will emit light for only a fraction of a second after being withdrawn from the sun's rays, while many seem not to possess it at all.

What is called phosphorescence in certain animals and in decaying animal and vegetable substances has no relation to that just described, and can not properly be considered here.

CHAPTER VII.

DOUBLE REFRACTION AND POLARIZATION.

439. Change of Vibrations in Polarized Light.—It has been stated (Art. 422) that the vibrations of the ether in the case of common light, must be supposed to be *transverse in all directions.* But, instead of this, we may conceive, what is mechanically equivalent to it, that the vibrations are made in *two* transverse directions at right angles to each other, and to the direction of the ray.

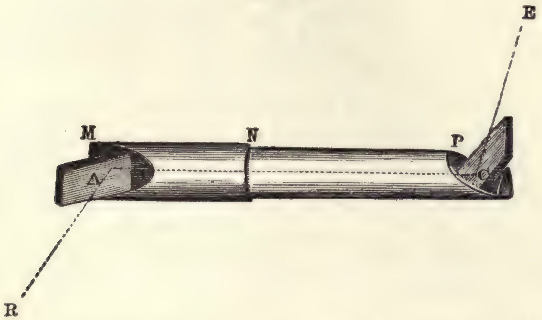
This being the nature of common light, it is easy to state what is meant by polarized light. It is that in which the vibrations are performed in only *one* of the transverse directions. It is, of course, immaterial what particular transverse motion is cut off, provided all the motion at right angles to it is retained.

440. Polarizing and Analyzing by Reflection.—When light is reflected, those vibrations of the ray which are *in* the plane of incidence are generally weakened in a greater or less degree, while those which are *perpendicular* to the same plane are not affected. How much the vibrations are weakened depends on the elasticity of the ether within the medium, and on the angle of incidence. But reflection of light rarely if ever takes place without diminishing the amplitude of those vibrations which are in the plane of incidence; so that a reflected ray is always polarized, at least, in a slight degree.

441. Polarization by Reflection.—Let two tubes, $M N$ and $N P$ (Fig. 273) be fitted together in such a manner that one can be revolved upon the other; and to the end of each let there be attached a plate of dark-colored glass, A and C, capable of reflect-

ing only from the first surface. These plates are hinged so as to be adjusted at any angle with the axis of the tube. Let the plane of each glass incline to the axis of the tube at an angle of 33°, and let the beam *R A* make an incidence of 57°, the complement

FIG. 273.



of 33°, on *A*; then it will, after reflection, pass along the axis of the tube, and make the same angle of incidence on *C*. If now the tube *N P* be revolved, the second reflected ray will vary its intensity, according to the angle between the two planes of incidence on *A* and *C*. The beam *A C* is *polarized light;* the glass *A*, which has produced the polarization, is called the *polarizing plate;* the glass *C*, which shows, by the effects of its revolution, that *A C* is polarized, is *the analyzing plate;* and the whole instrument, constructed as here represented, or in any other manner for the same purpose, is called a *polariscope.*

442. Changes of Intensity Described.—The changes in the ray *C E* are as follows:

Since all vibrations except those perpendicular to the plane of incidence have been destroyed, when the tube *N P* is placed so that the plane of incidence on *C* is coincident with the former plane of incidence, *R A C*, whether *C E* is reflected forward or backward in that plane, the intensity at *E* will be the same as if *A C* had been a beam of common light. If *N P* is revolved, *E* will begin to grow fainter, and reach its minimum of intensity when the planes *R A C* and *A C E* are at right angles, which is the position indicated in the figure; for only vibrations at right angles to the plane of incidence can be reflected, and there are no such vibrations with reference to this second plane of incidence.

Continuing the revolution, we find the intensity increasing through the second quadrant of revolution, and reaching its maximum, when the two planes of incidence again coincide, 180° from the first position.

No reflection polarizes perfectly, and hence there will be increase and decrease of the intensity of the reflected ray, without total extinction.

443. The Polarizing Angle.—The angle of 57° is called the polarizing angle for glass, not because glass will not polarize at other angles of incidence, but because at all other angles it polarizes the light in a less degree; and this is indicated by the fact that, in revolving the analyzing plate, there is less change of intensity, and the light at *E* does not become so faint. Different substances have different polarizing angles, and for that angle of incidence for any substance which will produce a maximum of polarization, the *reflected* and *refracted* rays will make with each other an angle of 90°. Hence the refractive power of opaque bodies may be determined. The polarization produced by reflection from the metals is very slight.

444. Polarization by a Bundle of Plates. — Light may also be polarized by *transmission* through a bundle of laminæ of a transparent substance, at an angle of incidence equal to its polarizing angle.

Since the *reflected* ray in perfectly polarized light consists of vibrations only at right angles to the plane of incidence, the *transmitted* light, being deprived of these vibrations, will consist of vibrations only *in* the plane of incidence and refraction. As no single reflection perfectly sifts out the vibrations at right angles to the planes of incidence and reflection, many reflections at successive surfaces are secured, so that finally there will remain in the *refracted* ray only vibrations in the plane of incidence and refraction.

Let a pile of twenty or thirty plates of transparent glass, no matter how thin, be placed in the same position as the reflector *A*, in Fig. 273, and a beam of light be transmitted through them in a direction toward *C*. In entering and leaving the bundle *A*, situated as in the figure, the angles of incidence and refraction are in a horizontal plane. When *C* is revolved, the beam undergoes the same changes as before, with this difference, that the places of greatest and least intensity will be reversed. If the light is reflected from *C* in the same plane in which it was refracted by *A*, its intensity is least, and it is greatest when reflected in a plane at right angles to it, as at *E* in the figure.

445. Polarization by Absorption.—The third and most perfect method of polarizing light, is by *transmission through certain crystals*. Some crystals polarize the transmitted light by *absorption*. If a thin plate be cut from a crystal of tour-

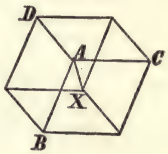
maline, by planes parallel to its axis, the beam transmitted through it is polarized, the vibrations parallel to the axis being transmitted and those perpendicular to the axis being absorbed, and, when received on the analyzing plate, will alternately become bright and faint, as the tube of the analyzer is revolved.

If the analyzer be a plate of tourmaline similar to the polarizer the rays will pass when the axes of the plates are parallel, but will be wholly absorbed when the axes are at right angles to each other.

446. Double Refraction.—There are many transparent substances, particularly those of a crystalline structure, which, instead of refracting a beam of light in the ordinary mode, *divide it into two beams.* This effect is called *double refraction,* and substances which produce it are called *doubly-refracting substances.*

This phenomenon was first observed in a crystal of carbonate of lime, denominated *Iceland spar.* It is bounded by six rhomboidal faces, whose inclinations to each other are either 105° 5′, or 74° 55′. There are two opposite solid angles, A and X (Fig. 274), each of which is formed by the meeting of three obtuse plane angles. A line drawn in such direction as to be equally inclined to the three edges of either of these obtuse solid angles, is called the axis of the crystal.

FIG. 274.

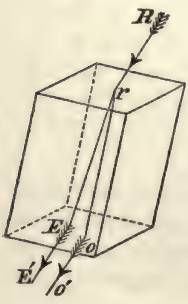


If the edges of the crystal were equal, the axis would be the diagonal $A\ X$ of the rhomb.

If a thick crystal of spar be laid on a line of writing, it appears as *two* lines, one of which seems not only thrown aside from the other, but brought a little nearer to the eye; and if the crystal be revolved in the plane of the paper, and the eye be placed vertically over it, one image of a letter will be seen to remain stationary while the other will revolve about it.

Therefore every ray of light, in passing through, is divided into two rays, which come to the eye in different directions. The double refraction may also be seen by letting a very slender sunbeam, $R\ r$ (Fig. 275), fall on the crystal; as it enters it takes two directions, $r\ O$, and $r\ E$, which on passing out describe the lines $O\ O'$, $E\ E'$, parallel to the incident beam, $R\ r$. One of these rays, $O\ O'$, is called the *ordinary* ray, because it is always refracted according to the ordinary law of refraction (Art. 369); that is, it remains in the plane of incidence, and the sines of incidence and refraction have a constant ratio to each other at all inclinations. The other, $E\ E'$, is called the

FIG. 275.



extraordinary ray, because in some positions it departs from this law of refraction in one or both particulars.

In the experiment above, the *ordinary* image seems nearest to the eye, and is stationary, when viewed vertically, while the crystal revolves.

The property of double refraction belongs to a large number of crystals, and also to some animal substances, as hair, quills, &c.; and it may be produced artificially in glass by heat or pressure.

447. Optical Relations of the Axis.—The axis of Iceland spar has been defined with reference to *form;* but it is also the axis with respect to its optical relations, for in the direction of that line a ray is never doubly refracted, while it is doubly refracted in all other directions.

Every plane which includes the axis of a crystal is called a *principal section.* In every principal section the extraordinary ray conforms to one part of the law of refraction, but not to the other; it remains in the plane of incidence, but does not preserve a constant ratio of sines at different inclinations.

In a plane at right angles to the axis, the extraordinary ray conforms to both parts of the law; but in all planes besides this and the principal sections, it conforms to neither part.

Crystals of a *positive* axis, are those in which the extraordinary ray has a *larger* index of refraction than the ordinary ray; crystals of a *negative* axis are those in which the index of the extraordinary ray is less than that of the ordinary ray. Iceland spar is a crystal of negative axis.

Some crystals have two axes of double refraction; that is, there are two directions in which light may be transmitted without being doubly refracted. A few crystals have more than two axes.

448. Polarizing by Double Refraction.—In doubly-refracting crystals, the ether possesses different degrees of elasticity in different directions; hence, so far as vibrations lie in *one* plane, they may be more retarded in their progress, and in a plane at right angles to that they may be less retarded, and the degree of refraction depends on the amount of retardation (Art. 424). Thus the two systems become separated, and emerge at different places.

If a beam is passed through a doubly-refracting crystal, and the two parts fall on the analyzing plate, they will come to their points of greatest and least brightness at alternate quadrants; indeed, when one ray is brightest, the other is entirely extin-

guished. Therefore the two rays which emerge from a doubly-refracting crystal are polarized *completely*, the ordinary ray in a principal plane and the extraordinary ray in a plane at right angles to a principal plane.

449. Different Kinds of Polarization.—Since the discovery was made that the ethereal atoms may by certain methods be thrown into circular movements, and by others into vibrations in an ellipse with the axis in a fixed direction, the polarization already described has been called *plane polarization*, since the atoms vibrate in a plane. *Circular polarization* is that in which the atoms revolve in circles; and *elliptical polarization* denotes a state of vibration in ellipses, whose major axes are confined to one plane.

The consideration of these various modes of polarization demands more space than can be spared here.

450. Every Polarizer an Analyzer.—We have seen that light is polarized by reflection from glass at an incidence of 57°, and analyzed by another plate at the same angle of incidence. This is but an instance of what is always true, that every method of polarizing light may be used to analyze, i. e., to test its polarization. Hence, a bundle of thin plates of glass may take the place of the analyzer *C*, as well as of the polarizer *A*. For, on turning it round, though the transmitted beam remains in the same place, yet it will, at the alternate quadrants, brighten to its maximum and fade to its minimum of intensity.

So, again, if light has passed through a tourmaline, and is received on a second whose crystalline axis is parallel to that of the former, the ray will proceed through that also; but if the second is turned in its own plane, the transmitted ray grows faint, and nearly disappears at the moment when the two axes are at 90° of inclination, and this alternation continues at each 90° of the whole revolution.

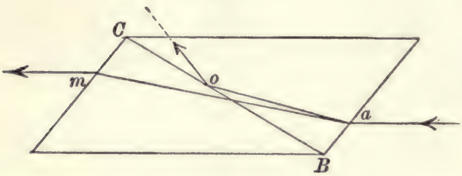
Finally, place a double-refractor at each end of the polariscope, and let a beam pass through them and fall on a screen. The first crystal will polarize each ray, and the second will doubly refract and also analyze each, exhibiting a very interesting series of changes. In general, four rays will emerge from the second crystal, producing four luminous spots on the screen. But, on revolving the tube, not only do the rays commence a revolution round each other, but two of them increase in brightness, and the other two at the same time diminish as fast, till two alone are visible, at their greatest intensity. At the end of the second quadrant, the spots before invisible are at their maximum of brightness, and the others are extinguished. This alternation

continues as long as the crystal is revolved. In the middle of each quadrant the four are of equal brightness.

451. Nicol's Prism.—As the four beams in the last case are an annoyance in investigations requiring polarized light, only one is retained, the others being turned aside.

A rhomb of Iceland spar is cut by a plane *A B X* passing through the obtuse angles (Fig. 274); the two halves are then joined together again, as they were before the cutting, by Canada balsam. This is a Nicol prism. If a beam enters the prism at *a* (Fig. 276), it will be separated into two beams, the ordinary and the extraordinary, and then will fall upon the film of Canada balsam *C B*. Now the refractive index of Canada balsam is less than the index of refraction for the ordinary ray, and greater than that for the extraordinary ray; hence the ordinary ray will be totally reflected at *o*, and will pass out at the side of the prism, while the extraordinary ray will be refracted through the film of balsam, emerging as polarized light at *m*. If a similar prism be used as an analyzer, one of the two rays into which the polarized ray is separated is again turned out of the prism by total reflection, and only a single ray emerges.

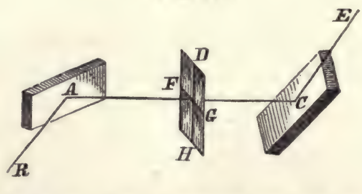
Fig. 276.



452. Color by Polarized Light.—The phenomena of *color* produced by polarized light are beautiful, and of great interest.

Let a thin principal section of some doubly-refracting crystal be placed perpendicularly across the axis of the polariscope (Fig. 277), whose analyzer is turned so that the reflected ray *C E* is at its minimum intensity. When the axis of the crystal, *D H*, coincides with the first plane of reflection, *R A C*, or is perpendicular to it, all the phenomena are the same as if no crystal was interposed. But let the film be revolved in its own plane till *D H* makes 45° with the plane *R A C*; then, instead of the dark spot at *E*, a brilliant color appears. That color may be any tint of the spectrum, according to the thickness of the interposed film. If now the revolution of the crystal is continued, the color fades out at the end of the next 45°, reappears

Fig. 277.



at 90°, and so on. But if the crystal be so placed as to give color, and the analyzing plate be revolved, a different series presents itself. The color observed at *E*, during the first 45°, gradually fades, and during the next 45° its *complement* appears and brightens to its maximum. The original color is restored at 180°, and the complementary color at 270°.

The most interesting form of this experiment is seen when the light is polarized and analyzed by means of double-refractors; since the polarization is more perfect, and the two pairs of oppositely polarized rays are on the screen at once. When two of the images are of a certain color, the other two have the complementary color.

453. Systems of Colored Rings.—Systems of irised bands and rings may also be produced by the polariscope. Let a plate be cut from a doubly-refracting crystal of one axis by planes perpendicular to that axis; and place it between the polarizer and analyzer. If now a pencil of sufficient divergency is transmitted, a system of colored circles will be formed, resembling Newton's rings between lenses. If a polariscope is formed of two tourmalines, and the crystal laid between them, and the whole combination, less than half an inch thick, is brought close to the eye, the pencil of light will consist of rays of various obliquity, and the rings may be seen beautifully projected on the sky. Or the ring systems may be projected on a screen by a polariscope furnished with concentrating lenses. Fig. 278 presents the system as seen through Iceland spar when the planes of reflection in the polariscope are at right angles. Two dark diameters cross the system and interrupt the rings. If the planes of reflection are coincident, the system is in every respect complementary to the other (Fig. 279). The colors of the rings are all reversed, and the crossing bands are white. If double-refractors of two axes are used instead of the spar, compound systems are shown, of various forms and great beauty.

FIG. 278.

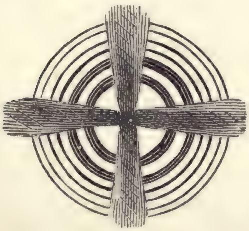


FIG. 279.

In treating of this subject it has been assumed that the vibra-

tions of the reflected polarized beam are *at right angles* to the plane of reflection, called also the plane of polarization; but this assumption is made merely to help the student to fix the facts in mind, since it is a point in regard to which the theory is not yet authoritatively settled.

CHAPTER VIII.

VISION.

454. Image by Light through an Aperture.—If light from an external object pass through a small opening of any shape in the wall of a dark room, it will form an ill-defined inverted image on the opposite wall. Imagine a minute square orifice, through which the light enters and falls on a screen several feet distant. A pencil of light, in the shape of a square pyramid, emanating from the highest point of the object, passes through the aperture, and forms a luminous square near the bottom of the screen. From an adjacent point another pencil, crossing the first at the aperture, forms another square, overlapping and nearly coinciding with the former. Thus every point of the object is represented by its square on the screen; and as the pencils all cross at the aperture, the image formed is every way inverted. It is also indistinct, because the squares overlap, and the light of contiguous points is mingled together. If the orifice is smaller, the image is less luminous, but more distinct, because the pencils which form it overlap in a less degree. If the hole is circular, or triangular, or of irregular form, there is no change in the appearance of the image, which is now composed of small circles, or triangles, or irregular figures, whose shape is completely lost by overlapping.

455. Effect of a Convex Lens at the Aperture.—The image will become distinct, and more luminous also, if the aperture be enlarged to a diameter of two or three inches, and then covered by a convex lens of the proper curvature. The image will be *distinct*, because the rays from each point of the object are converged to a point again, and *luminous* in proportion as the lens has a larger area than the aperture before employed. This is a real, and therefore an inverted image (Art. 385). A *scioptic ball* is a sphere containing a lens, and so fitted in a socket that it can be turned in any direction, and thus bring into the room the

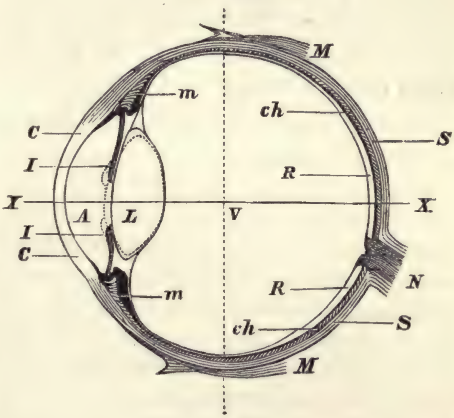
images of different parts of the landscape. The *camera obscura* is a darkened room furnished with a scioptic ball and adjustable screen for producing distinct pictures of external objects.

Instead of connecting the lens with the wall of a room, it is frequently attached to a portable box or case, within which the image is formed. The *Daguerreotype*, or *photograph*, is the image produced by the convex lens, and rendered permanent by the chemical action of light on a surface properly prepared. The lens for photographic purposes needs to be achromatic, and corrected, also, as far as possible, for spherical aberration.

456. The Eye.—The eye is a camera obscura in miniature; we find here the darkened room, the aperture, the convex lens, and the screen, with inverted images of external objects painted on it. A horizontal section of the eye is represented in Fig. 280.

The optical apparatus of the eye, and the spherical case which incloses it, constitute what is called the *eye-ball.* The case itself, except about a sixth part of it in front, is a strong white substance, called, on account of its hardness, the *sclerotic coat, S, S* (Fig. 280). In the front, this opaque coat changes to a perfectly transparent covering, called the *cornea, C, C,* which is a little more convex than the sclerotic coat. The increased convexity of the cornea may be felt by laying the finger gently on the eyelid when closed, and then rolling the eye one way and the other.

Fig. 280



The bony socket, which contains the eye, is of pyramidal form, its vertex being some distance behind the eyeball; room is thus afforded for the mechanism which gives it motion. This cavity, except the hemisphere in front occupied by the eye itself, is filled up with fatty matter and with the six muscles by which the eye-ball is revolved in all directions.

457. The Interior of the Eye.—Behind the cornea is a fluid, *A*, called the *aqueous humor.* In the back part of this fluid lies the *iris, I, I,* an opaque membrane, having in the centre of it a circular aperture, the *pupil,* through which the light enters.

The iris is the colored part of the eye ; the back side of it is black. Directly back of the aqueous humor and iris, is a flexible double convex lens, L, called the *crystalline lens*, or *crystalline humor*, having the greatest convexity on the back side. The large space back of the crystalline is occupied by the vitreous humor, V, a semi-liquid, of jelly-like consistency. Next to the vitreous humor succeed those inner coatings of the eye, which are most immediately concerned in vision. First in order is the *retina*, R, R, on which the light paints the inverted pictures of external objects. The fibres of the optic nerve, which enter the ball at N, are spread all over the retina, and convey the impressions produced there to the brain. Outside of the retina is the *choroid coat*, $c\ h$, $c\ h$, covered with a black pigment, which serves to absorb all the light so soon as it has passed through the retina and left its impressions. The choroid is inclosed by the sclerotic already described. The nerve-fibres, which are spread over the interior of the retina, are gathered into a compact bundle about one-tenth of an inch in diameter, which passes out through the three coatings at the back part of the ball, about fifteen degrees from the axis, $X\ X$, on the side toward the other eye. M, M represent two of the muscles, where they are attached to the eyeball.

458. Vision.—The index of refraction for the cornea, and the aqueous and vitreous humors, is just about the same as that for water; for the crystalline lens, the index is a little greater. The light, therefore, which comes from without, is converged principally on entering the cornea, and this convergency is a little increased both on entering and leaving the crystalline. If the convergency is just sufficient to bring the rays of each pencil to a focus on the retina, then the images are perfectly formed, and there is distinct vision. To prevent the reflection of rays back and forth within the chamber of the eye, its walls are made perfectly black throughout by a pigment which lines the choroid, the ciliary processes, and the back of the iris. Telescopes and other optical instruments are painted black in the interior for a similar purpose.

The cornea is prevented from producing spherical aberration by the form of a prolate spheroid which is given to its surface, and the crystalline, by a gradual increase of density from its edge to its centre.

459. Adaptations.—By the prominence of the cornea rays of considerable obliquity are converged into the pupil, so that the eye, without being turned, has a range of vision more or less perfect, through an angle of about 150°.

The quantity of light admitted into the eye is regulated by the size of the pupil. The iris, composed of a system of circular and

radial muscles, expands or contracts the pupil according to the intensity of the light. These changes are involuntary; a person may see them in his own eyes by shading them, and again letting a strong light fall upon them, while he is before a mirror.

The pupils in the eyes of animals have different forms according to their habits; in the eyes of those which graze, the pupil is elongated horizontally, and in the eyes of beasts and birds of prey, it is elongated vertically.

The eyes of animals are adapted, in respect to their refractive power, to the medium which surrounds them. Animals which inhabit the water have eyes which refract much more than those of land animals. The human eye being fitted for seeing in air, is unfit for distinct vision in water, since its refractive power is nearly the same as that of water, and therefore a pencil of parallel rays from water entering the eye would scarcely be converged at all. The effect is the same as if the cornea were deprived of all its convexity.

460. Accommodation to Diminished Distance.—It has been shown (Art. 385), that as an object approaches a lens, its image moves away, and the reverse. Therefore in the eye there must be some change in order to prevent this, and keep the image distinct on the retina while the object varies its distance. In a state of rest, the eye converges to the retina only the pencils of *parallel* rays, that is, those which come from objects at great distances. Rays from near objects diverge so much that, while the eye is at rest, it cannot sufficiently converge them so that they will meet on the retina; but each conical pencil is cut off before reaching its focus, and all the points of the object are represented by overlapping circles, causing an indistinct image. The change in the eye, which fits it for seeing near objects distinctly, is called *accommodation*. This is effected by increasing the convexity of the crystalline lens, principally the front surface. The *ciliary muscle, m, m,* surrounds the crystalline, and is attached to the sclerotic coat just on the circle where it changes into the cornea. This muscle is connected with the edge of the crystalline by the circular ligament which surrounds the latter and holds it in place. When the muscle contracts, it relaxes the ligament, and the crystalline, by its own elastic force, begins to assume a more convex form, as represented by the dotted line. The eye is then accommodated for the vision of objects more or less near, according to the degree of change in the lens. On the other hand, when the ciliary muscle relaxes, the ligament again draws upon the lens to flatten it, and adapt it for the view of distant objects. In Fig. 281 these two conditions of the crystalline are more distinctly

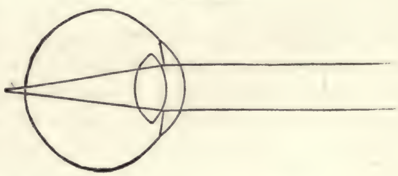
shown. The dotted line exhibits the shape of the lens when accommodated for seeing near objects. Accompanying this action of the ciliary muscle is that of the iris, which diminishes the pupil for near objects, so as to exclude the outer and more divergent rays. The dotted lines in front of the iris represent its situation when pushed forward by the crystalline accommodated for near objects.

FIG. 281.

461. Long-Sightedness.—As life advances, the crystalline becomes harder and less elastic. It therefore assumes a less convex form when the ligament is relaxed, and cannot be accommodated to so short distances as in earlier years; and at length it remains so flattened in shape that only very distant objects can be seen distinctly. The eye is then said to be long-sighted, and requires a convex lens to be placed before it, to compensate for insufficient convexity in the crystalline.

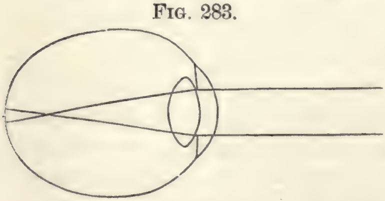
There are, however, cases of long-sightedness in early life. Such instances are found to be the result of an oblate form of the eyeball, as shown in Fig. 282; it is too short from front to back to furnish room for the convergency of the pencils, and they are cut off by the retina before reaching their focal points. In order to bring the distinct image forward upon the retina, convex glasses are needed in such cases, just as for the eyes of most people when advanced in life. As the term long-sightedness is now applied to this abnormal condition of the eye, the effect of age upon the sight is more properly called old-sightedness.

FIG. 282.



462. Short-Sightedness.—The eyes of the short-sighted have a form the reverse of that just described; the eyeball is elongated from cornea to retina (Fig. 283), resembling a prolate spheroid, so that rays parallel, or nearly so, are converged to a point before reaching the retina, and after crossing, fall on it in a circle; and the image, made up of overlapping circles instead of points, is indistinct. If this

FIG. 283.



elongation of the eyeball is extreme, an object must be brought very near, in order that its image may move back to the retina, and distinct vision be produced. This inconvenience is remedied by the use of concave lenses, which increase the divergency of the rays before they enter the eye, and thus throw their focal points further back.

In the normal condition of the eyes in early life, the nearest limit of distinct vision is about *five* inches. This limit slowly increases with advance of life, but much more slowly in some cases than others, till it is at an indefinitely great distance. The near limit of distinct vision for the short-sighted varies from *five* down to *two* inches, according to the degree of elongation in the eyeball.

463. Why an Object is Seen Erect and Single.—The image on the retina is *inverted;* and that is the very reason why the object is seen erect; the image is not the thing *seen*, but *that by means of which we see.* The impression produced at any point on the retina is referred outward in a straight line through a point near the centre of the lens, to something external as its cause; and therefore that is judged to be highest without us which makes its image lowest on the retina, and the reverse.

An object appears as *one*, though we see it by means of *two* images; but this is only one of many instances in which we have learned by experience to refer two or more sensations to one thing as the cause. Provided the images fall on parts of the retina, which in our ordinary vision *correspond* with each other, then by experience we refer both impressions to one object; but if we press one eye aside, the image falls in a new place in relation to the other, and the object seems double.

464. Indirect Vision.—The Blind Point.—To obtain a clear and satisfactory view of an object, the axes of both eyes are turned directly upon it, in which case each image is at the centre of the retina. But when the light from an object is exceedingly faint, it is better seen by *indirect vision,* that is, by looking to a point a little on one side, and especially by changing the direction of the eyes from moment to moment, so that the image may fall in various places *near* the centre of the retina. Many heavenly bodies are plainly discerned by indirect vision, which are too faint to be seen by direct vision.

In the description of the eye it was stated that the retina, as well as the choroid and the sclerotic, is perforated to allow the optic nerve to pass through. At that place there is no vision, and it is called the *blind point.* In each eye it is situated about 15° from the centre of the retina toward the other eye. Let a person

close his right eye, and with the left look at a small but conspicuous object, and then slowly turn the eye away from it toward the right; presently the object will entirely disappear, and as he looks still further to the right, it will after a moment reappear, and continue in sight till the axis of the eye is turned 70° or 80° from it. The same experiment may be tried with the right eye in the opposite direction. The reason why people do not generally notice the fact till it is pointed out, is that an object cannot disappear to both eyes at once, nor to either eye alone, when directed to the object.

465. Continuance of Impressions.—The impression which a visible object makes upon the retina continues about one-eighth or one-ninth of a second; so that if the object is removed for that length of time, and then occupies its place again, the vision is uninterrupted. A coal of fire whirled round a centre at the rate of eight or nine times per second, appears in all parts of the circumference at once. When riding in the cars, one sometimes gets a faint but apparently an uninterrupted view of the landscape beyond a board fence, by means of successive glimpses seen through the cracks between the upright boards. Two pictures, on opposite sides of a disk, are brought into view together, as parts of one and the same picture, by whirling the disk rapidly on one of its diameters. Such an instrument is called a *thaumatrope*. The *phantasmascope* is constructed on the same principle. Several pictures are painted in the sectors of a circular disk, representing the same object in a series of positions. These are viewed in a mirror through holes in the disk, as it revolves quickly in its own plane. Each glimpse which is caught whenever a hole comes before the eye, presents the object in a new attitude; and all these views are in such rapid succession that they appear like one object going through the series of movements.

466. Subjective Colors.—There are impressions on the retina of another kind, which are produced by intense lights; they continue longer, and are in respect to color unlike the objects which cause them. They are called *subjective colors:* If a particular part of the retina is for some time affected by the image of a bright colored object, and then the eyes are shut, or turned upon a white surface, the *form* appears to remain, but the *color* is complementary to that of the object; and its continuance is for a few seconds or several minutes, according to the vividness of the impression.

That portion of the retina upon which the bright colored image was formed loses sensitiveness to that particular color after a short time, and when white light falls upon the retina that particular spot is affected by the complementary color only.

467. Irradiation.—When small bodies are intensely illuminated the retina is affected somewhat beyond their proper images upon it, and the bodies consequently appear larger than they would if less bright. A white circle upon a black ground looks larger than an equal black circle upon a white ground. At new moon, when both the bright and the dark portions are visible, the crescent seems to be a part of a larger sphere than that which it accompanies.

This enlargement of the image is called irradiation.

468. Estimate of the Distance of Bodies.—

1. If objects are near, we judge of relative distance by the *inclination of the optic axes* to each other. The greater that inclination is, or, which is the same thing, the greater the change of direction in an object, as it is viewed by one eye and then by the other, the nearer it is. If objects are *very* near, we can with one eye alone judge of their distance by the degree of effort required to accommodate the eye to that distance.

2. If objects are known, we estimate their distance by the *visual angle* which they fill, having by experience learned to associate together their distance and their *apparent*, that is, their *angular* size.

3. Our judgment of distant objects is influenced by their *clearness* or *obscurity.* Mountains, and other features of a landscape, if seen for the first time when the air is remarkably pure, are estimated by us nearer than they really are; and the reverse, if the air is unusually hazy.

4. Our estimate of distance is more correct when *many objects intervene.* Hence it is that we are able to place that part of the sky which is near the horizon further from us than that which is over our heads. The apparent sky is not a hemisphere, but a flattened semi-ellipsoid.

469. Magnitude and Distance Associated.—Our judgments of distance and of magnitude are closely associated. If objects are known, we estimate their distance by their visual angle, as has been stated; but if unknown, we must first acquire our notion of their distance by some other means, and then their visual angle gives us a definite impression as to their size. And if our judgment of distance is erroneous, a corresponding error attaches to our estimate of their magnitude. An insect crawling slowly on the window, if by mistake it is supposed to be some rods beyond the window, will appear like a bird flying in the air. The moon near the horizon seems larger than above us, because we are able to locate it at a greater distance.

The apparent linear dimensions of objects are directly proportional to their actual dimensions and inversely proportional to their distances from the observer.

470. Binocular Vision.—The Stereoscope.—If objects are placed quite near us, we obtain simultaneously *two views,* which are essentially different from each other—one with one eye, and one with the other. By the right eye more of the right side, and less of the left side, is seen, than by the left eye. Also, objects in the foreground fall further to the left compared with distant objects, when seen with the right eye than when seen with the left. And we associate with these combined views the form and extent of a body, or group of bodies, particularly in respect to distance of parts from us. It is, then, by means of *vision with two eyes,* or *binocular vision,* that we are enabled to get accurate perceptions of prominence or depression of surface, reckoned in the visual direction. A picture offers no such advantage, since all its parts are on one surface, at a common distance from the eyes. But, if two perspective views of an object should be prepared, differing as those views do, which are seen by the two eyes, and if the right eye could then see only the right-hand view, and the left eye only the left-hand view, and if, furthermore, these two views could be made to appear on one and the same ground, the vision would then be the same as is obtained of the real object by both eyes. This is effected by the *stereoscope. Two* photographic views are taken, in directions which make a small angle with each other, and these views are seen at once by the two eyes respectively, through a pair of half-lenses, placed with their thin edges toward each other, so as to turn the visual pencils away from each other, as though they emanated from one object. An appearance of *relief* and *reality* is thus given to superficial pictures, precisely like that obtained from viewing the objects themselves.

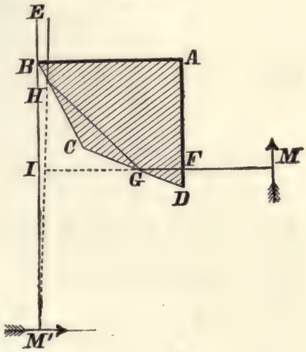
CHAPTER IX

OPTICAL INSTRUMENTS.

471. The Camera Lucida.—This is a four-sided prism, so contrived as to form an apparent image at a surface on which that image may be copied, the surface and image being both visible at the same time. It has the form represented by the section in

Fig. 284; $A = 90°$, $C = 135°$; B and D, of any convenient size, their sum of course $= 135°$. A pencil of light from the object M, falling perpendicularly on $A\ D$, proceeds on, and makes, with $D\ C$, an angle equal to the complement of D. After suffering total reflection at G, and again at H, its direction $H\ E$ is perpendicular to $M\ F$. For, produce $M\ F$, and $E\ H$, till they intersect in I; then, since $C = 135°$, $C\ G\ H + C\ H\ G = 45°$; but $I\ G\ H = 2\ C\ G\ H$, and $I\ H\ G = 2\ C\ H\ G$; $\therefore I\ G\ H + I\ H\ G = 90°$; $\therefore I = 90°$. Therefore $H\ E$ emerges at right angles to $A\ B$, and is not refracted. Now, if the pupil of the eye be brought over the edge B, so that, while $E\ H$ enters, there may also enter a pencil from the surface at M', then both the surface M' and the object M will be seen coinciding with each other, and the hand may therefore sketch M on the surface at M'. The reason for two reflections of the light is, that the inversion produced by one reflection may be restored by the second.

FIG. 284.



One of the most useful applications of the camera lucida is in connection with the compound microscope, where it is employed in copying with exactness the forms of natural objects, too small to be at all visible to the naked eye.

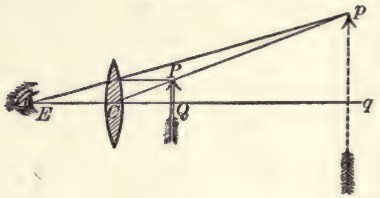
472. The Microscope.—This is an instrument for *viewing minute* objects. The nearer an object is brought to the eye, the larger is the angle which it fills, and therefore the more perfect is the view, provided the rays of each pencil are converged to a point on the retina. But if the object is nearer than the limit of distinct vision, the eye is unable to produce sufficient convergency. If the letters of a book are brought close to the eye, they become blurred and wholly illegible. But let a pin-hole be pricked through paper, and interposed between the eye and the letters, and, though faint, they are *distinct* and *much enlarged.* The *distinctness* is owing to the fact that the outer rays, which are most divergent, are excluded, and the eye is able to converge the few central rays of each pencil to a focus. The letters appear *magnified,* because they are so near, and fill a large angle. The microscope utilizes these excluded rays, and renders the image not only large and distinct, but luminous.

473. The Single Microscope.—The single microscope is merely a convex lens. It aids the eye in converging the rays,

which come from a very near object, so that a distinct and luminous image may be formed on the retina.

Let $P\,Q$ (Fig. 285) be the object, and $p\,q$ its image. We have $\frac{p\,q}{P\,Q} = \frac{C\,q}{C\,Q}$; but, by Art. 383, $C\,q = \frac{F \times C\,Q}{F - C\,Q}$, which gives $\frac{p\,q}{P\,Q} = \frac{F}{F - C\,Q}$. The visual angle of $p\,q$, at the distance $E\,q$, is $\frac{p\,q}{E\,C + C\,q}$; and of $P\,Q$, *at the distance of distinct vision*, v, is $\frac{P\,Q}{v}$; calling m their ratio we have

FIG. 285.



$$m = \frac{p\,q}{E\,C + C\,q} \div \frac{P\,Q}{v} = \frac{F}{E\,C + C\,q} \div \frac{F - C\,Q}{v}$$
$$= \frac{F}{E\,C + C\,q} \times \frac{v}{F - C\,Q}.$$

Making $E\,C = 0$, by bringing the eye close to the lens, we have $m = \frac{F}{C\,q} \times \frac{v}{F - C\,Q} = \frac{v}{C\,Q}$. If $C\,Q = F$, then $m = \frac{v}{F}$. If $C\,q = v$, then $C\,Q = \frac{v\,F}{v + F}$, whence $m = \frac{v + F}{F} = \frac{v}{F} + 1$, the maximum effect of the lens. Hence, *the magnifying power of a lens, the eye being close to it, is between the limits* $\frac{v}{F}$ *and* $\frac{v}{F} + 1$, *according to the position of the object.*

Though the focal distance of a lens may be made as small as we please, yet a practical limit to the magnifying power is very soon reached.

1. The *field of view*, that is, the extent of surface which can be seen at once, diminishes as the power is increased.

2. Spherical aberration increases rapidly, because the outer rays are very divergent. Hence the necessity of diminishing the aperture of the lens, in order to exclude the most divergent rays.

3. It is more difficult to illuminate the object as the focal length of the lens becomes less; and this difficulty becomes a greater evil on account of the necessity of diminishing the aperture in order to reduce the spherical aberration.

To lessen the spherical aberration, two or more plano-convex lenses are used, close together.

Magnifying glasses are single microscopes of low power, such as

are used by watchmakers. Lenses of still lower power and several inches in diameter are used for viewing pictures.

474. The Compound Microscope.—It is so called because it consists of two parts, an object-glass, by which a real and magnified image is formed, and an eye-glass, by which that image is again magnified. Its general principle may be explained by Fig. 286, in which *a b* is the small object, *c d* the object-glass, and *e f* the eye-glass. Let *a b* be a little beyond the principal focus of *c d*, and then the image *g h* will be real, on the opposite side of *c d*, and larger than *a b*. Now apply *e f* as a single microscope for viewing *g h*, as though it were an object of comparatively large size. Let *g h* be at the principal focus of *e f*, so that the rays of each pencil shall be parallel; they will, therefore, come to the eye at *k*, from an *apparent* image on the same side as the *real* one, *g h*; and the extreme pencils, *e k*, *f k*, if produced backward, will include the image between them, *e k f* being the angle which it fills.

FIG. 286.

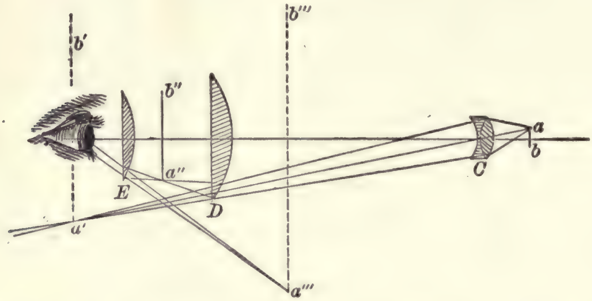
475. The Magnifying Power.—The *magnifying power* of the compound microscope is estimated by compounding two ratios; first, the distance of the image from the object-glass, to the distance of the object from the same; and secondly, the limit of distinct vision to the distance of the image from the eye-glass. For the image itself is enlarged in the first ratio (Art. 385); and the eye-glass enlarges that image in the second ratio (Art. 473). The advantage of this form over the single microscope is not so much that a great magnifying power is obtained, as that a given magnifying power is accompanied by a larger field of view.

476. Modern Improvements.—Great improvements have been made in the compound microscope, principally by combining lenses in such a manner as greatly to reduce the chromatic and spherical aberrations. The object-glass generally consists of one, two, or three achromatic pairs of lenses. The eye-piece usually contains two plano-convex lenses, a combination which is found to be the most favorable for diminishing the spherical aberration, and for enlarging the field of view.

In Fig. 287 let *a b* be the object, *C* an achromatic lens, called the *objective*, *D* the field lens (so called because it enlarges the field of view by bending the pencil which would come to a focus at *a′*, and pass below the eye lens, so that it may come to a focus at *a″*, and thence pass into the eye lens), *E* the eye lens which

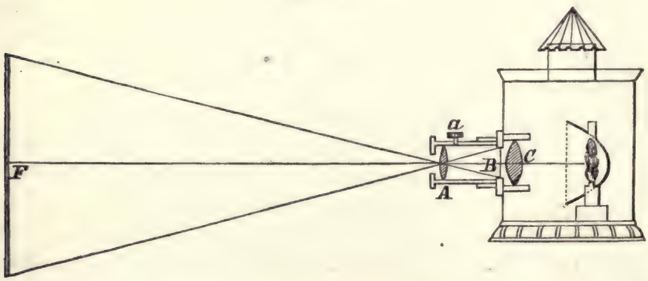
renders the rays of the pencil so nearly parallel that the eye receives them as though coming from the point a'''.

FIG. 287.



The method of determining the magnifying power given in the last paragraph, is not applicable in this form of instrument, but an experimental determination is made as follows: A very finely-divided scale is placed under the microscope; a mirror, from which a small part of the silvering has been removed, is placed near the eye-piece at an angle of 45° with the axis of the instrument, and behind this at the distance of ordinary distinct vision, about ten inches, and visible through the unsilvered part of the mirror, is placed a second scale like the first; the number of divisions of the second scale covered by one division seen through the microscope by reflection gives the power. Any change in the relative positions of the lenses, changes the magnifying power.

FIG. 288.



477. The Magic Lantern.—It consists of a box, represented in Fig. 288, containing a lamp, and having openings so arranged as to permit the air to pass freely through it, without letting light escape. In front of the lamp is a tube containing a concentrating lens, *C*, the painting on glass, *B*, and the lens, *A*, for producing the image; back of the lamp may be a concave

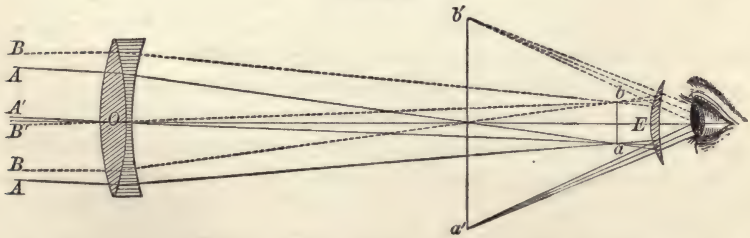
mirror for reflecting additional light on the lens *C*. The transparency *B* is a painting on glass, and the strong light which falls on it proceeds through the lens *A*, as from an original object brilliantly colored. It is a little further from *A* than its principal focus, and therefore the rays from any point are converged to the conjugate focus in a real image, *F*, on a distant screen. This image is of course inverted relatively to the object, and therefore, if the picture *B* is inverted, *F* will be erect. The lens may be placed at various distances from *B* by the adjusting screw *a*, so as to give the greatest distinctness to the image at any given distance of the screen. According to Art. 385, the diam. of *B* : diam. of *F* : : *A B* : *A F*; and therefore, theoretically, the image may be as large as we please.

478. The Telescope.—The telescope aids in *viewing distant* bodies. An image of the distant body is first formed in the principal focus of a convex lens or a concave mirror; and then a microscope is employed to magnify that image as though it were a small body. The image is much more luminous than that formed in the eye, when looking at the heavenly body, because there is concentrated in the former the large beam of light which falls upon the lens or mirror, while the latter is formed by the slender pencil only which enters the pupil of the eye. If the image in a telescope is formed by a lens, the instrument is called a *refracting telescope;* but if by a mirror, a *reflecting telescope.*

479. The Astronomical Telescope.—This is the most simple of the refracting telescopes, consisting of a lens to form an image of the heavenly body, and a single microscope for magnifying that image. The former is called the *object-glass,* the latter the *eye-glass.*

Let *a* (Fig. 289) be the image of some point of a heavenly body, the divergent rays from which, marked *A A′ A*, are practi-

FIG. 289.



cally parallel, and *b* the image of the point *B B′ B*. As the rays forming these images are parallel rays, *a b* is at the principal focus of the object glass *O*. The eye lens *E* receives the diver-

gent pencils from a and b, bends them so that they enter the eye as parallel or nearly parallel beams coming apparently from the direction of a' and b'. The image $a\ b$ is situated at the principal focus of E, the distance between the lenses O and E being the sum of their principal focal distances.

480. The Powers of the Telescope.—The *magnifying power* of the astronomical telescope is expressed by the ratio of the *focal distance of the object-glass to that of the eye-glass.* For (Fig. 289) the object as it would be seen by the eye if placed at O fills the angle $A'\ O\ B'$ between the axes of its extreme pencils. But, since the axes cross each other in straight lines at the optic centre of the lens, $A'\ O\ B' = a\ O\ b$. Therefore, to an eye placed at the object-glass, the image, $a\ b$, appears just as large as the object; while at the eye-glass it appears as much larger in diameter as the distance is less.

Since no simple eye lenses are used, and as the equivalent power of the compound eye-piece is not readily found, the following practical method of finding the power of an astronomical telescope is of use:

Adjust the eye-piece so that a sharp and clear view of some very distant object may be obtained; remove the object-glass, and in its place put an opaque card disk in which is cut an opening of the shape of a very flat isosceles triangle, whose base is nearly as long as the diameter of the object-glass; receive an image of this opening upon a translucent glass or paper screen held close to the eye-piece, and measure the base of the image very exactly; the length of the base of the opening divided by the base of its image is the power. If any of the rays from the opening are cut off by diaphragms in the tube the imperfection of the image will make known the difficulty, which must be removed.

The field of view is determined thus: Direct the telescope to a star on or near the celestial equator, and note the time in seconds which the star occupies in passing across the diameter of the field of view; divide this time by 4 and the quotient will be the diameter of the field in minutes of arc, because a star on the equator moves through one minute of arc in four seconds of time.

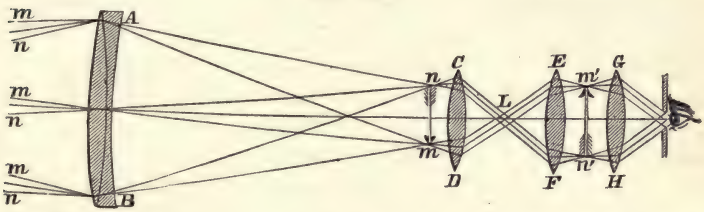
The *illuminating power* is important for objects which shed a very feeble light on account of their immense distance. This power depends on the size of the beam, that is, on the aperture of the object-glass.

The *defining power* is the power of giving a clear and sharply-defined image, without which both the other powers are useless. And it is the power of producing a well-defined image which limits both of the other powers. For every attempt to increase

the magnifying power by giving a large ratio to the focal lengths of the object-glass and the eye-glass, or to increase the illuminating power by enlarging the object-glass, increases the difficulties in the way of getting a perfect image. These difficulties are three —the spherical aberration (Art. 388), the chromatic aberration (Art. 402), and unequal densities in the glass. The third difficulty is a very serious one, especially in large lenses.

481. The Terrestrial Telescope.—In order to secure simplicity, and thus the highest excellence, in the astronomical telescope, the image is allowed to be *inverted*, which circumstance is of no importance in viewing heavenly bodies. But, for terrestrial objects, it would be a serious inconvenience; and, therefore, a *terrestrial telescope*, or *spy-glass*, has additional lenses for the purpose of forming a second image, inverted, compared with the first, and, therefore, erect, compared with the object. In Fig. 290, *m m m* represent a pencil of rays from the top of a distant object, and

FIG. 290.



n n n from the bottom; *A B*, the object-glass; *m n*, the first image; *C D*, the first eye-glass, which converges the pencils of parallel rays to *L*. Instead of placing the eye at *L*, the pencils are allowed to cross and fall on the second eye-glass, *E F*, by which the rays of each pencil are converged to a point in the second image, *m' n'*, which is viewed by the third eye-glass, *G H*. The second and third lenses are commonly of equal focal length, and add nothing to the magnifying power.

Such instruments are usually of a portable size, and hence the aberrations are corrected with comparative ease, by the methods already described. The spy-glass, for convenient transportation, is made of a series of tubes, which slide together in a very compact form.

If the lenses *C D* and *E F* are of the same power they do not affect the power of the telescope, which may then be represented as in the astronomical telescope by $\frac{F}{f}$.

To determine the power practically, look at some distant scale of equal parts, a brick wall for instance, and keeping both eyes

open, note how many bricks as seen by the unaided eye are covered by the image of one brick as seen through the telescope; this number so covered is the expression for the power. If one space, for instance, seen through the telescope, covers twenty spaces seen with the unaided eye, the telescope magnifies twenty diameters.

482. Galileo's Telescope.—This was the first form of telescope, having been invented by Galileo, whose name it therefore bears. It differs from the common astronomical telescope in having for the eye-glass a *concave* instead of a convex lens, which receives the rays at such a distance from the focus to which they tend, as to render them parallel.

Thus the rays *m m m* (Fig. 291), from a point at the top of the object, are converged by the object-glass *O* towards a focus *a*;

FIG. 291.

but before meeting at *a* they fall upon a concave eye-lens *E*, and are rendered parallel or slightly divergent, as though they came from a point in the direction indicated by *M*. The point from which the rays *m m m* proceeded, and its virtual image *M*, are both on the same side of the axis of the instrument, and there is no inversion.

It is obvious that, since the pencils diverge, only the central ones, within the size of the pupil, can enter the eye. This circumstance exceedingly limits the field of view, and unfits the instrument for telescopic use. It is employed for opera-glasses, having a power usually of only *two* or *three* diameters.

The expression for the power is $\frac{F}{f}$, as in the preceding forms. The power may be very readily determined practically as in the case of the terrestrial telescope.

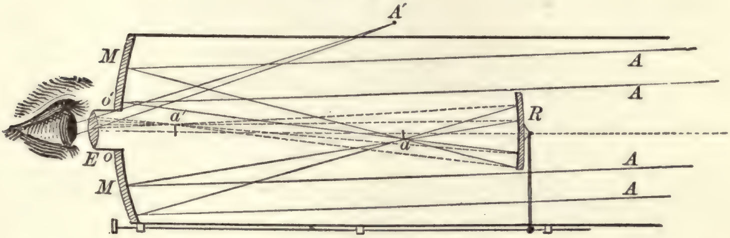
483. The Gregorian Telescope.—This is the most frequent form of reflecting telescope, and receives its name from the inventor, Dr. Gregory, of Scotland.

Let *A* (Fig 292) be a point of a very distant body from which rays, practically parallel, fall upon the large concave mirror *M*, which is perforated through the middle *o o′*; these are converged to the principal focus *a*, and passing this point, diverging again, are received by the small concave reflector *R* of short focus, and

are made to converge to a', forming a real image; thence the rays diverge once more, and falling upon the eye-glass E, are refracted as though they came from an object in the direction A'.

The *Cassegrainian* telescope differs from the Gregorian in having a *convex* reflector in place of the *concave* R; this is so

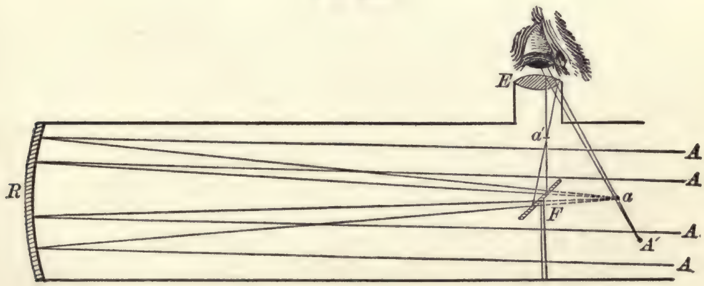
FIG. 292.



placed as to receive the rays before they reach the focus a, and, by rendering them less convergent, bring them to a focus at the place of the image a', but upon the same side of the axis as a.

484. The Newtonian Telescope.—R is a concave reflector (Fig. 293), F a plane mirror called a *flat*, E the convex eye-glass. Rays from some point A of a distant object are converged by R towards the principal focus a; they are intercepted by the *flat* F and turned aside, without change of convergency, to the

FIG. 293.



focus a', passing which they fall upon the eye-lens E, and enter the eye as though they came from the direction A'. The magnifying power =

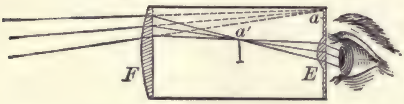
$$\frac{\text{focal length of reflector}}{\text{focal length of eye-glass}}.$$

485. The Herschelian Telescope.—Sir William Herschel modified the Newtonian by dispensing with the small reflector F, and inclining the large speculum R, so as to form the image near the edge of the tube, where the eye-glass is attached. Thus, the

observer is situated with his back to the object. The speculum of Herschel's telescope was about four feet in diameter, and weighed more than 2,000 pounds, and its focal length was forty feet. The Earl of Rosse has since constructed a Herschelian telescope having an aperture of *six* feet, and a focal length of *fifty* feet. The magnifying power is the same as in the Newtonian.

486. Eye-pieces, or Oculars.—The negative, or Huyghenian, eye-piece consists of two plano-convex lenses of crown glass, F and E (Fig. 294), the convex surfaces being turned towards the object-glass. A pencil of rays from the object-glass, converging to a principal focus a, is bent from its course by F and brought to a focus a', half way between the two lenses. The image found at a' is then viewed by the eye-lens E as usual.

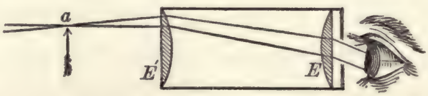
FIG. 294.



This eye-piece is called negative because it is adapted to rays already converging. The focal length of F is three times that of E, and the distance between the lenses is one-half the sum of the focal lengths. This combination is achromatic, for the following reasons: the deviation of a ray by a convex lens is greater, the greater the distance of the point of incidence upon the lens is from the axis of the lens; a ray refracted by F is separated into its component colors, the red ray being least bent towards the centre of the eye-lens E and the violet most; the red ray falls upon E at a greater distance from the centre than the violet, and being more bent from its course than the violet, they emerge parallel.

The *positive,* or Ramsden, eye-piece consists of two plano-convex lenses E' and E (Fig. 295) with the convex surfaces turned toward each other. The rays from the object-glass are focussed at a and thence pass to the eye as indicated in the figure. Two lenses are used instead of one in order more easily and perfectly to correct spherical aberration. E' and E are of equal focal lengths, and the distance between them is two-thirds the focal length of one of them. This combination is not achromatic. It is always used when spider lines are placed in the focus of the object-glass for purposes of exact measurement.

FIG. 295.



PART VI.

HEAT.

CHAPTER I.

EXPANSION BY HEAT.—THE THERMOMETER.

487. Nature of Heat.—There is abundant reason for believing that heat consists of exceedingly minute and rapid vibrations of ordinary matter and of the ether which fills all space. It is to be regarded as one of the modes of *motion*, which may be caused by any kind of force, and which may be made a measure of that force. Heat affects only one of our senses, that of feeling. Its increase produces the sensation of warmth, and its diminution that of cold.

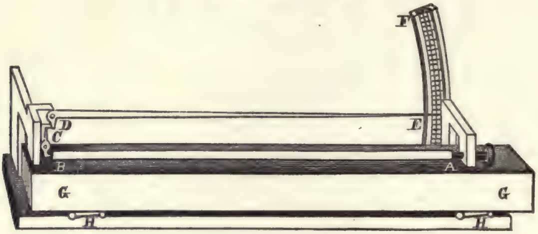
488. Expansion and Contraction by Heat and Cold.—It is found to be a fact almost without exception, that as bodies are heated they are expanded, and that they contract as they are cooled. It is easy to conceive that the vibratory motion of the several molecules of a body compels them to recede from each other, and to recede the more as the vibration becomes more violent. Although the change in magnitude is generally very small, yet it is rendered visible by special contrivances, and is made the means of measuring temperature.

489. Expansion of Solids.—When the expansion of a solid is considered simply in one dimension, it is called *linear* expansion; in two dimensions only, *superficial* expansion; in all three dimensions, *cubical* expansion.

The linear expansion of a metallic rod is readily made visible by an instrument called the *pyrometer*, which magnifies the motion. The end A of the rod $A\ B$ (Fig. 296) is held in place by a screw. The end B rests against the short arm of the lever C, the longer arm of which bears on the arm D of the long bent lever $D\ E$; this serves as an index to the graduated arc $E\ F$. The long metallic dish $G\ G$, being raised on the hinges $H\ H$, so as to

enclose the bar A B, and then filled with hot water, the bar instantly expands, and raises the index along the arc E F.

FIG. 296.



490. Coefficient of Expansion.—The *coefficient* of linear expansion of a given substance is the fractional increase of its length, when its temperature is raised *one degree.* But since this increase is generally somewhat greater at higher temperatures, the coefficients of expansion given in tables usually refer to a temperature at or near the freezing point of water. Thus the coefficient of expansion for silver is 0.000019097 ; by which is meant that a silver bar one foot long at 0° C. becomes 1.000019097 ft. in length at 1° C.

The coefficient of superficial expansion is *twice,* and that of cubical expansion *three times* as great as the coefficient of linear expansion. For, suppose c to be the coefficient of linear expansion ; then if the edge of a cube is 1, and the temperature is raised 1°, the edge becomes $1 + c$, and the area of one side becomes $(1 + c)^2 = 1 + 2c + c^2$, and the volume $(1+c)^3 = 1+3c+3c^2+c^3$. But as c is very small, the higher powers may be neglected, and the area is $1+2c$, and the volume is $1+3c$; that is, the coefficient of superficial expansion is $2c$, and that of cubical expansion is $3c$, as stated above.

491. The Coefficient of Expansion differs in different Substances.—Copper expands nearly twice as much as platinum for a given increase of temperature; the ratio of expansion in steel and brass is about as 61 to 100. This ratio is employed in the construction of the compensation pendulum (Art. 164).

If two thin slips of metal of different expansibility be soldered together so as to make a slip of double thickness, it will bend one way and the other by changes of temperature. If it is straight at a certain temperature, heating will bend it so as to bring the most expansible metal on the convex side ; and cooling will bend it in the opposite direction ; and the degree of flexure will be according to the degree of change in temperature. Compensation in clocks and watches is sometimes effected on this plan. If the

compound slip has the form of a helix, with the most expansible metal on the inside, heating will begin to uncoil it, and cooling, to coil it closer. A very sensitive thermometer, known as Breguet's thermometer, is constructed on this principle.

As notable exceptions to the general rule that solids expand when heated, may be mentioned stretched India-rubber, and also Rose's fusible metal, an alloy of 2 parts bismuth, 1 part lead, and 1 part tin; the latter compound expands up to 44° C., then rapidly contracts up to 69° C., which is the temperature of maximum density, and again expands till it melts at 94° C.

492. The Strength of the Thermal Force.—It is found that the force exerted by a body, when expanding by heat or contracting by cold, is equal to the mechanical force necessary to expand or compress the body to the same degree. The force is therefore very great. If the rails were to be fitted tightly end to end on a railroad, they would be forced out of their places by expansion in warm weather, and the track ruined. The tire of a carriage wheel is heated till it is too large, and then put upon the wheel; when cool, it draws together the several parts with great firmness. In repeated instances, the walls of a building, when they have begun to spread by the lateral pressure of an arched roof, have been drawn together by the force of contraction in cooling. A series of iron rods being passed across the building through the upper part of the walls, and broad nuts being screwed upon the ends, the alternate bars are expanded by the heat of lamps, and the nuts tightened. Then, when they cool, they draw the walls toward each other. The remaining bars are then treated in the same manner, and the process is repeated till the walls are restored to their vertical position and secured.

493. Expansion of Liquids.—As liquids have no permanent form, the coefficient of expansion for them is always understood to be that of cubical expansion. There is a practical difficulty in the way of finding the coefficient for liquids, because they must be enclosed in some solid, which also expands by heat. Hence, the *apparent* expansion must be corrected by allowing for the expansion of the inclosing solid, before the coefficient of *absolute* expansion is known.

This fact is illustrated by the following experiment. Fill the bulb and part of the stem of a large thermometer tube with a colored liquid, and then plunge the bulb quickly into hot water; the first effect is, that the liquid *falls*, as if it were cooled; after a moment it begins to rise, and continues to do so till it attains the temperature of the hot water. The first movement is caused by the expansion of the glass, which is heated so as to enlarge its

capacity and let down the liquid before the heat has penetrated the latter. It is obvious that what is rendered visible in this case, must always be true when a liquid is heated—namely, that the vessel itself is enlarged, and therefore that the rise of the liquid shows only the difference of the two expansions. Ingenious methods have been devised for obtaining the coefficients of absolute expansion of liquids, and the results are to be found in tables on this subject.

From the examination of such tables we learn: (1) That liquids expand more than solids for a given increase of temperature; (2) that the coefficient of expansion increases with the rise of temperature; (3) that the more volatile the liquid the more rapidly will it expand for a given rise of temperature.

494. Exceptional Case.—There is a very important exception to the general law of expansion by heat and contraction by cold, in the case of water just above the freezing point. If water be cooled down from its boiling point, it continually contracts till it reaches 39.1° F. or 3.94° C., when it begins to expand, and continues to expand till it freezes at 32° F. or 0° C. On the other hand, if water at 32° F. be heated, it contracts till it reaches 39.1° F. or 3.94° C., when it commences to expand. Therefore the density of water is greatest at the point where this change occurs. Different experimenters vary a little as to its exact place, but it is usually called 4° C., or 39° F.

The importance of this exception is seen in the fact that ice forms on the *surface* of water, and continues to float until it is again dissolved. As the cold of winter comes on, the upper stratum of a lake grows more dense and sinks; and this process continues till the temperature of the surface reaches 39° F., when it is arrested. Below that temperature the surface grows lighter as it becomes colder, till ice is formed, which shields the water beneath from the severe cold of the air above.

As in solids so in liquids, the thermal force is very great. Suppose mercury to be expanded by raising its temperature one degree, it would require more than 300 pounds to the square inch to compress it to its former volume.

495. Expansion of Gases.—The gases expand by heat more rapidly and more regularly than solids and liquids. The large expansion and contraction of air is made visible by immersing the open end of a large thermometer tube in colored liquid. When the bulb is warmed, bubbles of air are forced out and rise to the top of the liquid; when it is cooled, the air contracts and the liquid rises rapidly in the tube.

Gases, *at a constant pressure,* expand much more than liquids

or solids for a given increment of temperature. All gases, at temperatures much above that of liquefaction, have almost exactly the same coefficients of expansion. The coefficient of expansion for air is $\frac{1}{273}$ from 0° C. to 1° C., or $\frac{1}{490}$ from 32° F. to 33° F. This coefficient increases *slightly* with increase of temperature and pressure.

To find the volume of any gas at 0° C., let v be the known volume at t° C., also let v' be the required volume at 0° C., then

$$v = v'\left(1 + \tfrac{1}{273}t\right),$$

from which we have $v' = \dfrac{v}{1 + \frac{1}{273}t}$.

If there is a change of pressure, then, since the tensions or pressures are inversely as the volumes, the temperatures being the same (Art. 237), have

$$v'' : v' :: p' : p,$$

in which v'' is the volume at 0° C. and barometric pressure of 30 inches, v' the volume at pressure p', and p the normal pressure 30 inches; from which we get, by substituting for v' its value above,

$$v'' = \frac{v}{1 + \frac{1}{273}t} \times \frac{p'}{p}.$$

496. The Thermometer.—This instrument measures the degree of heat, or the *temperature*, of the medium around it, by the expansion and contraction of some substance. The substance commonly employed is mercury. The liquid, being inclosed in a glass bulb, can expand only by rising in the fine bore of the stem, where very small changes of volume are rendered visible. A scale is attached to the stem for reading the degrees of temperature.

The graduation of the thermometer must begin with the fixing of two important points by natural phenomena, the melting of ice and boiling of water. When the bulb is plunged into powdered ice, the point at which the column settles is the *freezing point* of the thermometer. And if it is placed in steam under the mean atmospheric pressure, the mercury indicates the *boiling point*. Between these two points, namely 32° and 212° F., there must be 180°, and the scale is graduated accordingly. As the bore of the tube is not likely to be exactly equal in all parts, the length of the degrees should vary inversely as the area of the cross-section. The necessary correction is determined by moving a short column of mercury along the different parts and comparing the lengths occupied by it. The degrees in the several parts must vary in the ratio of these lengths.

The zero of the scale tends to rise for some time after the thermometer is made, the change amounting to more than 2° in

some instances, and therefore the instrument should not be used for at least six months after construction. The zero may also be displaced by subjecting the instrument to high temperatures.

497. Different Systems of Graduation.—There are in use three kinds of thermometer scale, Fahrenheit's, Reaumur's, and the Centigrade or Celsius. In Fahrenheit's, the freezing point of water is called 32°, and the boiling point, 212°; in Reaumur's, the freezing point is called 0°, and the boiling point 80°; in the Centigrade, the freezing point 0°, and the boiling point 100°. In a scientific point of view, the Centigrade is preferable to either of the others, but Fahrenheit's is generally used in this country. The letter F., R., or C., appended to a number of degrees, indicates the scale intended. In this country, F. is understood if no letter is used.

498. To Reduce from one Scale to Another.—Since the zero of Fahrenheit's scale is 32° below the freezing point, while in both of the others it is at the freezing point, 32° must always be subtracted from any temperature according to Fahrenheit, in order to find its relation to the zero of the other scales. Then, since 212° − 32° (= 180°) F. are equal to 80° R., and to 100° C., the formula for changing F. to R. is $\frac{4}{9}$ (F. − 32) = R.; and for changing F. to C., it is $\frac{5}{9}$ (F. − 32) = C. Hence, to change R. to F., we have $\frac{9}{4}$ R. + 32 = F.; and to change C to F., $\frac{9}{5}$ C. + 32 = F.

Mercury congeals at about −38.8° C.; therefore, for temperatures lower than that, alcohol is used, which does not congeal at any known temperature.

Above 100° C. the indications of the mercurial thermometer are not exact.

499. Absolute Zero of Temperature.—At a temperature of 273° C. the volume of a gas is double its volume at 0° C. (Art. 495). Suppose that instead of raising the temperature, we lower it; for a fall from 0° to −1° C. the volume contracts $\frac{1}{273}$, and for a fall of 273° it must contract $\frac{273}{273}$; that is to say, the volume would disappear entirely. That the contraction *would* go on to −273° C. is not asserted; but on the supposition that the law of contraction would hold, we fix the temperature −273° as that at which all vibrations would cease, and at which consequently there could be no heat whatever. The absolute zero more exactly given is −273.7° C., and −460.66° F. The absolute temperature is found by adding these readings, with signs changed, to the respective readings of the mercurial thermometer. As both Fahrenheit and

Centigrade thermometers are in use, both will be referred to in the text, as indicated, that the student may become familar with both systems of graduation.

CHAPTER II.

PASSAGE OF HEAT THROUGH MATTER AND SPACE.

500. Heat is Communicated in Several Ways.—

1. By *conduction.* This is the slow progress of the vibratory motion from places of higher to places of lower temperature in the same body.

2. By *convection.* This mode of communication takes place only in *fluids.* When the particles are expanded by heat, they are pressed upward by others which are colder and therefore specifically heavier. Heat is thus conveyed from place to place by the motion of the heated matter, though the ultimate transfer of heat may still take place by conduction.

3. By *radiation.* Heat is said to be *radiated* when the vibratory motion is transmitted from the source with great swiftness through the ether which fills space. Its velocity is the same as that of light. The motion is propagated in straight lines in every direction, and each line is called a *ray* of heat. We feel the rays of heat from the sun or a fire, when no object intervenes between it and ourselves.

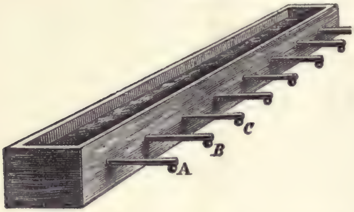
501. Conduction of Heat by Solids.—Conducted heat passes through bodies very slowly, and yet at very different rates in different bodies. Those in which heat is conducted most rapidly, are called good conductors, as the common metals; those in which it passes slowly, are called poor conductors, as glass and wood. In general, the bodies which are good conductors of heat, are also good conductors of electricity; thus calling the conductivity for electricity *E,* and for heat *H,* and using silver as the standard, we find—

Silver	E = 100	H = 100	Iron	E = 13	H = 12
Copper	" 73	" 74	Lead	" 11	" 9
Gold	" 59	" 53	German silver	" 6	" 6
Brass	" 22	" 24	Bismuth	" 2	" 2

Let rods of different metals and other substances, *A, B, C,*

&c. (Fig. 297), all of the same length, be inserted with water-tight joints in the side of a wooden vessel. Then attach by wax a marble under the end of each rod, and fill the vessel with boiling water. The marbles will fall by the melting of the wax, not at the same, but at different times, showing that the heat reaches some of them sooner than others. It will be seen, however, in the chapter on specific heat, that the order in which they fall is not necessarily the order of conducting power.

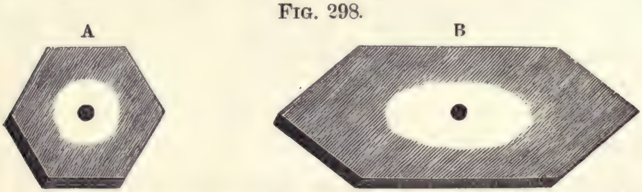
FIG. 297.



The amount of heat conducted through a thin lamina is directly proportional to the area, to the time during which it flows, to the difference of temperature at the two surfaces, and to the conductivity of the substance; and is inversely proportional to the thickness of the lamina.

502. Effects of Molecular Arrangement.—Organic substances usually conduct heat poorly; and bodies having a structural arrangement which differs in different directions, are not likely to conduct equally well in all directions. Thus, let two thin plates be cut from the same crystal, one, *A* (Fig. 298), perpendicular, and the other, *B*, parallel to the optic axis. Let a hole be drilled through the centre of each, and after a lamina of wax has been spread over the crystal, let a hot wire be inserted in it. On the plate *A*, the melting of the wax will advance in a circle, showing equal conducting power in all directions in the transverse section. In the plate *B*, it will advance in an elliptical form, the major axis being parallel to the optic axis of the crystal, proving the best conduction to be in that direction.

FIG. 298.



A block of wood cut from one side of the trunk of a tree, conducts most perfectly in the direction of the fibre, and least in a direction which is tangent to the annual rings and perpendicular to the fibre, and in an intermediate degree in the direction of the radius of the rings.

503. Conduction by Fluids.—Fluids, both liquid and gaseous, are in general very poor conductors. Water, for example, can be made to boil at the top of a vessel, while a cake of ice is fastened within it a few inches below the surface. If thermometers are placed at different depths, while the water boils at the top, there is discovered to be a very slight conduction of heat downward. The gases conduct even more imperfectly than liquids.

It will be seen hereafter (Art. 505) that a mass of fluid becomes heated by convection, not by conduction.

504. Illustrations of Difference in Conductive Power.—In a room where all articles are of equal temperature, some feel much colder than others, simply because they conduct the heat from the hand more rapidly; painted wood feels colder than woolen cloth, and marble colder still. If the temperature were higher than that of the blood, then the marble would seem the hottest, and the cloth the coolest, because of the same difference of conduction *to* the hand.

Our clothing does not impart warmth to us, but, by its non-conducting property, prevents the vital warmth from being wasted by radiation or conduction. If the air were hotter than our blood, the same clothing would serve to keep us cool.

A pitcher of water can be kept cool much longer in a hot day, if wrapped in a few thicknesses of cloth; for these prevent the heat of the air from being conducted to the water. In the same way ice may be prevented from melting rapidly.

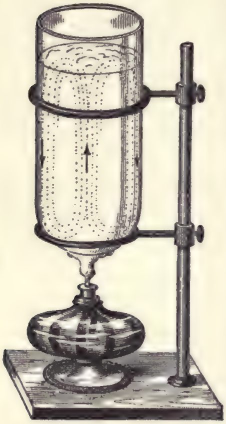
The vibrations of heat, like those of sound, are greatly interrupted in their progress by want of continuity in the material. Any substance is rendered a much poorer conductor by being in the condition of a powder or fibre. Ashes, sand, sawdust, wool, fur, hair, &c., owe much of their non-conducting quality to the innumerable surfaces which heat must meet with in being transmitted through them.

Davy's safety lamp is a practical application of conduction. A wire gauze surrounds the lamp, and the air which supplies the flame with oxygen can only reach it by passing through the gauze. A naked flame would ignite the *fire damp* of the mines; but though the fire damp may ignite after passing the gauze and may fill the whole lamp with a body of flame, yet, owing to the cooling effected by the conduction of the wires, the gases on the outside are not raised to the temperature of ignition; thus warning, and time for escape from danger, are given.

505. Convection of Heat.—Liquids and gases are heated almost entirely by convection. As heat is applied to the sides and bottom of a vessel of water, the heated particles become

specifically lighter, and are crowded up by heavier ones which take their place. There is thus a constant circulation going on which tends to equalize the temperature of the whole, by bringing the hot portions into contact with the colder, and thus greatly facilitating the *conduction* of heat among the molecules.

Fig. 299.

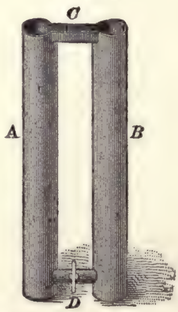


This motion is made visible in a glass vessel, by putting into the water some opaque powder of nearly the same density as water. Ascending currents are seen over the part most heated, and descending currents in the parts farthest from the heat, as represented in Fig. 299. The ocean has perpetual currents caused in a similar manner. The hottest portions flow away from the tropical toward the polar latitudes, while at greater depth the cold waters of high latitudes flow back toward the tropics.

For a like reason, the air is constantly in motion. The atmospheric currents on the earth have been considered in Chapter III. of Pneumatics.

506. Determination of the Temperature of Water at its Maximum Density.—The apparatus used by Joule in his research is represented in outline in Fig. 300, in which *A* and *B* are cylinders 4½ feet high and 6 inches in diameter; the open trough *C* connects them at top, and a large tube, with stop-cock *D*, connects them at the bottom. When the cylinders were filled so that there was a free flow through the trough *C*, any difference of density in *A* and *B* would produce a convection current through *D* and *C*, and the existence of such current in *C* was made known by the motion of a small glass bulb, of nearly the specific gravity of water, floating in the trough. A very slight difference of density between the water in *A* and *B*, gave motion to the bulb in *C*. The cock *D* being closed, the temperatures of *A* and *B* were adjusted so that one should be above and the other below that of maximum density. Having recorded the temperatures, *D* was opened, and any difference of density would be shown by a motion of the bulb towards the denser column. By carefully adjusting the temperatures, so that upon opening *D*

Fig. 300.



no motion in the trough C should result, a pair of temperatures was obtained, corresponding to the same density. From a series of such pairs, the differences of which were made successively smaller, Joule fixed the temperature of maximum density at 39.1° F. or 3.94° C., very nearly.

507. Radiation of Heat.—*Radiation* of heat is the communication of the vibrations of the heated body to the ether surrounding it, by which the waves of heat are transmitted in the manner already explained in the article Light. Heat rays differ from rays of light only in wave length, and are capable of reflection, refraction, interference, and polarization. A body not hot enough to send forth rays affecting the optic nerve, still sends out heat rays, nor can any body be so cold as not to radiate heat at all.

The intensity of heat radiated from a given source, is governed by the three following laws :

1. *The intensity of radiated heat varies as the temperature of the source.*

2. *It varies inversely as the square of the distance.*

3. *It grows less, while the inclination of the rays to the surface of the radiant grows less.*

The truth of these laws is ascertained by a series of careful experiments. But the second may be proved mathematically from the fact of propagation in straight lines, as in sound and light. For the heat, as it advances in every direction from the radiant, is spread over spherical surfaces which increase as the squares of the distances ; therefore the intensities must grow less in the same ratio; that is, the intensities vary inversely as the squares of the distances.

The radiating power of a given body depends on the condition of its surface.

If a cubical vessel filled with hot water have one of its vertical sides coated with lamp black, another with mica, a third with tarnished lead, and the fourth with polished silver, and the heat radiated from these several sides be concentrated upon a thermometer bulb, the ratio of radiation will be found nearly as follows :

Lamp black	100	Tarnished lead	45
Mica	80	Polished silver	12

Polished metals generally radiate feebly ; and this explains the familiar fact that hot liquids retain their temperature much better in bright metallic vessels than in dark or tarnished ones.

When the temperature of a body is gradually raised, not only

are new kinds of radiations produced, whose wave lengths are smaller than those already emitted, but the intensity of existing radiations also increases. A white-hot body emits more red rays than a red-hot body, and more non-luminous rays than a non-luminous body.

508. Equalization of Temperature.—Radiation is going on continually from all bodies, more rapidly in general from those most heated ; and therefore there is a constant tendency toward an equal temperature in all bodies. A system of exchange goes on, by which the hotter bodies grow cool, and the colder ones grow warm, till the temperature of all is the same. But this equality does not check the radiation; it still goes forward, each body imparting to every other as much heat as it receives from it, the radiations emitted and absorbed by either body being equal not only in total heating effect, but being the same in the intensity, wave length, and plane of polarization of every component part of either radiation.

509. Reflection of Heat.—When rays of heat meet the surface of a body, some of them are *reflected,* passing off at the same angle with the perpendicular on the opposite side. But others pass *into* the body, and are said to be absorbed by it. It is true of waves of heat as of all other kinds of vibration, that when they meet a new surface and are reflected, the angle of incidence equals the angle of reflection, and that their intensity after reflection is weakened.

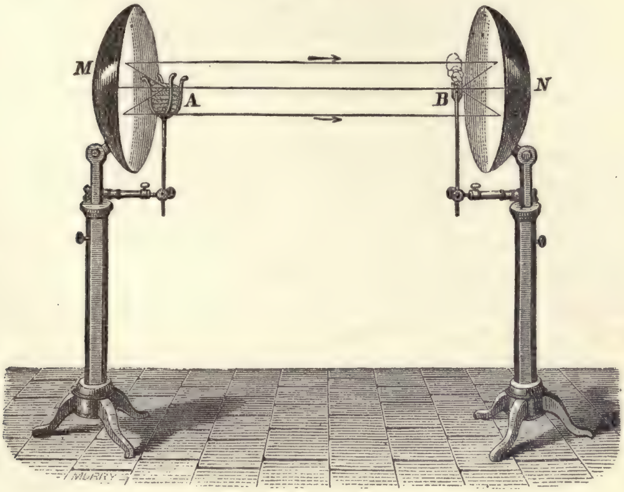
If a person, when near a fire, holds a sheet of bright tin so as to see the light of the fire reflected by it, he will plainly perceive that heat is reflected also. And if any *sound* is produced by the fire, as the crackling of combustion, or the hissing of steam from wood, the reflection of the sound is likewise heard. This simple experiment proves that waves of sound, of heat, and of light, all follow the same law of reflection.

510. Heat Concentrated by Reflection.—Let two polished reflectors, M and N (Fig. 301), having the form of concave paraboloids, be placed ten or fifteen feet apart, with their axes in the same straight line, and let a red-hot iron ball be in the focus A of one, and an inflammable substance, as phosphorus, in the focus B of the other; then the latter will be set on fire by the heat of the ball. The rays diverging from A to M are reflected in parallel lines to N, and then converged to B.

If, instead of phosphorus, the bulb of a thermometer is put in the focus B, a high temperature is of course indicated on the scale. Now remove the hot ball from A, and put in its place a lump of

ice; then the thermometer at *B* sinks far below the temperature of the room. This last experiment does not prove that *cold* is reflected as well as heat, but confirms what was stated (Art. 508),

FIG. 301.



that all objects radiate to one another till their temperatures are equalized. The ice radiates only a little heat, which is reflected to the thermometer, but the latter radiates much more, which is reflected to the ice, so that the temperature of the thermometer rapidly sinks.

511. Absorption of Heat.—So much of the radiant heat as falls on a body and is not reflected, is absorbed. The absorbing power in a body is found to be in general equal to its radiating power. It is very noticeable that bodies equally exposed to the radiant heat of the sun or a fire, become very unequally heated. A white cloth on the snow, under the sunshine, remains at the surface; a black cloth sinks, because it absorbs heat, and melts the snow beneath it. Polished brass before a fire remains cold; dark, unpolished iron, is soon hot.

Lamp black reflects little of the radiation which falls on it; nearly the whole is absorbed.

Polished silver reflects the greater part of the radiations falling upon it, absorbs only about 2½ per cent., and transmits none.

Rock salt reflects less than 8 per cent. of the radiation it receives, absorbs almost none, and transmits 92 per cent.

512. Diathermancy.—Substances which transmit heat rays, without themselves becoming hot, are called *Diathermanous;*

those which are heated by the transmission are said to be *Athermanous.*

Radiant heat passes freely through the atmosphere as well as through vacant space. The air is therefore said to be *diathermal;* it is also transparent, since it permits light to pass freely through it. But there are substances which allow the free transmission of the waves of light, but not those of heat; and there are others through which waves of heat can freely pass, but not those of light.

Water and glass, which are almost perfectly transparent to the faintest light, will not transmit the vibrations of heat unless they are very intense. If an open lamp-flame shines upon a thin film of ice, while nearly the whole of the *light* is transmitted, only 6 per cent. of the *heat* can pass through.

A plate of rock salt, one-tenth of an inch thick, will, as shown in the last paragraph, transmit 92 per cent. of the heat of a lamp; and if it be coated with lampblack so thick as to stop light completely, the heat is still transmitted with almost no diminution.

Prisms and lenses of rock salt have been used in illustrating refraction of heat, just as glass prisms and lenses are used in the case of luminous rays.

CHAPTER III.

SPECIFIC HEAT.—CHANGES OF CONDITION.—LATENT HEAT.

513. Specific Heat.—The heat which is absorbed by a body is not wholly employed in raising its temperature. While a part of the thermal force which is communicated, throws the atoms into vibration, that is, heats the body, another part performs interior work of some other kind, such as urging the atoms asunder, or forcing them into new arrangements. This latter portion is lost to our sense and to the thermometer, until the body is again cooled, when it reappears. The relative quantity of the force thus hidden from view is different in different substances. Hence the phrase *specific heat,* is used to express the amount of heat required to raise a given weight of a substance one degree of temperature as compared with the amount required to raise an equal weight of water one degree. The specific heats of nearly all solids and liquids increase with the temperature.

The specific heat of a gas is nearly constant at all temperatures and under all pressures.

The mean specific heats of a few substances, between 0° and 100° C. are given below to show how greatly they differ:

Water	1.0000	Silver	.0570
Ice	.5040	Tin	.0562
Sulphur	.2026	Mercury	.0333
Iron	.1138	Gold	.0324
Copper	.0951	Lead	.0314

The specific heat of water is greater than that of every other substance known, and therefore it is made the standard of comparison.

The great specific heat of water moderates the changes of temperature upon islands and upon the sea-coast. A pound of water, losing one degree of temperature, would raise about 4.2 lbs. of air one degree, the specific heat of air being 0.2379. But water is 770 times as heavy as air; hence the 4.2 lbs. of air is 3234 times the volume of one pound of water.

The *Thermal unit* is variously given as the amount of heat required to raise 1 lb. of water 1° F., or 1 lb. of water 1° C., or 1 kilogramme of water 1° C.

The thermal units which we shall use are the amounts of heat necessary to raise the temperature of a pound of water 1° C. or 1° F., as may be indicated in the text.

514. Method of Finding Specific Heat.—The following is one of several methods of finding the specific heat of a substance; it is called the *method of mixtures.*

Let a pound of mercury at 100° C. be poured into a pound of water at 0° C., and suppose the temperature of the mixture to be 3.2° C. Let x equal the specific heat of mercury. Now one pound of water has been raised from 0° C. to 3.2° C., requiring for this change $3\frac{1}{5}$ thermal units; one pound of mercury has cooled from 100° C. to 3.2° C., thus giving out $x \times 96.8$ thermal units, x representing that fraction of a thermal unit which would raise one pound of mercury one degree; as no heat has been lost, these two amounts must be equal, and we have $3.2 = (96.8)\, x$, from which we find $x = .033$.

Suppose five pounds of iron at 100° C. to be put into ten pounds of water at 10° C., what will be the temperature of the mixture? Let $x°$ be the resulting temperature. The 10 lbs. of water will absorb $10\ (x - 10)$ units of heat; the 5 lbs. of iron will give out $5\ (100 - x) \times .1138$ units of heat, and these two amounts must be equal; hence, $10\ (x - 10) = 5\ (100 - x) \times .1138$, whence $x = 14.8°$ C.

The specific heats of substances are also found by determining the amounts of ice at 0° C., or 32° F., which they will melt in cooling from a given temperature to that of melting ice.

The specific heat of a substance in a liquid state is generally greater than in the solid form. The specific heats of the more perfect gases are nearly equal to that of air, which is 0.237.

515. Apparent Conduction Affected by Specific Heat.—The conducting power of different substances cannot be correctly compared, without making allowance for their specific heat (Art. 501). For the heat which is communicated to one end of a rod, will collect at the other end more slowly, if a great share of it disappears on the way. For instance, at the same distance from the source of heat, wax is melted quicker on a rod of bismuth than on one of iron, though iron is the best conductor, because the specific heat of iron is three times as great as that of bismuth ; the heat actually reaches the wax soonest through the iron, but not enough to melt it, because so much is required to raise the iron to a given temperature.

516. Changes of Condition.—Among the most important effects produced by heat, are the changes of condition from solid to liquid and from liquid to gas, or the reverse, according as the temperature of a body is raised or lowered. Increase of heat changes ice to water, and water to steam, and the diminution of heat reverses these effects. A large part of the simple substances, and of compound ones not decomposed by heat, undergo similar changes at some temperature or other ; and probably it would be found true of all if the requisite temperature could be reached.

The *melting point* (called also *freezing point*, or *point* of *congelation*) of a substance is the temperature at which it changes from a solid to a liquid or the reverse.

The *boiling point* is the temperature at which it changes from a liquid to a gas or the reverse.

517. Latent Heat.—Whenever a solid becomes a liquid, or a liquid become a gas, a large amount of heat disappears, and is said to become *latent*. The thermal force is expended in sundering the atoms, and perhaps in putting them into new relations and combinations, so that there is not the slightest increase of temperature after the change begins till it ends. The force is not *lost*, but is treasured up in the form of *potential energy*, which becomes available whenever a change is made in the opposite direction. Using the force of heat to turn water into steam, is like using the strength of the arm in coiling up a spring, or lifting a weight from the earth. The spring and the weight are each in

a condition to perform work. They have potential energy, which can be used at pleasure.

It has been already noticed that much heat disappears in bodies of great specific heat, as their temperature rises. But the amount which becomes latent, while a change of condition takes place, is vastly greater.

Let heat be supplied at a uniform rate to a mass of water, n pounds, at 0° C., and note the time required to raise it to 100° C.; continuing the same uniform supply of heat, it will take 5.37 times as long to change the n pounds into steam at 100° C. In raising the temperature from 0° to 100° the number of thermal units required was $100 \times n$, and in changing into steam 5.37 times as many thermal units were added, or $537 \times n$ thermal units. The whole of the $537 \times n$ thermal units have been employed in rearranging the atoms, without producing any change of temperature. The latent heat of water is 80; that is to say, it will require $n \times 80$ thermal units to change n pounds of ice at 0° C. into water also at 0° C.

The latent heat of steam is not the same for all temperatures; the total number of heat units, Q, required to change n pounds of water at $t°$ C. into steam at $t°$ C. is expressed by the formula

$$Q = (606.5 + 0.305\ t)\ n.$$

For 100° C., $Q = 637\ n$; for 150° C., $Q = 651\ n$; for 200° C., $Q = 667.5\ n$.

518. Fusion or Melting.—The change from the solid to the liquid state may be either very gradual or very abrupt. As the temperature rises, many substances become pasty, like wrought iron at white heat, and for a considerable range of temperatures such substances are neither solid nor liquid, and no definite melting point can be assigned. Ice passes very abruptly from the solid to the liquid state, probably during a rise of temperature not greater than 0.1° C.

From the beginning of fusion till the end of the change of condition there is no rise of temperature, the heat which does internal work being termed *latent heat of fusion*.

The latent heat of fusion of ice has already been given (Art. 517). The melting points of a few substances are given below:

Ice	0° C.	Tin	235° C.
Spermaceti	49° C.	Lead	325° C.
White Wax	65° C.	Silver	1000° C.
Sulphur	111° C.	Iron	1500° C.

The melting point of a substance which expands on solidifying is lowered by great increase of pressure above the ordinary

pressure of the atmosphere, while that of a substance which contracts in solidifying is raised. The melting point of wax was raised from 65° C. to 75° C. by a pressure of 520 atmospheres, while the melting point of ice is lowered about 0.0074° C. for every additional pressure of one atmosphere.

Alloys are generally more fusible than the metals of which they are composed.

519. Vaporization.—The change from the liquid to the gaseous state is termed *vaporization.* This change is sometimes effected quietly without the formation of bubbles, then termed *evaporation,* and sometimes in a violent manner with the formation of bubbles, to which action the term *ebullition* is applied.

Vaporization is more rapid as the pressure upon the surface of the liquid is diminished.

The boiling point of water at one atmosphere, at the level of the ocean, is 100° C.; but upon the tops of high mountains the boiling point is 90° and 85° C., and in the air pump vacuum it is as low as 23° C.

The effect of diminished pressure to lower the boiling point is well shown by the following familiar experiment: In a thin glass flask, boil a little water, and after removing it from the fire, cork and invert the flask. The steam which is formed will soon press so strongly upon the water as to stop the boiling. When this happens, pour a little cold water upon the flask; the water within will immediately commence boiling violently, because the vapor is condensed and the pressure removed. This effect may be reproduced several times before the water in the flask is too cool to boil in a vacuum.

520. Other Causes Affecting the Boiling Point.—The boiling point is raised by substances in solution, provided they are less volatile than the liquid in which they are dissolved.

Water saturated with common salt boils at 109° C., and when chloride of calcium replaces the salt the boiling point is raised to 179° C. Substances held in suspension, but not dissolved, have no effect upon the boiling point.

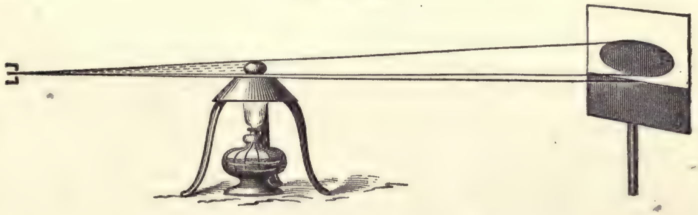
Water from which the dissolved air has been removed by previous ebullition, has been raised to 112° C. before boiling, the elastic air seeming to act as a spring to aid ebullition.

Water boils at a higher temperature in glass vessels than in metallic ones, rising as high as 105° C. before ebullition begins. If metal clippings or filings, or any angular fragments whatever which may serve as a nucleus, be dropped into the flask, the boiling point is brought down to 100° C., and the violent bumping which accompanies ebullition at the higher temperatures is prevented.

521. Spheroidal Condition.—When a little water is placed in a red-hot metallic cup, instead of boiling violently, and disappearing in a moment, as might be expected, it rolls about quietly in the shape of an oblate spheroid, and wastes very slowly. So drops of water, falling on the horizontal surface of a very hot stove, are not thrown off in steam and spray with a loud hissing sound, as they are when the stove is only moderately heated, but roll over the surface in balls, slowly diminishing in size till they disappear.

In such cases, the water is said to be in the *spheroidal state.* Not being in contact with the metal, it assumes the shape of an oblate spheroid, in obedience to its own molecular attractions and the force of gravity, as small masses of mercury do on a table. The reason why the water does not touch the hot metal is, that the heat causes a coat of vapor to be instantly formed about the drop, on which it rests as on an elastic cushion ; and as the vapor is a poor conductor of heat, further evaporation proceeds very slowly. It is easily seen that the spheroid does not touch the metal, by so arranging the experiment that a beam of light may shine horizontally upon the drop, and cast its shadow completely separated from that of the hot plate below it, as in Fig. 302.

Fig. 302.



If the heated surface is cooling, the temperature may become so low that the drop at length touches it, when in an instant violent ebullition takes place, and the water quickly disappears in vapor.

522. Evaporation.—Many liquids and even solids pass into the gaseous state by a slow and almost insensible process, which goes on at the *surface.* This is called *evaporation ;* and it takes place at all temperatures, but more rapidly as the temperature is higher. Ice and snow waste away gradually at temperatures far below 0° C., and the odor of brass, copper, and iron is attributed to an insensible evaporation of these metals.

523. Condensation.—The change from the condition of vapor to the liquid state is called *condensation.* This change of state may be caused by cooling and by compression. A saturated

vapor at any given temperature and pressure will be partially condensed by either lowering the temperature or by increasing the pressure. Those gases which have usually been called *permanent gases,* because, under ordinary conditions they are very far removed from their point of condensation, have been reduced to the liquid state by very low temperatures and great pressures combined.

Vapors give up their latent heat of vaporization during the process of condensation ; the latent heat of steam may be determined by passing a known weight of steam at 100° C. into a given quantity of water at a known temperature, and taking the resulting temperature.

Suppose 1 lb. of steam at 100° C. to be condensed by 6 lbs. of water at 0° C., and that the resulting temperature is 91° C. The 6 lbs. of water raised from 0° to 91° required $6 \times 91 = 546$ heat units ; the pound of steam, after condensation at 100°, gave up 9 of these 546 units in cooling from 100° to 91°, leaving 537 heat units as the latent heat of vaporization.

524. Solidification.—Substances which have been melted and which cool slowly while passing into the solid state usually assume a regular crystalline structure. If they expand on solidifying, the solid will float in the liquid, but if they contract the solid will sink.

A liquid may be cooled below its normal temperature of solidification. A hot saturated solution of Glauber's salt, cooled slowly and at rest, will remain liquid at the ordinary temperature of the atmosphere ; but upon being suddenly jarred, or when a crystal of the salt is dropped into the liquid, the molecular equilibrium is destroyed and solidification ensues at once. Water which has been boiled, to free it from air, may be cooled to —10° C., or even lower, without freezing; but any vibration causes instant crystallization. In all such cases the latent heat of fusion becomes sensible, and may be felt by placing the hands upon the containing vessel.

The freezing point of water containing salt in solution is lower than that of pure water. Sea water freezes at —2.5° C. to —3° C. ; the ice is pure, containing none of the salt.

525. Freezing Produced by Melting.—Since a great amount of heat disappears in a substance as it passes from the solid to the liquid state, the loss thus occasioned may produce freezing in a contiguous body. When salt and powdered ice are mixed, their union causes liquefaction. And if this mixture is surrounded by bad conductors, and a tin vessel containing some

liquid be placed in the midst of it, the latter is frozen by the abstraction of heat from it, by the melting of the ice and salt. In this way ice creams and similar luxuries are easily prepared in hot as well as in cold weather.

526. Freezing by Evaporation.—In like manner, freezing by evaporation is explained. Put a little water in a shallow dish of thin glass, and set it on a slender wire-support under the receiver of an air pump. Beneath the wire-support place a broad dish containing sulphuric acid. When the air is exhausted, the water in a few moments is found frozen. As the pressure of the air is taken off, evaporation proceeds with increased rapidity, and the requisite heat for this change of condition can be taken only from the dish of water. But the atmosphere of vapor retards the process by its pressure ; hence the sulphuric acid is placed in the receiver, so as to seize upon the vapor as fast as formed, and thus render the vacuum more complete. The water is frozen by giving up its heat to become latent in the vapor, so rapidly formed ; but when this vapor becomes liquid again in combining with the acid, the same heat reappears in raising the temperature of the acid.

Thin cakes of ice may sometimes be procured, even in the hottest climates, by the evaporation of water in broad shallow pans under the open sky, where radiation by night aids in reducing the temperature. The pans should be so situated as to receive the least possible heat by conduction.

Various ice-making machines have been devised in which the vaporization of some volatile liquid, such as ether, liquid ammonia, liquid sulphurous acid, &c., abstracts sufficient heat from the water to freeze it.

527. Regelation.—If two pieces of ice at 0° C., having smooth surfaces, be pressed together, they will soon adhere, and will do this in air, in water, or in vacuo. This freezing together again is called *regelation.*

The interior of a block of melting ice is a little colder than the surface : now when the two surfaces are pressed together, the very thin film of water which covers them is removed from the warmer air, and is in the same condition as though transferred to the interior of a block, the lower temperature of which freezes it.

CHAPTER IV.

TENSION OF VAPOR.—THE STEAM-ENGINE.—MECHANICAL EQUIVALENT OF HEAT.

528. Dalton's Laws.—

1. Whatever be the temperature of a liquid which partly fills a vessel, vaporization will go on till the vessel is filled with vapor, of a density determined solely by the temperature, after which vaporization will cease.

2. If the space occupied by the vapor be made larger, the temperature being the same, then vaporization will again go on till the density is the same as before. If the space be made smaller, the temperature remaining constant, a part of the vapor returns to the liquid state, and the remaining vapor will have the same density as before.

3. If, besides the liquid and its vapor, the vessel contains any gas, *not capable of chemical action on the liquid*, then exactly the same amount of vapor, of the same density as before, will be formed; but the time required to reach the maximum density will be greater because of the mechanical obstruction to a rapid diffusion, which the gas offers.

Fig. 303.

A vapor at the maximum density and pressure for the given temperature is called a *saturated* vapor.

529. Experimental Illustration.—Fill a barometer tube $A\ B$ (Fig. 303) full of mercury; close the open end with the finger and invert into the cup H of the deep mercury cistern $H\ K$. With a pipette, the tube of which is bent upwards at the end, transfer enough ether to the barometer tube to leave a thin film of liquid $c\ d$, after the space $A\ d$ is filled with saturated vapor. Measure the height $c\ H$ of the mercury column. If the tube $A\ B$ be raised, tending to increase the space $A\ d$ above the liquid, more vapor will form and $c\ H$ will remain unaltered; if the tube $A\ B$ be depressed, tending to diminish the space $A\ d$, vapor will condense to liquid again, and $c\ H$ will still be unaltered.

To show the effect of change of temperature use a barometer

tube bent at its closed end as in Fig. 304, so that a portion of the bend may either be surrounded with cooling mixtures, as at *A*, or may be warmed by a flame. Upon raising the temperature of the contained vapor its tension will increase and the mercury column *C B* will be shortened; upon lowering the temperature the tension will decrease, and *C B* will lengthen as the mercury rises.

FIG. 304.

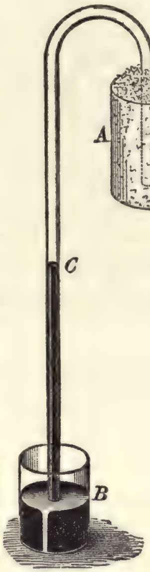
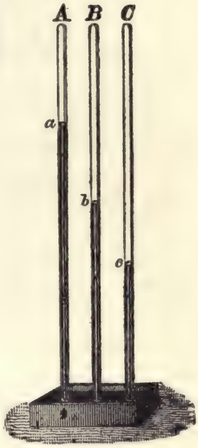


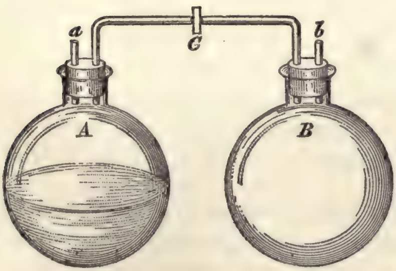
FIG. 305.



530. Tensions of Different Vapors. — Transfer to three barometer tubes, *A*, *B*, and *C* (Fig. 305), water, alcohol and ether respectively, and the mercury columns will stand at different heights *a*, *b* and *c*, showing that the tensions of the three vapors are not the same at the same temperature.

531. Tension in Generator and Condenser.— Let two vessels, *A* and *B* (Fig. 306), be connected by a pipe furnished with a stop-cock *C*. Let the tubes *a* and *b* be connected with separate manometers to indicate the tensions of the vapor in *A* and *B* when *C* is closed. Having partly filled *A* with water, cause it to boil until all air has been driven from both flasks through *C* and the loosened stopper of *B*; now close *B* and remove the lamp. The two manometers will indicate the same tension in both flasks.

FIG. 306.



Now close *C* and surround *B* with cold water; part of the vapor in it will be condensed, the remainder having a greatly reduced tension as shown by the fall of the mercury column in its manometer. Apply heat to *A*, thus forming new vapor of higher tension than before, as will be shown by its manometer reading.

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If now C be opened the manometer connected with A will fall to the same reading as that of B, and the two will indicate this same reading just as long as the temperature of B is kept constant and below that of A.

The liquid in A merely distills over to B, at the tension of the vapor in the colder vessel.

532. Thermal Force in Steam.—It has been already noticed that while water is heated, and especially while it is converted into steam by boiling, the heat apparently lost is so much *force* treasured up ready for use, as truly as when strength is expended in lifting great weights, which by their descent can do the work desired. In modern engineering, the force of steam is employed more extensively, and for more varied purposes, than any other. Every steam-engine is a machine for transforming the internal motion of heated steam into some of the visible forms of motion.

533. Tension of Steam.—When steam is formed by boiling water in the open air, its tension is equal to that of the air, and therefore ordinarily about fifteen pounds to the square inch. But when it is formed in a tight vessel, so that it cannot expand, as the temperature of the water is raised the tension is increased in a much greater ratio; because the same steam has greater tension at a higher temperature, and besides this, new steam is continually added.

The following table gives the temperature corresponding to various atmospheres of tension:

Atmosphere.	Temperature.		Atmosphere.	Temperature.	
1	F. 212	C. 100	14	F. 384	C. 195.5
2	249	120.6	15	390	198.8
4	291	144.0	19	411	210.4
5	306	152.2	20	415	213.0
9	348	175.8			
10	356	180.3			

It is seen by the above table that 37° F. or 20.6° C. are required to add the second atmosphere of tension, while only 4° F. or 2.6° C. are required to add the twentieth atmosphere.

534. Relation of Temperature, Pressure and Volume.—The specific heat of water is not constant, but varies with the temperature (Art. 513), and hence we cannot assume that 180° F. heat units are required to raise 1 lb. of water from 32° F. to 212° F., nor that 250 heat units will suffice to raise 1 lb. of water from 32° F. to 282° F.; in the first case 180.9 heat units are required, and in the second 252.2 units.

The latent heat of vaporization is also variable, being 965.7 heat units at 212° F., and 939.4 units at 249° F.

A formula for the latent heat of vaporization of water, derived from Regnault's experiments, is $R = 1091.7 - 0.695\ (t - 32)$, in which R = number of heat units (F.) required to convert 1 lb. of water at $t°$ F. into steam at that temperature. The total heat units (F.) required to raise 1 lb. of water from 32° F. to $t°$ F., and evaporate it at that temperature is

$$L = 1091.7 + 0.305\ (t - 32).$$

The volume of steam at $t°$ F. as compared with the volume of water at 39.2 F. which gave it is found, very nearly, from

$$V = \frac{1 + 0.00204\ (t - 32)}{0.000055\ p},$$

p being the pressure, in pounds per square inch, corresponding to the temperature $t°$ F.

The following table illustrates the various relations which have just been discussed:

Pressures in Atmospheres.	Pressures in lbs. per sq. inch.	Temperatures in F.°	No. of Heat Units (F°) to raise 1 lb. of Water from 32° to given Temperature.	No. of Heat Units (F°) to vaporize 1 lb. of Water at Temperature given in Column 3.	Volume of Steam at given Temperature as compared with the Volume of Water at 39.2° F. which gave it.
1	14.7	212.0	180.90	965.7	1691
2	29.4	249.1	218.55	939.4	892
3	44.1	273.1	242.98	922.2	615
4	58.8	291.2	261.56	909.2	473
5	73.5	306.0	276.73	898.5	386
6	88.2	318.6	289.69	889.4	327
7	102.9	329.6	301.04	881.4	284
8	117.6	339.4	311.20	874.3	252

535. The Steam-Engines of Savery and Newcomen. —The only steam-engines that were at all successful before the great improvements made by Watt, were the engine of Savery and that of Newcomen. No other purpose was proposed by either than that of removing water from mines.

In the engine of Savery, steam was made to raise water by acting on it directly, and not through the intervention of machinery.

It consisted of a boiler B (Fig. 307); a cylinder A, with a valve at c opening inward, and one at d opening outward; a pipe e to discharge cold water upon the cylinder, and a steam pipe f, from the boiler to the cylinder.

First the steam-cock at f is opened and steam fills the cylinder A, driving the air out through the valve d. Next f is closed and

the cock *e* is opened, allowing cold water to flow over the cylinder from the delivery pipe *O*, thus condensing the steam in *A*, and creating a vacuum, into which the atmosphere forces water from the supply *P*, through the valve *c*. Now *e* is closed and *f* opened

Fig. 307.

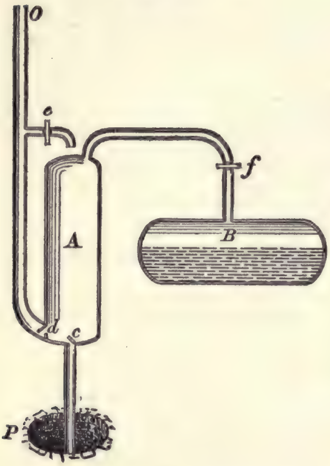
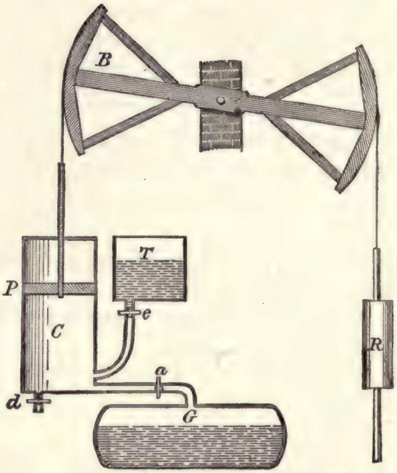


Fig. 308.



again, and steam enters the cylinder *A* and drives the water out through *d*. When *A* is full of steam the operation is repeated as before.

Newcomen used steam to work a common pump. The weighted pump rod *R* (Fig. 308) was attached to one end of a working beam *B*, while at the other end of *B* was hung the piston *P*, working steam tight in the cylinder *C*. Steam at atmospheric pressure from the boiler *G* enters *C* through the cock *a*, and *P* being pressed upon equally on both sides is drawn to the top of *C* by the weight of the pump rod *R*. Now *a* is closed and *e* is opened, permitting cold water from a tank *T* to flow into *C*, which condenses the steam, creating a vacuum, and allows the piston to descend under atmospheric pressure. When *P* has reached the bottom of *C*, *e* having been closed, *a* and *d* are opened and steam enters the cylinder, while the injection water from *T* flows out through *d*, and the piston *P* rises as at first. On closing *a* and *d* and opening *e* the stroke is repeated.

As the water was raised by the direct pressure of the atmosphere, this invention of Newcomen was called the *atmospheric engine.*

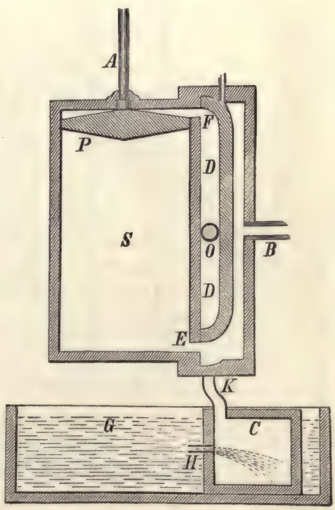
In these diagrams of Savery's and Newcomen's engines, all details of valves or other working parts have been omitted, that the principle alone might claim attention.

In neither of these methods was steam used economically as a power. The movements in both cases were sluggish, and a large part of the force was wasted, because the steam was compelled to act upon a cold surface, which condensed it before its work was done.

536. The Steam-Engine of Watt.—Steam did not give promise of being essentially useful as a power till Watt, in the year 1760, made a change in the atmospheric engine, which prevented the great waste of force. Newcomen introduced the cold water which was to condense the steam into the steam cylinder itself; and the cylinder must be cooled to a temperature below 100° F., else there would be steam of low tension to retard the descent of the piston. But when the piston was to be raised, the cylinder must be heated again to 212° F., in order that the admitted steam might balance the pressure of the air.

In the engine of Watt, the steam is condensed in a separate vessel called the *condenser*. The steam cylinder is thus kept at the uniform temperature of the steam. In the first form which he gave to his engine, he so far copied the atmospheric engine as to allow the piston, after being pressed down by steam, to be raised again by the load on the opposite end of the great beam, while the steam circulates freely below and above the piston. This was called the *single-acting* engine, and might be successfully used for the only purpose to which any steam-engine was as yet applied, namely, pumping water from mines. But he almost immediately introduced the change by which the whole force of the steam was brought to act on the upper and the under side of the piston. It thus became *double-acting*, and the steam force was no longer intermittent.

Fig. 309.

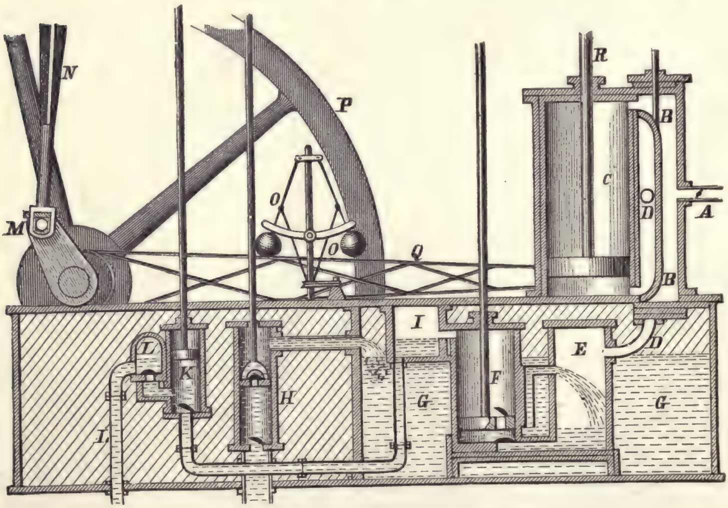


537. The Double-acting Engine.—Let S (Fig. 309) be the steam cylinder, P the piston, A the piston rod, passing with steam-tight joint through the top of the cylinder, C the condenser, kept cold by the water of the cistern G, B the steam pipe from the boiler, K the eduction pipe, which opens into the valve chest at O, D D the D-valve,

E F the openings from the valve-chest into the cylinder. As the D-valve is situated in the figure, the steam can pass through *B* and *E* into the cylinder below the piston, while the steam above the piston can escape by *F* through *O* and *K* to the condenser, where it is condensed as fast as it enters; so that in an instant the space above the piston is a vacuum, while the whole force of the steam is exerted on the under side. The piston is therefore driven upward without any force to oppose it. But before it reaches the top, the D-valve, moved by the machinery, begins to descend, and shut off the steam from *E* and admit it to *F*, and, on the other hand, to shut *F* from the eduction pipe *O*, and open *E* to the same. The steam will then press on the top of the piston, and there will be a vacuum below it, so that the piston descends with the whole force of the steam, and without resistance. To render the condensation more sudden, a little cold water is thrown into the condenser at each stroke through the pipe *H*.

538. Condensing Engine.—The principle of the condensing engine is illustrated by the figure and description of the preceding article. But the condensing apparatus of this kind of engine requires many other parts, most of which are presented in Fig. 310. *C* is the steam cylinder; *R* the rod connecting its

FIG. 310



piston with the end of the working beam, not represented; *A* the steam-pipe and throttle-valve; *B B* the D-valve; *D D* the eduction pipe, leading from the valve-chest to the condenser *E*; *G G*

the cold water surrounding the condenser; F the air-pump, which keeps the condenser clear of air, steam, and water of condensation; I the hot well, in which the water of condensation is deposited by the air-pump; K the hot-water pump, which forces the water in the hot well through L to the boiler; H the cold-water pump, by which water is brought to the cistern G G; the rods of all the pumps, F, K, and H, are moved by the working beam; P the fly-wheel; M the crank of the same, N the connecting-rod, by which the working beam conveys motion to the fly-wheel; Q the excentric rod, by which the D-valve is moved; O O the governor, which regulates the throttle-valve in the steam-pipe A.

There is much economy of fuel and saving of wear in the machinery, arising from the proper adjustment of the valves. If the steam enters the cylinder during the whole length of a stroke of the piston, its motion is *accelerated;* and is therefore swiftest at the instant before being stopped; thus the machinery receives a violent shock. If the valve is adjusted to *cut off* the steam when the piston has made one-third or one-half of its stroke, the diminishing tension may exert about force enough, during the remaining part, to keep up a uniform motion. The *cut-off*, however, should be regulated in each engine, according to friction and other obstructions.

539. Non-condensing Engine. — For many purposes, especially those of locomotion, it is advantageous to dispense with the large weight and bulk of machinery necessary for condensation, and do the work with steam of a higher tension. If (Fig. 310) the condenser, cistern, and all the pumps are removed, then the steam is discharged from E and F at each stroke into the air. Therefore the steam in that part of the cylinder which is open to the air, will have a tension of 15 lbs. per inch; and, consequently, the steam on the opposite side of the piston must have a tension 15 lbs. per inch greater than before, in order to do the same work.

Steam of a pressure not greater than 45 lbs. per inch (above the atmosphere) is called *low pressure* steam, or *low steam; high steam* is at a pressure above this, and not uncommonly runs higher than 200 lbs. per inch by the gauge.

540. Estimation of Steam Power.—It is customary to express the power of a steam-engine by comparing the work done by it with that which horses can do. In making this comparison Watt took as a measure of *one horse-power* the ability to raise 2,000,000 lbs. through the height of *one foot* in an hour; or

2,000,000 *foot-pounds* per hour. The modern expression for one horse-power is 33,000 ft. lbs. per minute, or 1,980,000 ft. lbs. per hour. The horse-power of an engine varies with the pressure of steam used, and with the speed at which it is run ; hence it is absurd to assign a special horse-power to an engine unless the conditions of the pressure and speed are given also.

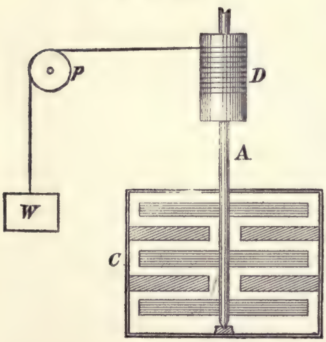
541. Mechanical Equivalent of Heat.—In all cases in which mechanical force produces heat, and again in all those in which heat produces visible motion, careful experiment proves that heat and mechanical force may each be made a measure of the other. Forces of any kind may be compared, by observing the weights which they will lift through a given distance. The *mechanical equivalent of heat* (commonly called, from the name of an English experimenter, Joule's equivalent) is given in the following statement:

The force required to heat one pound of water one degree F., is equal to that which would lift 772 pounds the distance of one foot, or is equal to 772 foot-pounds.

The mode of determining this value of the mechanical equivalent is the following:

A weight W (Fig. 311), by means of a cord passing over a pully p and around a drum D, gives to the vertical axis A a rapid rotation. Attached to this axis are a number of radial arms, or paddles, as shown in the figure; projecting from the sides of the cylinder C, in which these arms rotate, are fixed arms, as shown, to arrest any tendency to a rotary motion of the water in the cylinder.

FIG. 311.



If one pound of water at 60° F. be put into the cylinder C, it will require the expenditure of 772 foot-pounds of energy on the part of the falling weight W to raise its temperature by agitation to 61° F.

The force requisite to raise one pound of water 1° F., is sometimes called the *thermal unit* (Art. 513), and all forces may be brought to this as a standard of comparison. Thus, one horse-power (2,000,000 foot-pounds per hour) is 2,590 thermal units per hour, or about 43 per minute.

Since a force of 772 foot-pounds is expended in heating a pound of water 1° F., therefore to heat the same from 32° to 212°

requires a force of 138,960 foot-pounds; and to change the same pound of water into steam of atmospheric tension requires an additional force of 746,900 foot-pounds (Art. 517).

The efficiency of the best types of condensing *engines* does not exceed about 65 per cent. of the work of the steam *delivered to them;* while the engine and boiler combined do not realize more than 20 per cent. of the energy of the fuel consumed.

CHAPTER V.

TEMPERATURE OF THE ATMOSPHERE.—MOISTURE OF THE ATMOSPHERE.—DRAFT AND VENTILATION.

542. Manner in which the Air is Warmed.—The space through which the earth moves around the sun is intensely cold, probably 75° below zero; and the one or two hundred miles of height occupied by the atmosphere is too cold for animal or vegetable life, except the lowest stratum, three or four miles in thickness. This portion receives its heat mainly by convection. The radiated heat of the sun passes through the air, warming it but little, and on reaching the earth is partly absorbed by it. The air lying in contact with the earth, and thus becoming warmed, grows lighter and rises, while colder portions descend and are warmed in their turn. So long as the sun is shining on a given region of the earth, this circulation is going on continually. But the heated air which rises is expanded by diminished pressure, and thus cooled. Hence the circulation is limited to a very few miles next to the earth.

543. Limit of Perpetual Frost.—At a moderate elevation, even in the hottest climate, the temperature of the air is always as low as the freezing point. Hence the permanent snow on the higher mountains in all climates. The limit at the equator is about three miles high, and, with many local exceptions, it descends each way to the polar regions, where it is very near the earth. The descent is more rapid in the temperate than in the torrid or frigid zones.

544. Isothermal Lines.—These are imaginary lines on each hemisphere, through all those points whose mean annual temperature is the same. At the equator, the mean temperature is about 82° F., and it decreases each way toward the poles, but not equally

on all meridians. Hence the isothermal lines deviate widely from parallels of latitude. Their irregularities are due to the difference between land and water, in absorbing and communicating heat, to the various elevations of land, especially ranges of mountains, to ocean currents, &c. In the northern hemisphere, the isothermal lines, in passing westward round the earth, generally descend toward the equator in crossing the oceans, and ascend again in crossing the continents. For example, the isothermal of 50° F., which passes through China on the parallel of 44°, ascends in crossing the eastern continent, and strikes Brussels, lat. 51°; and then on the Atlantic, descends to Boston, lat. 42°, whence it once more ascends to the N. W. coast of America. The lowest mean temperature in the northern hemisphere is not far from zero, but it is not situated at the north pole. Instead of this, there are *two* poles of greatest cold, one on the eastern continent, the other on the western, near 20° from the geographical pole. There are indications, also, of two south poles of maximum cold.

545. Moisture of the Atmosphere.—By the heat of the sun all the waters of the earth form above them an atmosphere of vapor, or invisible moisture, having more or less extent and tension, according to several circumstances. Even ice and snow, at the lowest temperatures, throw off some vapor.

At a given temperature, there can exist an atmosphere of *vapor* of the same height and tension, whether there is an atmosphere of *oxygen* and *nitrogen* or not (Art. 528). Vapor is not *suspended* in the air, or *dissolved* by it, but exists independently. And yet it is by no means always true that there is actually the same tension of vapor as there would be if it existed alone, because of the time required for the formation of vapor, on account of mechanical obstruction presented by the air; whereas, if no air existed, the vapor would form almost instantly.

546. Temperature and Tension of Vapor.—The degree of tension of vapor forming without obstruction, depends on its temperature, but varies far more rapidly, increasing pretty nearly in a geometrical ratio, while the heat increases arithmetically (Art. 534); thus the tension at 212° F. is 1 atmosphere, at 249° F. is 2 atmospheres, at 291° F. is 4 atmospheres, and at 339° F. is 8 atmospheres. Hence, if vapor should receive its full increment of tension, while the thermometer rises 10 degrees from 80° to 90°, a vastly greater quantity would be added than when it rises 10 degrees from 40° to 50°. On the contrary, if vapor is at its full tension in each case, much more water will be precipitated in cooling from 90° to 80° than from 50° to 40°.

547. Dew-point.—This is the temperature at which vapor, in a given case, is precipitated into water in some of its forms. If there was no air, the dew-point would always be the same as the existing temperature; since lowering the temperature in the least degree would require a diminished tension or quantity of vapor, some must therefore be condensed into water. But in the air the tension may not be at its full height, and therefore the temperature may need to be reduced several degrees before precipitation will take place. A comparison of the temperature with the dew-point is one of the methods employed for measuring the humidity of the air.

548. Measure of Vapor.—The measure of the vapor existing at a given time, is expressed by two numbers, one indicating its *tension,—i. e.,* the height of the column of mercury which it will sustain; the other, *humidity,—i.e.,* its quantity per cent., as compared with the greatest possible amount at that temperature. Thus, tension = 0.6, humidity = 83, signifies that the quantity of vapor is sufficient to support six-tenths of an inch of mercury, and is 83 hundredths of the quantity which *could* exist at that temperature. The greatest tension possible at zero F., is 0.04; at the freezing point, 0.18; at 80° F., 1.0. At the lowest natural temperatures, the maximum tension is doubled every 12° or 14°; at the highest, every 21° or 22°.

549. Hygrometers. — This is the name usually given to instruments intended for measuring the moisture of the air. But the one most used of late years is called the *psychrometer,* which gives indication of the amount of moisture by the degree of *cold* produced in evaporation; for evaporation is more rapid, and therefore the cold occasioned by it the greater, according as the air is drier. The psychrometer consists of two thermometers, one having its bulb covered with muslin, which is kept moistened by the capillary action of a string dipping in water.

The wet-bulb thermometer will ordinarily indicate a lower temperature than the dry-bulb; if, in a given case, they read alike, the humidity is 100. The instrument is accompanied by tables, giving tension and humidity for any observation.

Various formulæ and complete tables may be found in the "Smithsonian Meteorological and Physical Tables."

550.—Dew.—Frost.—The deposition called *dew* takes place on the surface of bodies, by which the air is cooled below its dew-point. It is at first in the form of very small drops, which unite and enlarge as the process goes on. Dew is formed in the evening or night, when the surfaces of bodies exposed to the sky

become cold by radiation. As soon as their temperature has descended to the dew-point, the stratum of air contiguous to them deposits moisture, and continues to do so more and more as the cold increases.

Of two bodies in the same situation, that will receive most dew which radiates most rapidly. Many vegetable leaves are good radiators, and receive much dew. Polished metal is a poor radiator, and ordinarily has no dew deposited on it.

Sometimes, however, good radiators have little dew, because they are so situated as to obtain heat nearly as fast as they radiate it. Dew is rarely formed on a bed of sand, though it is a good radiator, because the upper surface gets heat by conduction from the mass below. Dew is not formed on water, because the upper stratum sinks and gives place to warmer ones.

Bodies most exposed to the open sky, other things being equal, have most dew precipitated on them. This is owing to the fact, that in such circumstances, they have no return of heat either by reflection or radiation. If a body radiates its heat to a building, a tree, or a cloud, it also gets some in return, both reflected and radiated. Hence, little dew is to be expected in a cloudy night, or on objects surrounded by high trees and buildings.

Wind is unfavorable to the formation of dew, because it mingles the strata, and prevents the same mass from resting long enough on the cold body to be cooled down to the dew-point.

When the radiating body is cooled below the freezing point, the water deposited takes the solid form in fine crystals, and is called *frost*. Frost will often be found on the best radiators, or those exposed to the open sky, when only dew is found elsewhere.

551. Fog.—This form of precipitation consists of very small globules of water sustained in the lower strata of the air. Fog occurs most frequently over low grounds and bodies of water, where the humidity is likely to be great. If air thus humid mixes with air cooled by neighboring land, even of less humidity, there will probably be more vapor than can exist at the intermediate temperature, for the reason mentioned in Art. 546. The case may be illustrated thus. Let two masses of air of equal volumes be mixed, the temperature of one being 40° F., the other 60° F., and each containing vapor at the highest tension. Then the mixture will have the mean temperature of 50°, and the vapor of the mixture will also be the *arithmetical* mean between that of the two masses. But, according to the law (Art. 546), the vapor can only have a tension which is nearly a *geometrical* mean between the two, and that is necessarily lower than the *arithmetical* mean ; hence the excess must be precipitated. If 8 lbs. of vapor were in one volume

and 18 lbs. in the other, an equal volume of the mixture would have $\frac{1}{2}(8 + 18) = 13$ lbs. of moisture; but at the mean temperature of 50°, only $\sqrt{8 \times 18} = 12$ lbs. could exist as vapor; therefore *one pound* must be precipitated. And even if one of the masses had a humidity somewhat below 100, still some precipitation is likely to take place.

552. Cloud.—The same as fog, except at a greater elevation. Air rising from heated places on the earth, and carrying vapor with it, is likely to meet with masses much colder than itself, and depositions of moisture are therefore likely to take place. Mountain-tops are often capped with clouds, when all around is clear. This happens when lower and warmer strata are driven over them, and thus cooled below the dew-point. The same air, as it continues down the other side, takes up its vapor again, and is as transparent as it was before ascending. A person on the summit perceives a chilly fog driving by him, but the fog was an invisible vapor a few minutes before reaching him, and returns to the same condition soon after leaving him. The cloud *rests* on the mountain; but all the particles which compose it are swiftly *crossing over*. Clouds are often above the limit of perpetual frost; they then consist of crystals of ice.

553. Classification of Clouds.—The aspects of clouds are various, and depend in some measure at least on the circumstances of their formation. The usual classification is the following:

1. *Cirrus.*—This cloud is fibrous in its appearance, like *hair* or flax, sometimes straight, sometimes bent, and frequently at one end is gathered into a confused heap of fibres. The cirrus is high, and often consists of frozen particles, even in summer.

2. *Cumulus.*—This consists of compact rounded heaps, which often resemble mountain-tops covered with snow. This form of cloud is confined mostly to the summer season; it usually begins to form after the sun rises, and to disappear before it sets, and is rarely seen far from land. The cumulus is generally not so high as the cirrus.

3. *Stratus.*—*Sheets* or stripes of cloud, sometimes overspreading the whole sky, or as a fog covering the surface of the earth or water. The stratus is the most common, and usually lies lowest in the air.

4, 5, 6. *Cirro-cumulus, cirro-stratus, cumulo-stratus.*—Intermediate or combined forms.

7. *Nimbus.*—A cloud, which forms so fast as to fall in rain or snow, is called by this name.

554. Rain, Mist.—Whether the precipitated moisture has the

form of cloud or rain, depends on the rapidity with which precipitation takes place. If currents of air are in rapid motion, if the temperature of masses, brought into contact by this motion, are widely different, and if their humidity is at a high point, the vapor will be precipitated so rapidly, that the globules will touch each other, and unite into larger drops, which cannot be sustained. Globules of fog and cloud, however, are specifically as heavy as drops of rain; but they are sustained by the slightest upward movements of the air, because they have a great surface compared with their weight. A globule whose diameter is 100 times less than that of a drop of rain, meets with 100 times more obstruction in descending, since the weight is diminished a million times $(\frac{1}{100})^3$, and the surface only ten thousand times $(\frac{1}{100})^2$. So the dust of even heavy minerals is sustained in the air for some time, when the same substances, in the form of sand, or coarse gravel, fall instantly.

If a cloud of fine dust contains so much matter as to make the *mass* of a cubic foot of the dusty air greater than that of a cubic foot of pure air, it will descend. If the mean density of a fog is greater than that of the purer surrounding air, it will settle down into hollows and valleys; if its mean density is less than the air, it will rise as cloud.

Mist is fine rain; the drops are barely large enough to make their way slowly to the earth.

555. Hail, Sleet, Snow.—When the air in which rapid precipitation occurs, is so cold as to freeze the drops, hail is produced. As hailstones are not usually in the spherical form when they reach the earth, it is supposed that they are continually receiving irregular accretions in their descent through the vapor of the air. Hail-storms are most frequent and violent in those regions where hot and cold bodies of air are most easily mixed. Such mixtures are rarely formed in the torrid zone, since there the cold air is at a great elevation; in the frigid zone, no hot air exists at any height; but in the temperate climates, the heated air of the torrid, and the intensely cold winds of the frigid zone, may be much more easily brought together; and accordingly, in the temperate zones it is that hail-storms chiefly occur. Even in these climates, they are not frequent except on plains and in valleys contiguous to mountains which are covered with snow during the summer. The slopes of the mountain sides give direction to currents of air, so that masses of different temperature are readily mingled together.

Sleet is frozen mist, that is, it consists of very small hailstones.

Snow consists of the small crystals of frozen cloud, united in flakes. Like all transparent substances, when in a pulverized

state, it owes its whiteness to innumerable reflecting surfaces. A cloud, when the sun shines upon it, is for the same reason intensely white (Art. 371).

556. Theories of Precipitation.—It is probable that clouds and rain are caused not only by the mixing of air of different temperatures, but also by the changes which take place in the condition of the air as it ascends.

In the lower strata, the air is about one degree colder for every 300 feet of elevation. If, therefore, a mass of air is transferred from the surface of the earth to a height in the atmosphere, it will be cooled to the temperature of the stratum which it reaches; not principally by giving off its heat, but by *expanding*, and thus having its own heat reduced by being diffused through a larger space. Now, if the rising mass was saturated with moisture, this moisture would begin at once to be precipitated by the cooling which it undergoes in consequence of expansion. If, instead of being saturated, its dew-point is a certain number of degrees below its temperature, it must ascend far enough to be cooled to the dew-point, before precipitation of its moisture will take place. Suppose, for instance, the temperature at the earth is 70° F., and the dew-point is 65° F.; then after the warm air has risen 1500 feet (5×300 ft.), it will become 5° cooler, and contain all the moisture which is possible at that temperature. At that point precipitation begins, and forms the base of a cloud. The clouds, called *cumulus*, which are seen forming during many summer forenoons, are the precipitations of columns rising from warm spots of earth so high that they are cooled below their dew-point. But the movement and the precipitation do not stop here; for, as moisture is precipitated, its latent heat is given off in large quantities, which elevates the temperature of the mass, and causes it to rise still higher, and precipitate still more of its moisture. As it becomes rarer, it spreads laterally, and causes the cumulus often to assume the overhanging form which distinguishes that species of cloud.

557. Cyclones.—The late Mr. Redfield investigated with great success the phenomena of violent storms, especially of *Atlantic hurricanes,* and showed that they are generally, if not always, great whirlwinds, called *cyclones.* They usually take their rise in the equatorial region eastward of the West India Islands; they rotate on a vertical axis, advancing slowly to the northwest, until they approach the coast of the United States near the latitude of 30°, and then gradually veer to the northeast, running nearly parallel to the American coast, and finally spend themselves in the northern Atlantic. Their rotary motion is always in one

direction, namely, from the east through the north to the west, or against the sun. This motion is also far more violent, especially in the central parts of the storm, than the progressive motion. The rotary motion may amount to 50 or 100 miles per hour, while the forward motion of the storm is not more than 15 or 20 miles.

In the southern hemisphere also, cyclones occur, having a progressive and a rotary motion, both symmetrical with those of the northern cyclones. On the axis they revolve *with* the sun, not against it; and they first advance toward the southwest, and gradually veer toward the southeast, as they recede from the equator.

558. Draught of Flues.—The effect of the sun's heat in causing circulation of the air has been already considered (Art. 268–272). Similar movements on a limited scale are produced whenever a portion of the air is heated by artificial means. Thus, the air of a chimney is made lighter by a fire beneath it, than a column of the outer air extending to the same height. It is therefore pressed upward by the heavier external air, which descends and moves toward the place of heat. The difference of weight in the two columns is greater, and therefore the draught stronger, if the chimney is high, provided the supply of heat is sufficient to maintain the requisite temperature. Chimneys are frequently built one or two hundred feet high for the uses of manufactories. The high fireplaces and large flues of former times were unfavorable for draught, both because much cold air could mingle with that which was heated, and because there was room for external air to descend by the side of the ascending column. For good draught, no air should be allowed to enter the flue except that which has passed through the fire.

559. Ventilation of Apartments.—The air of an apartment, as it becomes vitiated by respiration, may generally be removed, and fresh air substituted, by taking advantage of the same inequality of weight in air-columns, which has been mentioned. If opportunity is given for the warm impure air to escape from the top of a room, and for external air to take its place, there will be a constant movement through the room, as in the flue of a chimney, though at a slower rate. If the external air is cold, the weight of the columns differs more, and therefore the ventilation is more easily effected. But in cold weather, the air, before being admitted to the room, is warmed by passing through the air-chambers of a furnace. When there is a chimney-flue in the wall of a room, with a current of hot air ascending in it, the ventilation is best accomplished by admitting the air into the

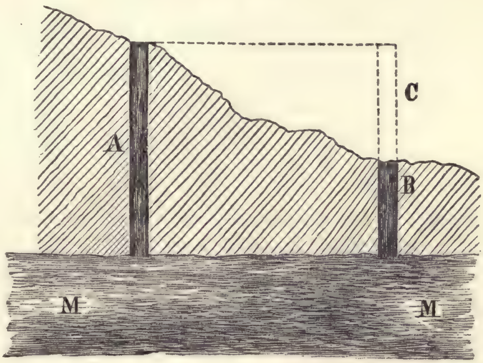
flue at the upper part of the room; since it will then be removed with the velocity of the hot-air current.

The tendency of the air of a warm room to pass out near the top, while a new supply enters at the lower part, is shown by holding the flame of a candle at the top, and then at the bottom, of a door which is opened a little distance. The flame bends outward at the top and inward at the bottom.

The impure air of a large audience-room is sometimes removed by a mechanical contrivance, as, for instance, a fan-wheel placed above an opening at the top, and driven by steam.

The ventilation of mines is accomplished sometimes by a fire built under a shaft, fresh air being supplied by another shaft, and sometimes by a fan-wheel at the top of the shaft. If there happen to be two shafts which open to the surface at very different elevations, ventilation may be effected by the inequality of temperature which is likely to exist within the earth and above it. Let *M M* (Fig. 312) be the vertical section of a mine through two shafts *A* and *B*, which open at different heights to the surface of the earth. If the external air is of the same temperature as the air within the earth, then the column *A* in the longer shaft has the same weight as *B* and *C* together, measured upward to the same level. In that case, which is likely to occur in spring and fall, there is no circulation without the use of other means. But in summer the air *C* is warmer than *A* and *B*; therefore *A* is heavier than $B + C$. Hence there is a current of air down *A* and up *B*. In winter, *C* is colder than air within the earth; therefore $B + C$ are together heavier than *A*, and the current sets in the opposite direction, down *B* and up *A*.

FIG. 312.

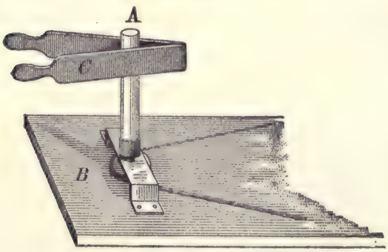


560. Sources of Heat.—*The sun,* although nearly a hundred millions of miles from the earth, is the source of nearly all the heat existing at its surface. The interior of the earth, except a thickness of forty or fifty miles next to the surface, is believed to be in a condition of heat so intense that all the materials composing it are in the melted state. But the earth's crust is so poor a

conductor that only an insensible fraction of all this heat reaches the surface.

Mechanical operations are usually attended by a development of heat. For example, if a broad surface of iron were made to revolve, rubbing against another surface, nearly all the force expended in overcoming the friction would appear as heat, a comparatively small part being conveyed through the air as sound. The cutting tool employed in turning an iron shaft has been known to generate heat enough to raise a large quantity of cold water to the boiling point, and to keep it boiling for an indefinite time. It is a fact familiar to all, that violent friction of bodies against each other will set combustibles on fire. The axles of railroad cars are made red-hot if not duly oiled ; boats are set on fire by the rope drawn swiftly over the edge by a whale after he is harpooned ; a stream of sparks flies from the emery wheel when steel is polished, &c.

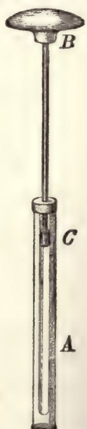
Fig. 313.



A lecture illustration devised by Tyndall will show the conversion of motion into heat.

Screw a brass tube A, about 4 inches long and $\frac{1}{2}$ inch in diameter, upon the spindle of a whirling table B (Fig. 313). Nearly fill the tube with water and insert a cork ; press the tube between the jaws of a wooden clamp C, while A is rapidly rotating. Heat is developed by the friction, and this communicated to the water causes it to boil, and finally to eject the cork.

Fig 314.



The heat developed by sudden compression of air may be rendered visible by igniting vapor of carbon bisulphide in a " Fire Syringe." A thick glass tube A (Fig. 314) closed at the lower end, has a well-fitted piston c, whose rod is terminated by a wide cap, or button, B, upon which the palm of the hand may strike forcibly without injury. If a bit of tinder, or a small tuft of cotton moistened with carbon bisulphide, be placed at the bottom of the tube, it will be ignited when the piston is driven down by a sudden blow upon it.

The heat due to percussion may be found thus : Let w ($=10$ lbs.) be the weight of a lead ball and

v (= 1000 ft. per sec.), be its velocity; then its energy in foot-pounds is (Art 37) $\frac{W v^2}{2 g} = 155279$ foot-pounds. Since 772 foot-pounds is the equivalent of one thermal unit (F) we find $\frac{155279}{772}$ = 201 thermal units, to be the heat developed by the sudden stopping of the ball. Suppose none of this heat to be transmitted to the body struck; then taking the specific heat of lead as .0314 (Art. 513), we find the temperature of the ball, assuming its original temperature to be 60° F., to be

$$60° + \frac{201}{10 \times .0314} = t = 760° \text{ F.},$$

a temperature above that of the melting point of lead, which is 635° F.

Indeed, wherever the full equivalent of any force is not obtained in some other form, the deficiency may be detected in the heat which is developed.

Chemical action is another very common source of heat. Combustion is the effect of violent chemical attraction between atoms of different natures, when both light and heat are manifested. If the union goes on slowly, as in the rusting of iron, the amount of heat is the same, but it is diffused as fast as developed. The molecular forces, expended in most cases of chemical combination, as measured by their heating effects, are enormously great.

The warmth produced by the vital processes in plants and animals is supposed by many physicists to be caused by chemical action. In breathing the air, some of its oxygen is consumed, which becomes united with the blood. This process is in some respects analogous to a slow combustion, by which heat is evolved in the animal system.

PART VII.

MAGNETISM.

CHAPTER I.

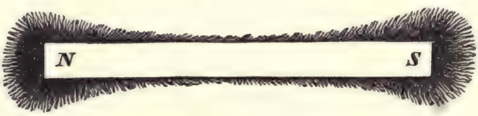
THE MAGNET AND ITS PROPERTIES.

561. The Magnet.—Fragments of iron ore are sometimes found which strongly attract iron, and bars of steel are artificially prepared which exhibit the same property. These bodies are called *magnets;* the ore is the *natural magnet,* commonly called *lodestone;* the prepared steel bar is an *artificial magnet.*

Ores of iron which are attracted by the magnet are abundant, both massive and in grains constituting magnetic iron sand.

562. The Attraction between a Magnet and Iron.—The magnetic property which is likely to be first noticed is the attraction of iron. If a lodestone or a bar magnet be rolled in iron filings (Fig. 315), there are two opposite points to which the

FIG. 315.



filings attach themselves in thick clusters, arranged in diverging filaments. These opposite points of greatest action are called *poles.* The *straight line* joining these poles is called the *axis* of the magnet.

The mutual attraction between a magnet and iron is shown by bringing a piece of iron toward either pole of the magnetic needle; the needle instantly turns so as to bring its pole as near as possible to the iron (Fig. 316). On the other hand, an iron needle being suspended in like manner, the same movement takes place, when either pole of a magnet is brought near to it.

That this attraction diminishes from the poles towards the middle point of the axis is indicated by the arrangement of the iron filings in Fig. 315. The variation in the force may be illustrated by means of a light balance beam, from one end of which is suspended a short piece of soft iron wire as in Fig. 317, counterpoised by weights in the scale-pan. As the bar magnet *M* is moved along below the wire *A*, at a convenient distance from it, the weights required to keep the beam horizontal indicate the variations in the force.

Fig. 316.

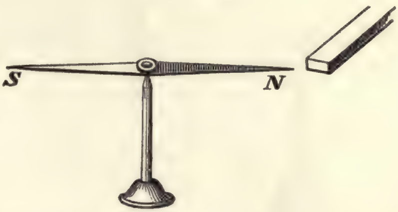
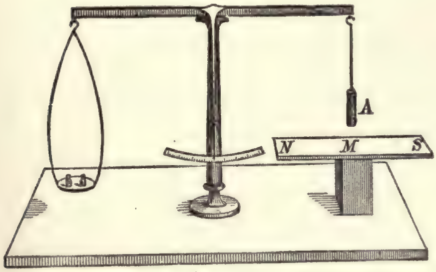
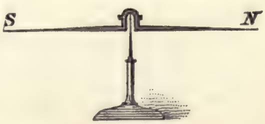


Fig. 317.



563. Polarity.—If a light magnet is delicately suspended on a pivot at the neutral point, as in Fig. 318, it is called a *magnetic needle.* When thus placed and left to itself, it oscillates for a time, and finally settles with its axis in a certain fixed direction, which in most places is nearly north and south. The end which points in a northerly direction is called the *north pole;* the other, the *south pole.* These poles are usually marked on the larger magnets by the letters *N* and *S*, so that they may be instantly distinguished. If a magnetic needle has simply a mark or stain on one end, that end is understood to be the north pole.

Fig. 318.



Instead of using the terms *north pole* and *south pole* it is better to speak of the *marked pole* and *unmarked pole,* to prevent confusion when discussing terrestrial magnetism. The pole is not at the extremity of the needle, but at a little distance from the end, and is not to be regarded as a point but rather as a resultant centre of forces.

564. Action of Magnets on Each Other.—While either pole of a magnet attracts and is attracted by a piece of iron, it is otherwise when the pole of one magnet is brought near the pole

of another. There is attraction in some cases, and repulsion in others. If the magnets are properly marked, and one of them suspended so as to move freely, it is readily discovered that the law of action is the following :

FIG. 319.

Poles of the same name repel, and those of contrary name attract each other.

Thus, the pole *S* of the magnet (Fig. 319) repels *s* of the needle, and attracts *n* ; and if the magnet were inverted, and the pole *N* brought near to *n*, the latter would be repelled, and *s* be attracted.

565. Magnets and Magnetic Substances.—All substances which *are attracted* by the magnet are classed as *magnetic* substances. Among these iron, nickel, cobalt, chromium, and manganese are the most marked.

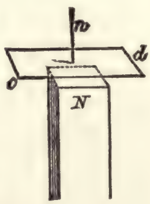
Magnetic substances show no polarity and exert no influence upon each other.

Magnets, on the contrary, manifest polarity and attract all magnetic substances, and attract or repel other magnets. Permanent magnets may be made of steel, cast-iron, nickel, and cobalt, the magnetism of the last being very feeble.

To determine whether a bar is a *magnet*, or merely *magnetic*, repulsion is a surer test than attraction; for a magnet will *attract* any magnetic substance, but *repulsion* can only be due to similar polarities.

To find experimentally whether a body is magnetic, lay a piece of stiff paper upon the end of a powerful bar-magnet held vertically, and upon the paper place a fine, short needle, the eye-end having been broken off, so that it may rest upon its point as in Fig. 320, in which *N* is the marked pole of the magnet, *c d* the paper or card, and *n* the needle. Move the paper until a point on the face of the magnet is found at which the needle will stand nearly vertical, and then raise the paper as far as possible without causing the needle to incline, when it will be very sensitive to magnetic influence. Any substance having even a trace of magnetic force, will cause the needle to sway aside on being brought near its upper end.

FIG. 320.



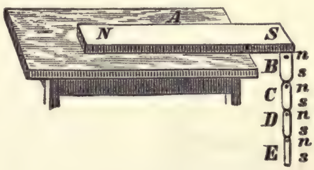
566. Magnetic Induction.—When a bar of iron is brought

near to the pole of a magnet, though attraction is the phenomenon first observed, as stated (Art. 562), yet it is readily proved that this attraction results from a change which is previously produced in the iron. It becomes a *magnet* through the influence of the magnet which is near it. That end of the iron bar which is placed near one pole of a magnet becomes a pole of the opposite name, and the remote end a pole of the same name. Hence, according to the law (Art. 564), the poles which are contiguous attract each other, because they are unlike. The influence by which the iron becomes a magnet is called *induction.* A magnet, when brought near to a piece of iron, *induces* upon the iron the magnetic condition, without any loss of its own magnetic properties. This influence is more powerful according as the two are nearer to each other; it is, therefore, greatest when the two bars are in contact.

That the iron is truly a magnet for the time being may be proved by sifting iron filings over it as in Art. 562, when the poles and the neutral line will be made apparent.

567. Successive Inductions.—Let a bar of iron, B (Fig. 321), be suspended from the unmarked pole of the magnet A; then the upper end of B is a marked pole, and the lower end an unmarked pole. Now, as B is a magnet, it will induce the magnetic state on another bar, C, when brought in contact; and, as before, the poles of opposite name will be contiguous. Therefore, the upper end of C is marked, and the lower end unmarked. D is also a magnet by the inductive power of C. Thus, there is an indefinite series of inductions, growing weaker, however, from one to another, as the number is greater, and as the bars are longer.

FIG. 321.



The filaments of iron filings which attach themselves to the pole of a magnet (Art. 562) are so many series of small magnets formed in the same manner as just described. Every particle of iron is a complete magnet, having its poles so arranged that the opposite poles of two successive particles are always contiguous.

568. Reflex Influence.—When a magnet exerts the inductive power upon a piece of iron which is near it, *its own magnetic intensity is increased.* The end of the piece of iron contiguous to the pole of the magnet is no sooner endued with the opposite polarity than it reacts upon the magnet, and increases its inten-

sity; so that, if fragments of iron are attached to a magnet, as many as it will sustain, then after a time another may be added, and again another, till there is a very sensible increase of its original power.

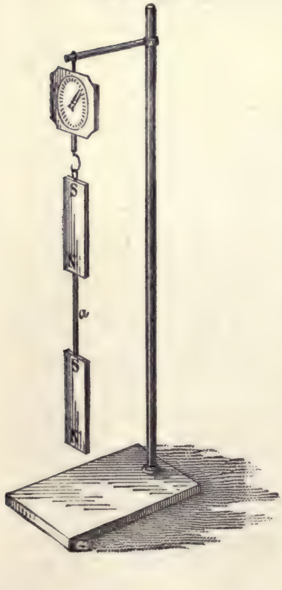
Hence, too, the force of attraction of the dissimilar poles of two magnets is greater than the force of repulsion of the similar poles; because, when the poles are unlike, each acts inductively on the other to develop its poles more fully; but when they are alike, the influence which they reciprocally exert tends to make them unlike, and of course to diminish their repulsive force.

An extreme case of this diminution of repulsive force occurs when the like poles of two very unequal magnets are brought into contact. The small magnet immediately clings to the large one, as though the poles were unlike; and if examined, it is found that they are unlike. The powerful magnet has in an instant reversed the poles of the weak one by its strong inductive power, the latter not having force enough to diminish sensibly the strength of the other.

569. Double Induction.—The effects of *two* inductions at once on a bar of iron are various.

1. The bar may become a single magnet of double strength.
2. It may consist of two distinct magnets.
3. It may have no magnetic power at all.

Fig. 322.



The first case is illustrated by bringing the north pole of a magnet to one end of the iron, and the south pole of another magnet to the other end. Each magnet will form two poles by induction, and it is evident that the two pairs of poles will coincide. Even one magnet produces the same effect when laid by the side of a bar of iron of the same length.

This case may also be experimentally illustrated by hanging a bar-magnet upon a spring balance, as in Fig. 322, and noting the force required to detach a soft iron rod, *a*, suspended from the magnet; having again brought the rod into contact with the magnet, touch the lower end of the rod *a* with a second magnet, equal in all respects to the first, the poles being arranged

as in the figure, and note the force required to detach the second magnet; the latter force will be found to be about double the former, the exactness of the result depending upon the degree of approximation to theoretically perfect conditions.

To show the second effect, apply one pole of a magnet to the middle of the iron bar; then an opposite pole is formed at the middle, and a like pole at each end, each half of the bar being a separate magnet. The same effect is produced by bringing the like poles of two magnets in contact with the ends of the bar; for both ends will be of the opposite kind, and the middle of the same kind, as the poles applied. If a pole is applied to the middle of a star of iron, the extremity of each ray is a pole of the same kind; if to the middle of a circle of iron, the same polarity is found at every point of the circumference.

As an example of the third case, suspend a bar of iron from the pole of a magnet, and then bring the opposite pole of an equal magnet to the point of contact; the two poles induced by one are contrary to the two induced by the other, and they are found to be completely neutralized, as is indicated by the fall of the bar.

This last case shows that two opposite and equal magnetic poles formed at the same point destroy each other.

570. Coercive Force.—If in the several experiments on iron bars, which have been already described, pure annealed iron is used, the iron rapidly acquires its maximum of magnetic intensity, and on being removed from the magnet its induced magnetism as rapidly disappears. In ordinary soft iron bars a trace of magnetism will remain for hours, or perhaps until a new induction of opposite magnetism destroys it. The magnetism so retained is called residual magnetism. It is least in very pure iron; but if the iron is hard, magnetic poles are slowly developed, and on removing the inducing magnet the iron as slowly returns to its former neutral state.

This property of iron which obstructs the development of magnetism in it, and which retards its return to a neutral state, is called *coercive force.* In pure iron, well annealed, the coercive force is feeble. It appears in wrought iron not carefully prepared; it is very great in cast iron, and is greatest in steel of the hardest temper.

It is, therefore, difficult to make a strong magnet of a steel bar by ordinary induction, unless it is quite thin; but after the development has once been made, the bar becomes a permanent magnet, and may by care be used as such for years.

571. Change in the Coercive Force.—The coercive force

is weakened by any cause which excites a tremulous or vibratory motion among the particles of the steel. This happens when the bar is struck by a hammer, so as to produce a ringing sound, which indicates that the particles are thrown into a vibratory motion. The passage of an electric discharge through a steel bar under the influence of a magnet, overcomes the coercive force for the time being, and permanent magnetism is developed. Heat produces the same effect; and hence a steel bar is conveniently magnetized by heating it to redness, placing it under a powerful inductive influence, and then hardening it by sudden cooling. The coercive force is thus neutralized by heat, till the development takes place, when it is restored, and the bar is a permanent magnet.

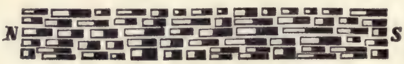
A magnet, however, loses its power by the same means as, during the process of induction, were used to develop it. Accordingly, any mechanical concussion or rough usage impairs or destroys the power of a magnet. By falling on a hard floor, or by being struck with a hammer, it is injured. Heat produces a similar effect. A boiling heat weakens, and a red heat totally destroys the magnetism of a needle.

572. Magnetism not Transferred, but only Developed.—This is strikingly proved by the fact that if a magnet be divided, even at the neutral point, where there is no sign of magnetism, the parts instantly become complete magnets, two unlike poles manifesting themselves at the place of fracture, whose strength is equal to that of the poles of the original magnet.

Both polarities seem to exist at every point, each neutralizing any external manifestations of the other (Art. 569); but when the bar is divided at any point the two polarities at that point are free to act upon magnetic bodies.

It is not necessary that the particles should be united by cohesion into a solid bar. A magnet can be formed by filling a brass tube with iron filings and sand, or by forming a rod of cement mixed with filings, and then subjecting them to inductive influence. Fig. 323 will give an idea of the probable structure of

FIG. 323.



every magnet. Each particle of it is a complete magnet, the like poles of all are turned the same way, and unlike poles are therefore contiguous to each other, and each acts inductively on the next.

573. Magnetic Intensity.—To deduce an expression for

the mutual attraction of two poles, it is necessary to adopt some unit of measure.

The British unit is the force which, acting upon one grain of matter during one second, will give to it a velocity of one foot per second. This is found to be about .03106 grain (Art. 34).

The *centimetre gramme-second* unit, usually written *C. G. S. unit,* is that force which, acting during one second upon a mass of one gramme, imparts to it a velocity of one centimetre per second. This latter unit is called a *dyne* and is nearly $\frac{1}{981}$ gramme. Adopting either of these units, we define a *unit magnetic pole* to be a pole of such strength that it will attract another equal pole, placed at a unit's distance, with the unit of force.

Suppose each of three unit poles, *a, b, c* (Fig. 324), to be distant one foot from each of two other unit poles, *d* and *e*. The attraction between *a* and *d* is one unit, and between *a* and *e* one unit also, hence the attraction between *a* and *d e,* taken together, is two units; in like manner the attraction between *b* and *d e,* is two units, and also between *c* and *d e,* it is two units. Hence the total attraction between the two groups is $3 \times 2 = 6$ units. If now we conceive the three poles *a, b, c,* to be united into one pole, it will have a strength three times that of any one of the unit poles; and *d* and *e* may be united into one pole of strength twice that of one pole alone. The mutual attraction will be the same as before, $3 \times 2 = 6$. In general, calling the strength of any pole as compared with the unit pole, *m*, and that of any other pole *m'*, the mutual attraction will be represented by their product, $f = m\,m'$.

Fig. 324.

574. Magnetic Intensity and Distance.—The law of the magnetic force is the following:

The intensity of the magnetic force, whether attraction or repulsion, varies inversely as the square of the distance.

The law in the case of the repulsion of like poles is readily proved by Coulomb's *torsion balance,* which is figured and described under Electricity. The angle of torsion is used as a measure of the repulsion, and it is found that the wire must be twisted through four times as large an angle to bring the poles to one-half the distance, and nine times as large an angle to bring them to one-third the distance, &c., the force increasing as the square of the distance diminishes.

To prove the law for the attraction of opposite poles, the vibrations of a needle are counted, when it is placed at different distances from a magnet. The square of the number made in a

given time is a measure of the attractive force, just as the square of the number of vibrations of a pendulum is a measure of the force of gravity (Art. 163).

In each of these experiments, the magnetic influence of the earth upon the needle must be eliminated, in order to obtain a correct result.

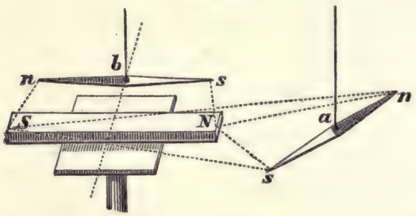
To determine the mutual attraction or repulsion between two magnets of strength m and m', and at a distance from each other of d feet, it will only be necessary to combine the results of this and the preceding Article. Calling F the attraction between the two poles, under this new condition we have

$$F = \frac{m\,m'}{d^2}.$$

In applying this formula *any increase of the strength of either pole by induction* must be borne in mind ; thus, if a magnet and a piece of soft iron be considered, it must be remembered that any change in the strength of the magnet produces a like change in the induced magnetism of the iron.

575. Equilibrium of a Needle near a Magnet. — If a small needle, free to revolve, be placed near the pole of a magnet, so that its centre is in the axis of the magnet produced, it will place itself in the line of that axis. For suppose that $N\,S$ (Fig. 325) is a large magnetic bar, and $n\,s$ a small needle suspended near the north pole of the magnet, with its centre in the axis of the bar produced at a; it will be seen that the action of the pole of the magnet is such as to bring the needle into a line with the magnet. The action of the pole N upon the needle tending to give it this direction (since it repels n and attracts s), is equal to the sum of its actions upon both poles. The pole S, by repelling s, and attracting n, tends to reverse this position, but on account of greater distance, its force is less than that of N.

FIG. 325.

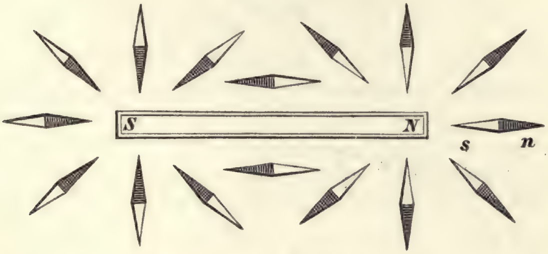


If the centre of the needle is in a line perpendicular to the bar at its middle point, the needle will be in equilibrium when parallel to the bar with its poles in contrary order. Thus supposing the needle to be suspended at b, it will be seen that the actions of both poles of the magnet conspire to move n to the left, and s to

the right; and as these forces are equal, equilibrium takes place only when the needle is parallel to the bar.

At intermediate points the needle will assume all possible inclinations to the axis of the bar, each position being determined by the resultant of the four forces which act on the needle. In Fig. 326 are indicated some of the positions which the needle takes

FIG. 326.

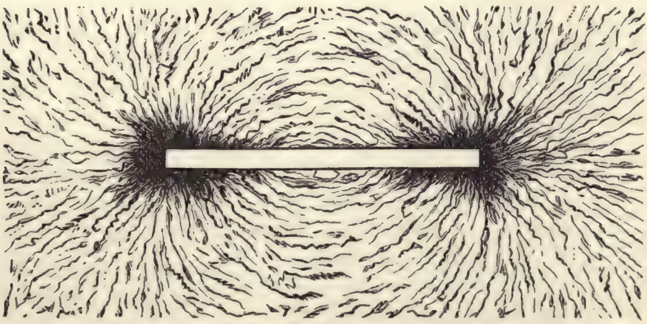


in being carried round the magnet. While it goes once round the magnet, it makes two revolutions on its own axis.

It is to be observed that in all positions the needle tends, as a whole, to move toward the bar, since the attractions always exceed the repulsions.

576. Magnetic Curves.—All the foregoing cases are shown at once by iron filings strewn on paper or parchment, which is stretched on a frame and placed near a magnet. Let the paper be slightly jarred, while the magnet lies parallel to it, either above or below, and all the inclinations of the needle will be represented by the particles of iron arranged in curves from pole to pole (Fig. 327). Near the poles of the magnet the filings stand up on the

FIG. 327.



paper at various inclinations. These are the extremities of still other curves, which would be formed in all possible planes passing through the axis of the magnet, provided the filings could float

suspended in the air, while the magnet is placed in the midst of them. These are called *magnetic curves.*

When the magnet is *below* the paper, the particles move away from the area over the poles, as in Fig. 327 ; but when it is *above,* they gather in a cluster under each pole. This singular difference arises from the force of gravity acting on the filaments, which are raised up on the paper, and which lean, in the former case, *from* each other, and in the latter, *toward* each other.

577. Lines of Force.—Magnetic Field.—Upon considering the action of one magnet upon others, as described in Art. 575, and as graphically exhibited in Fig. 326, we discover that in the space surrounding the magnet exists a force which either repels or attracts another magnet, and that the centre of this force is at the pole of the magnet which we are considering. The space within which this force is manifested is called the *magnetic field,* and the intensity of the attraction or repulsion of the *magnet* at any point in this surrounding space, is called also the *intensity of the field* at that point.

The curved and straight lines (Fig. 327) in which the iron filings arrange themselves, are the *lines of force ;* that is to say, the lines along which the force acts. If we could conceive of a single magnetic pole, without its opposite pole, existing in space not influenced by other magnets, the *lines of force* in that case would be straight lines radiating from such pole. The points of equal intensity in a magnetic field would, taken together, constitute a *surface* of uniform intensity, called an *equipotential surface,* which in the case supposed would be the surface of a sphere. The expressions *magnetic field, lines of force, equipotential surface,* are used in the sense which has been illustrated, however the magnetic field may be modified by other fields brought near the first. The lines of force are everywhere perpendicular to the equipotential surface.

FIG. 328.

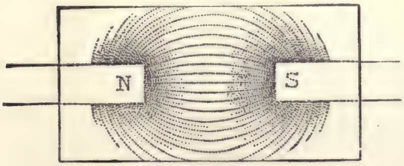


FIG. 329.

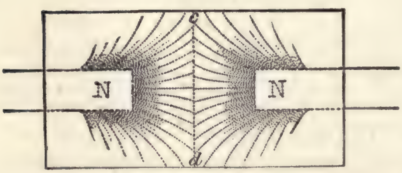


Fig. 328 illustrates the lines of force in the magnetic field between the unlike poles of two magnets, the plane of the paper being a section of the field through the common axis of the magnets. Fig. 329 gives a similar section of the field between two

like poles of equal intensity, the repulsive forces being in equilibrium along the line $c\ d$.

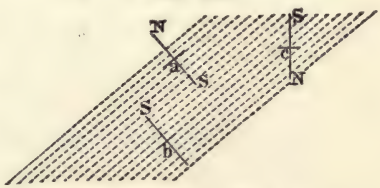
The direction of a line of force at any given point is the direction in which a single pole would move if free; and the intensity of the field at any point is proportional to the force which it exerts upon the free pole. In Art. 573 we learn that the mutual action of two magnets is represented by $m\ m'$; now call the intensity of the field at any point M, and that of the free pole m', and we have as before $f = M\ m'$.

A field of unit intensity is that which acts with the unit of force upon the unit pole. The intensity of the field is equal to the strength of the pole divided by the square of the distance, or $M = \frac{m}{d^2}$; hence, a unit field is at a unit distance from a unit pole.

If the lines of force are parallel, equal, and equidistant, the field is called a *uniform field*, and the equipotential surfaces become parallel planes. A small portion of a magnetic field, at a great distance from a pole, may be considered as uniform; the magnetic field due to terrestrial magnetism may be considered as uniform at any one place, the lines of force being indicated by the position assumed by a magnetized needle perfectly balanced upon its centre of gravity.

578. Position of a Needle Restrained by a Rigid Axis.—Suppose a needle to be mounted upon an axis through its middle point, at right angles to its length, so as to be free to move only in a plane perpendicular to such axis. If it be placed with this axis *perpendicular* to the lines of force of a field, it will place itself parallel to them (Fig. 326); if the axis be placed *parallel* to the lines of force, the needle will remain in any position, in its plane of motion, which we give to it, as in a and b, Fig. 330. If the axis be inclined to the lines of force, since the needle will take that position which most nearly approaches parallelism to them, it must turn till it makes the least possible angle with them; such position in the case represented in c, Fig. 330, being evidently in the plane of the section of the field represented by the page.

Fig. 330.



579. Diamagnetism.—Thus far only those substances which are *attracted* by either pole of a magnet have been called *magnetic;*

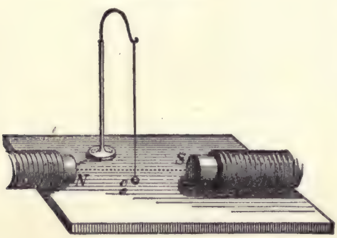
but as all substances seem to be influenced by magnetic force, a new definition becomes necessary.

Those substances which are attracted by a magnetic pole, or which in a magnetic field tend to move from places of less to places of greater intensity, are called Paramagnetic.

Those substances which are repelled by either pole indifferently, or which move from places of greater intensity to places of less intensity in the field, are called Diamagnetic.

All substances may be arranged in one or the other of these two classes. A very powerful magnet is needed to show the different effects produced upon paramagnetic and diamagnetic substances. Let *N* and *S*, Fig. 331, be the opposite poles of a large electro-magnet, and let a very small sphere of iron *c* be suspended by a thread, two or three feet long, so as to rest at *c*, midway between the poles, but not in the axial line *N S*. Upon making *N* and *S* magnets, the sphere will be drawn into the line *N S*, and will swing towards whichever pole happens to be nearest. Remove the iron sphere *c*, and replace it by one of bismuth; now upon again magnetizing the poles *N* and *S*, the sphere *c* will be repelled from the axial line *N S* a perceptible distance.

Fig. 331.



Replace the sphere by an iron needle suspended with its centre in the axial line *N S*. It will place itself with its longest dimension in the line *N S*, or *axially*. Substitute for the iron needle one of bismuth, or phosphorus, and the new needle will place itself with its length at right angles to the line *N S*, or *equatorially*. The iron needle, under the attraction of the poles, approaches as near them as possible, and to do this must set axially; while the bismuth needle being repelled must set equatorially in order to recede as far as possible from the poles.

Among diamagnetic substances bismuth, phosphorus, and antimony are the most marked; but the force of *repulsion*, in the case of bismuth even, is only a very small fraction of the force of *attraction* of iron under like conditions, being less than $\frac{1}{2000000}$ of the latter, according to Weber.

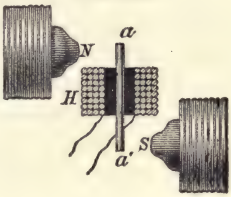
580. Influence of the Surrounding Medium.—If a paramagnetic needle be suspended in a fluid more strongly paramagnetic than itself, it will seem to be diamagnetic; and if a diamagnetic needle be suspended in a fluid more diamagnetic than itself, it will seem to be paramagnetic.

The air is less strongly paramagnetic than a weak solution of iron, and hence when a tube containing such solution is properly suspended between the poles of a magnet it will arrange itself axially; but if the same tube be now suspended in a stronger solution of iron it will set equatorially.

581. Diamagnetic Polarity.—Faraday's experiments to detect diamagnetic polarity, owing to lack of delicacy in his instruments, gave no evidence of its existence. Other physicists also failed to detect any manifestations of polarity, till Tyndall in 1855 established the fact of diamagnetic polarity by series of experiments, essentially as follows:

Let N and S (Fig. 332) be the unlike poles of two electromagnets, and H a helix, or coil of wire, of internal diameter sufficient to allow some vibration of a needle $a\ a'$ suspended within it. Now if $a\ a'$ be a needle of iron it will become a magnet when a current of electricity flows through the wire, as will be explained under Dynamic Electricity. Suppose a to be a marked pole, and a' an unmarked pole, under the given conditions; now if the flow of the current be reversed a becomes an unmarked pole and a' a marked pole, the poles being reversed by the reversal of the current, as shown by the corresponding attractions and repulsions between N and a, or S and a'. When a needle of bismuth is substituted for the iron, analogous but directly opposite effects are produced; that direction of the current which in iron produced attraction, now causes repulsion in the case of bismuth, and that current which produced repulsion now produces attraction. Later experiments with a different arrangement of helices and needles gave much more striking results. Liquids were also experimented upon, inclosed in thin glass tubes. Water was found to exhibit diamagnetic polarity.

FIG. 332.



582. Axis in Line of Greatest Density.—If the body experimented upon is not homogeneous, *then the magnetic axis coincides with the line of greatest density.*

In crystalline substances the axis is parallel to the planes of cleavage, so that a paramagnetic body will assume a position with its planes of cleavage axial, while a diamagnetic substance will place its planes of cleavage equatorially.

If by pressure the density of a substance be made greatest in a line at right angles to the planes of cleavage this line will determine the set of the body; and by repeated pressures in various

directions successive changes in the direction of the axis may be produced.

In such experiments the most striking effects are secured when the three dimensions of the body do not differ greatly from each other.

583. Molecular Changes.—A soft iron bar may be made a powerful magnet by causing a current of electricity to circulate around it, as will be more fully explained under Electro Magnetism.

At the instant of completing the circuit of the electric current around the bar, a sharp click is audible, and at the instant when the circuit is broken the sound is heard again.

If a bar be carefully measured before making the circuit, and again during the passage of the current around it, it will be found that magnetization has caused an increase of *length* and a decrease of *cross-section* of the bar, the ratio of these changes being such that the volume of the bar has been unaltered. These dimensional changes are very slight, and can be shown only by very delicate experiments.

To account for these effects, we may consider the bar to be made up of separate particles, as in Art. 572 ; now when the bar is magnetized these particles tend to arrange themselves with their longest dimensions parallel to the magnetic axis. Instead of the separate particles of Art. 572 substitute the infinitely minute crystals of the iron bar, with a tendency to place the longest axes parallel to the length of the bar, and we have De La Rive's explanation of the phenomena.

CHAPTER II.

RELATIONS OF THE MAGNET TO THE EARTH.

584. Declination of the Needle.—When the needle is balanced horizontally, and free to revolve, it does not generally point exactly north and south ; and the angle by which it deviates from the meridian is called the *declination.* A vertical circle coincident with the direction of the needle at any place is called the *magnetic meridian.* As the angle between the magnetic and the geographical meridians is generally different for different places, and also varies at different times in the same place, the

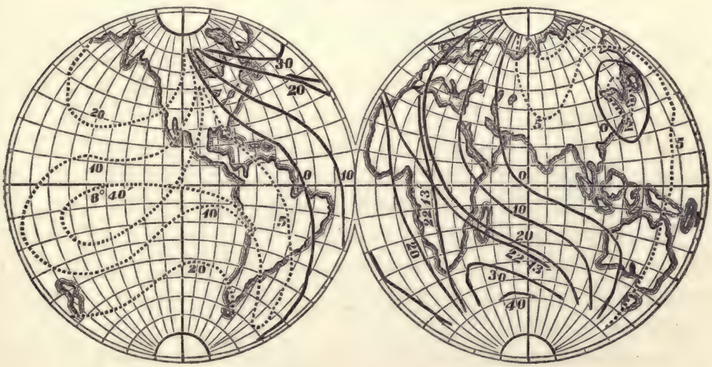
word *variation* expresses these *changes* in declination, though it is much used as synonymous with declination itself.

The force which causes the needle to set in the magnetic meridian is *merely directive.*

If the needle be weighed before it is magnetized and again after it has been made a magnet, no change of weight can be detected, proving that the earth's attraction for one pole is exactly equal to its repulsion of the other. This may also be shown by attaching a magnet to a cork and thus floating it upon water. It will set in the magnetic meridian but will show no tendency to move across the water towards the north, nor in any other direction. This effect is due to the earth's uniform magnetic field. The magnetic pole of the earth being practically at an infinite distance, the forces of attraction and repulsion, being equal, constitute a couple (Art. 56).

585. Isogonic Curves.—This name is given to a system of lines imagined to be drawn through all the points of equal declination on the earth's surface. We naturally take as the standard line of the system that which connects the points of *no declination,* or the isogonic of 0° (Fig. 333). Commencing at the north

FIG. 333.



pole of dip, about Lat. 70°, Lon. 96°, it runs in a general direction E. of S., through Hudson's Bay, across Lake Erie, and the State of Pennsylvania, and enters the Atlantic Ocean on the coast of North Carolina. Thence it passes east of the West India Islands, and across the N. E. part of South America, pursuing its course to the south polar regions. It reappears in the eastern hemisphere, crosses Western Australia, and bears rapidly westward across the Indian Ocean, and then pursues a northerly course across the Caspian Sea to the Arctic Ocean. There is also a

detached line of no declination, lying in eastern Asia and the Pacific Ocean, returning into itself, and inclosing an oval area of 40° N. and S. by 30° E. and W. Between the two main lines of no declination in the Atlantic hemisphere, the declination is *westward*, marked by continued lines, in Fig. 333; in the Pacific hemisphere, outside of the oval line just described, it is *eastward*, marked by dotted lines. Hence, on the American continent, in all places east of the isogonic of 0°, the marked pole of the needle declines westward, and in all places west of it, the marked pole declines eastward; on the other continent this is reversed, as shown by the figure.

Among other irregularities in the isogonic system, there are two instances in which a curve makes a wide sweep, and then intersects its own path, while those within the loop thus formed return into themselves. One of these is the isogonic of 8° 40′ E., which intersects in the Pacific Ocean west of Central America; the other is that of 22° 13′ W., intersecting in Africa.

In the northeastern part of the United States the declination has long been a few degrees to the west, with very slow and somewhat irregular variations.

586. Secular and Annual Variation. — The declination of the needle at a given place is not constant, but is subject to a slow change, which carries it to a certain limit on one side of the meridian, when it becomes stationary for a time, and then returns, and proceeds to a certain limit on the other side of it, occupying two or three centuries in each vibration. At London, in 1580, the declination was $11\frac{1}{4}°$ E.; in 1657, it was 0°; after which time the needle continued its western movement till 1818, when the declination was $24\frac{1}{2}°$ W.; since then the needle has been moving slowly eastward, and in 1879, at Kew, the declination was 19° 7′ *west*.

The entire secular vibration will probably last more than three centuries. The average variation from 1580 to 1818 was 9′ 10″ annually. But, like other vibrations, the motion is slowest toward the extremes.

There has also been detected a small *annual* variation, in which the needle turns its marked pole a few minutes to the east of its mean position between April and July, and to the west the rest of the year. This annual oscillation does not exceed 15 or 18 minutes.

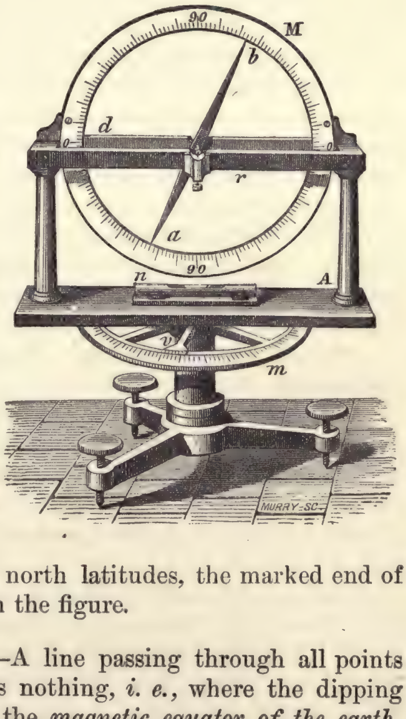
587. Diurnal Variation.—The needle is also subject to a small *daily* oscillation. In the morning the marked end of the needle has a variation to the east of its mean position greater than at any other part of the day. During winter this extreme point is attained at about 8 o'clock, but as early as 7 o'clock in the sum-

mer. After reaching this limit it gradually moves to the west, and attains its extreme position about 3 o'clock in winter, and 1 o'clock in summer. From this time the needle again returns eastward, reaching its first position about 10 P. M., and is almost stationary during the night. The whole amount of the diurnal variation rarely exceeds 12 minutes, and is commonly much less than that. These diurnal changes of declination are connected with changes of *temperature,* being much greater in summer than in winter. Thus, in England the mean diurnal variation from May to October is 10 or 12 minutes, and from November to April, only 5 or 6 minutes.

588. Dip of the Needle.—A needle first balanced on a horizontal axis, and then magnetized and *placed in the magnetic meridian,* assumes a fixed relation to the horizon, one pole or the other being usually depressed below it.

The axis of the needle must be placed very accurately at right angles to the plane of the magnetic meridian, or false indications will be given; if the axis of suspension were placed *in* the plane of the meridian the angle of depression would be 90° at all places on the earth's surface (Art. 578).

FIG. 334.

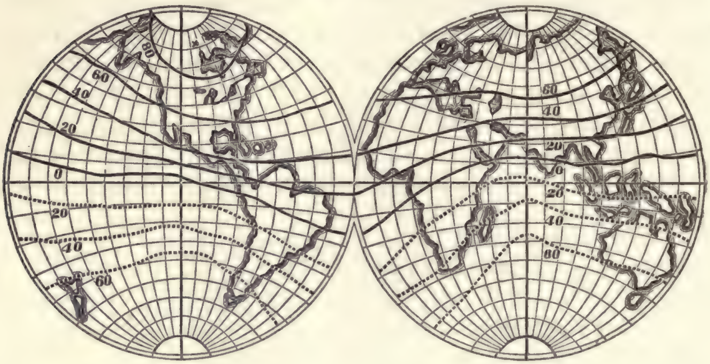


The angle of depression is called the *dip* of the needle. Fig. 334 represents the *dipping needle,* with its adjusting screws and spirit-level; and the depression may be read on the graduated scale. After the horizontal circle *m* is leveled by the foot-screws, the frame *A* is turned horizontally till the vertical circle *M* is in the magnetic meridian. For north latitudes, the marked end of the needle is depressed, as *a* in the figure.

589. Isoclinic Curves.—A line passing through all points where the dip of the needle is nothing, *i. e.,* where the dipping needle is horizontal, is called the *magnetic equator of the earth.* It can be traced in Fig. 335 as an irregular curve around the

earth in the region of the equator, nowhere departing from it more than about 15°. At every place north of the magnetic

Fig. 335.



equator the marked pole of the needle descends, and south of it the unmarked pole descends; and, in general, the greater the distance, the greater is the dip. Imagine now a system of lines, each passing through all the points of equal dip; these will be nearly parallel to the magnetic equator, which may be regarded as the standard among them. These magnetic parallels are called the *isoclinic curves ;* they somewhat resemble parallels of latitude, but are inclined to them, conforming to the oblique position of the magnetic equator. In the figure, the broken lines show the dip of the south pole of the needle; the others, that of the north pole. The points of greatest dip, or dip of 90°, are called the *poles of dip.* There is one in the northern hemisphere, and one in the southern. The north pole of dip was found, by Capt. James C. Ross, in 1831, to be at or very near the point, 70° 14′ N.; 96° 40′ W., marked x in the figure. The south pole is not yet so well determined.

At the poles of dip the horizontal needle loses all its directive power, because the earth's magnetism tends to place it in a vertical line, and, therefore, no component of the force can operate in a horizontal plane. The isogonic lines in general converge to the two dip-poles; but, for the reason just given, they cannot be traced quite to them.

The dip of the needle, like the declination, undergoes a variation, though by no means to so great an extent.

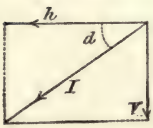
In 1576, the date of its discovery, the dip at London was 71° 50′; it increased to a maximum of 74° 42′ in 1723, since which time it has gradually decreased. In 1879 the dip at Kew was 67° 42′.

In the course of 250 years, it has diminished about five degrees in London. In 1820 it was about 70°, and diminishes from two to three minutes annually.

Since the dip at a given place is changing, it cannot be supposed that the poles are fixed points; they, and with them the entire system of isoclinic curves, must be slowly shifting their locality.

590. Magnetic Intensity of the Earth.—The force exerted by the magnetism of the earth varies in different places, being generally least in the region of the equator, and greatest in the polar regions. The ratio of intensity in different places is measured by the number of vibrations which the needle makes in a given time. In the discussion of the pendulum, it was proved (Art. 163) that gravity varies as the square of the number of vibrations. For the same reason the magnetic force at any place varies as the square of the number of vibrations of the needle at that place, provided the axis of suspension of the needle be perpendicular to the lines of force. As it is not convenient to use the dipping needle in this determination, the oscillations of a horizontal needle are used instead, and the intensity is computed from these. Thus, let I (Fig. 336) represent the *direction* and *intensity* of the earth's magnetic attraction at any place, and d the angle of *dip*. The horizontal component of the intensity will be $I \times \cos d$, and it is this component which varies as the square of the number of oscillations of a *horizontal* needle. Calling n and n' the number of oscillations per second at two different places, d and d' the dip, and I and I' the relative intensities, we have

FIG. 336.



$$I \cos d : I' \cos d' :: n^2 : n'^2,$$

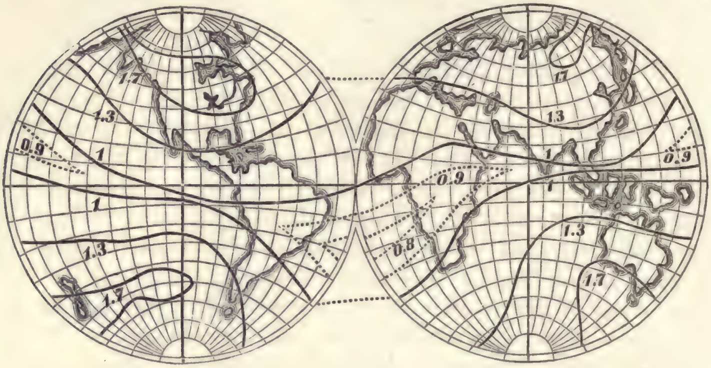
from which is deduced

$$\frac{I'}{I} = \frac{\cos d}{\cos d'} \times \frac{n'^2}{n^2}.$$

591. Isodynamic Curves.—After ascertaining, by actual observation, the intensity of the magnetic force in different parts of the earth, lines are supposed to be drawn through all those points in which the force is the same; these lines are called isodynamic curves, represented in Fig. 337. These also slightly resemble parallels of latitude, but are more irregular than the isoclinic lines. There is no one standard equator of minimum intensity, but there are two very irregular lines surrounding the earth in the equatorial region, in some places almost meeting each other, and in others spreading apart more than two thousand miles, on which the

magnetic intensity is the same. These two are taken as the standard of comparison, because they are the lowest which extend entirely round the globe. The intensity on them is therefore

FIG. 337.



called *unity,* marked 1 in the figure. In the wide parts of the belt which they include—lying one in the southern Atlantic, and the other in the northern Pacific oceans—there are lines of lower intensity which return into themselves, without encompassing the earth. In approaching the polar regions, both north and south, the curves, retaining somewhat the form of the unit lines, are indented like an hour-glass, as those marked 1.7 in the figure, and at length the indentations meet, forming an irregular figure 8 ; and at still higher latitudes, are separated into two systems, closing up around two poles of maximum intensity. Thus there are on the earth four poles of maximum intensity, two in the northern hemisphere and two in the southern. The American north pole of intensity is situated on the north shore of Lake Superior. The one on the eastern continent is in northern Siberia. The ratio of the least to the greatest intensity on the earth is about as 0.7 to 1.9 ; that is, as 1 to $2\frac{5}{7}$. In the figure, intensities less than 1 are marked by dotted lines.

592. Magnetic Charts.—These are maps of a country, or of the world, on which are laid down the systems of curves which have been described. But for the use of the navigator, only the isogonic lines, or lines of equal declination, are essential. There are large portions of the globe which have as yet been too imperfectly examined for the several systems of curves to be accurately mapped. It must be remembered, too, that the earth is slowly but constantly undergoing magnetic changes, by which, at any given place, the declination, dip, and intensity are all essentially

altered after the lapse of years. A chart, therefore, which would be accurate for the middle of the nineteenth century, will be, to some extent, incorrect at its close.

593. Magnetic Observatories.—In accordance with a suggestion of Humboldt, in 1836, systematic observations have been since made upon terrestrial magnetism, in various parts of the world, in order to deduce from them the laws of its changes. Buildings have been erected without any iron in their construction, to serve as magnetic *observatories;* and the most delicate *magnetometers* have been devised and used for detecting minute oscillations both in the horizontal and vertical planes. By these means has been discovered a class of phenomena called *magnetic storms,* in which the needle suffers numerous and rapid disturbances, sometimes to the extent of several degrees; and it is a remarkable and interesting fact that these disturbances occur at the same absolute time in every part of the earth.

594. Aurora Borealis.—This phenomenon is usually accompanied by a disturbance of the needle, thus affording visible indications of a magnetic storm; but the contrary is by no means generally true, that a magnetic storm is accompanied by auroral light. The connection of the aurora borealis with magnetism is manifested not only by the disturbance of the needle, but also by the fact that the streamers are parallel to the dipping needle, as is proved by their apparent convergence to that point of the sky to which the dipping-needle is directed. This convergence is the effect of perspective, the lines being in fact straight and parallel.

595. Source of the Earth's Magnetism.—If a needle is carried round the earth from north to south, it takes approximately all the positions in relation to the earth's axis which it assumes in relation to a magnetic bar, when carried round it from end to end (Art. 575). At the equator it is nearly parallel to the axis, and it inclines at larger and larger angles as the distance from the equator increases; and in the region of the poles, it is nearly in the direction of the axis. The earth itself, therefore, may be considered a magnet, since it affects a needle as a magnet does, and also induces the magnetic state on iron. But it is necessary, on account of the attraction of opposite poles, to consider the northern part of the earth as being like the south pole of a needle, and the southern part like the north pole. To avoid this, the words *boreal* and *austral* are applied to the two magnetic states, and the *boreal magnetism* is the name given to that development found in the northern hemisphere, and the *austral magnetism* to that in the southern. Hence, it becomes necessary, in using

these names for a magnet, to reverse their order, and to speak of its north pole as exhibiting the austral, and its south pole the boreal magnetism.

Modern discoveries in electro-magnetism and thermo-electricity furnish a clew to the hypothesis which generally prevails at this day. Attention has been drawn to the remarkable agreement between the *isothermal* and the *isomagnetic* lines of the globe. The former descend in crossing the Atlantic Ocean toward America, and there are two poles of maximum cold in the northern hemisphere. The isoclinic and the isodynamic curves also descend to lower latitudes in crossing the Atlantic westward; so that, at a given latitude, the degree of *cold*, the magnetic *dip*, and the magnetic *intensity*, is each considerably greater on the American than on the European coast. This is only an instance of the general correspondence between these different systems of curves. It has likewise been noticed (Art. 587) that the needle has a movement diurnally, varying westward during the middle of the day, and eastward at evening, and that this oscillation is generally much greater in the hot season than the cold. It is obvious, therefore, that the development of magnetism in the earth is intimately connected with the temperature of its surface. Hence it has been supposed that the heat received from the sun excites electric currents in the materials of the earth's surface, and these give rise to the magnetic phenomena.

The actual phenomena, while presenting the analogies given above, also indicate peculiar conditions such that no simple law of magnetic distribution can be enunciated.

CHAPTER III.

MAGNETIZATION.

596. Formation of Permanent Magnets.—Needles and small bars may be more or less magnetized by the following methods, the reasons for which will be readily understood:

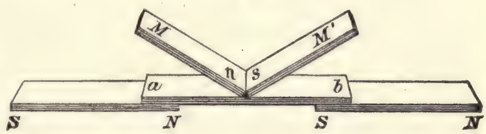
1. A feeble magnetism may be developed in a steel bar, by causing it to ring while held vertically. The earth's influence upon it, however, is stronger if it is held, not precisely vertical, but leaning in a direction parallel to the dipping needle. The inductive influence of the earth explains the fact often noticed, that rods of iron or steel that have stood for many years in a position nearly vertical, as, for instance, lightning-rods, iron

pillars, stoves, &c., are found somewhat magnetic, with the marked pole downward.

2. A needle may be magnetized by simply suffering it to remain in contact with the pole of a strong magnet, or better, in contact with the opposite poles of two magnets.

3. Place two opposite poles of equally strong magnets *M M'*, (Fig. 338), upon the middle of the bar to be magnetized *a b*, and draw them from the middle to the ends. Return the magnets *M M'* to the middle of the bar again, carrying them around at a distance from the bar, and repeat the first stroke. Do this several times upon each face of *a b*. The effects will be intensified if *a b* be placed upon two other magnets, as in Fig. 338. This is called the method of separate touch.

FIG. 338.



4. If, instead of moving the magnets from the middle of the needle towards each end simultaneonsly, they be moved *together*, first towards *a* (Fig. 338) and then back again to *b*, and so on, ending the strokes at the middle, the magnetization will be very strong. This is the method of double touch.

Either of these two methods may be applied to several bars at once, or to two horseshoe magnets. The variations in the mode of application are numerous.

FIG. 339.

In order to take advantage of the earth's inductive influence, along with that of steel magnets, place the needle parallel to the dipping needle, and draw the unmarked pole of one magnet over the lower half, and the marked pole of another over the upper half, with repeated and simultaneous movements.

If the steel bar be not homogeneous, or if any of the processes of magnetization be not carefully carried on, the bar may have one or more poles between the extreme poles. These are called *consequent points* or *poles*, and are more likely to be developed in very long bars than in short ones.

None of these methods, however, are of great practical value at the present day, since the galvanic circuit affords a far readier and more efficient means of magnetizing bars.

The *horseshoe magnet*, sometimes called the *U-magnet* (Fig. 339), is for many purposes a very convenient form,

and originated in the practice of *arming* the lodestone ; that is, furnishing it with two pieces of soft iron, which are confined by brass straps to the poles of the stone, and project below it, so that a bar and weight may be attached. When a magnet has this form, both poles may be applied to a body at once. The *U*-magnet, *A N S*, being suspended, and the keeper, *B*, made of soft iron, being attached to the poles, weights may be hung upon the hook *C*, to show the strength.

597. Compound Magnets.—Thin steel plates can be most readily magnetized. Many of these bound together by suitable clamps of a non-magnetic substance, with their like poles side by side, constitute a compound magnet. Since the mutual action of the like poles in juxtaposition tends to weaken them, the strength of the compound magnet will never equal the sum of the strengths of its component parts, but it will far exceed that of any one of them, and also that of a solid bar of a mass equal to the sum of their masses. The best effects are produced by using an odd number of plates, of lengths such that the successive pairs may be shorter than the preceding, giving tapered poles as in Fig. 340. This rounded or tapered form of pole is also best for single bar and horseshoe magnets.

FIG. 340.

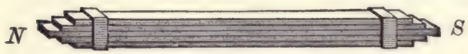
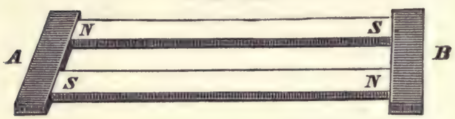


FIG. 341.



598. Armatures.—The keeper *B* (Fig. 339), is called an armature. If two bar-magnets (Fig. 341) be placed parallel, and their poles be joined by soft iron bars *A* and *B*, these are called armatures also.

The armature of a magnet, when in contact with the poles, tends to diminish the intensity of the surrounding field. This may be readily shown by bringing a small horseshoe magnet near a declination needle ; on removing the armature, the deviation of the needle increases, and on replacing the *armature* the needle returns to its former position. An electro-magnetic *ring* produces no sensible field in its neighborhood.

599. Preservation of Magnets.—That magnets may be preserved in good condition care should be exercised in handling them. They should never be subjected to blows, nor to any jarring action. They should not be greatly heated. Bar-magnets

should always be placed in the plane of the magnetic meridian, the marked poles north, when laid away after use. Like poles of magnets of unequal strength should not be brought into contact, lest the stronger should reverse the polarity of the weaker. Horseshoe magnets, and bar-magnets in pairs, should always have their armatures in contact, when laid away. The armature should not be jerked away, but should be removed by sliding off.

600. Saturation of Magnets.—Up to a certain point the magnetism induced in a steel bar increases with the strength of the magnets used and with the increase of the number of *touches;* but soon a limit is reached, and then the bar is said to be magnetized to saturation. It is possible to give a temporary increase to the intensity of a permanent magnet, but when the inducing cause is removed the intensity falls again, rapidly at first, and then more slowly for days and even weeks. If a horseshoe magnet be suspended, and weights be hung upon the armature (Fig. 339), it will be found that after a time still other weights may be added, till finally a much larger load will be sustained than the magnet will ordinarily "pick up." When this loaded armature is torn away the strength of the magnet decreases to its normal condition.

601. Power of Magnets.—The strength of a saturated horseshoe magnet is given by the following formula from Haecker, $P = a\sqrt[3]{p^2}$, in which P is the maximum weight which the magnet can lift, p the weight of the magnet itself, and a a constant depending upon the quality of the steel and the mode of magnetization. By means of this formula we derive the relative powers of two magnets, subject to the same constant a. Suppose one magnet to be eight times as heavy as the other, and we have

$$P : P' :: a\sqrt[3]{1} : a\sqrt[3]{64},$$

whence

$$P' = 4 \times P.$$

Thus we see that small magnets are stronger proportionally than large ones.

The most powerful permanent magnet yet constructed, according to Gordon, weighs about 110 pounds, and sustains ten times its own weight. It is said that small magnets have been constructed which would carry twenty-five times their own weight.

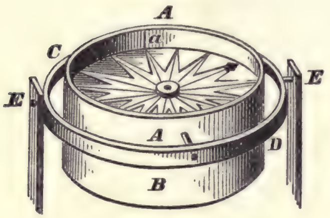
602. The Declination Compass.—This instrument consists of a magnetic needle suspended in the centre of a cylindrical brass box covered with glass; on the bottom of the box within is fastened a circular card, divided into degrees and minutes, from 0° to 90° on the several quadrants. On the top of the box are two

uprights, either for holding sight-lines or for supporting a small telescope, by which directions are fixed. The quadrants on the card in the box are graduated from that diameter which is vertically beneath the line of sight.

When the axis of vision is directed along a given line, the needle shows how many degrees that line is inclined to the magnetic meridian. In order that the angle between the line and the geographical meridian may be found, the declination of the needle for the place must be known.

603. The Mariner's Compass.—In the mariner's compass (Fig. 342) the card is made as light as possible, and attached to the needle, so that the north and south points marked on the card always coincide with the magnetic meridian. The index, by which the direction of the ship is read, consists of a pair of vertical lines, diametrically opposite to each other, on the interior of the box. These lines, one of which is seen at *a*, are in the plane of the ship's keel. Hence, the degree of the card which is against either of the lines shows at once both the angle with the magnetic meridian and the quadrant in which that angle lies.

FIG. 342.



In order that the top of the box may always be in a horizontal position, and the needle as free as possible from agitation by the rolling of the ship, the box, *B*, is suspended in *gimbals*. The pivots, *A*, *A*, on opposite sides of the box, are centred in the brass ring, *C*, *D*, while this ring rests on an axis, which has its bearings in the supports, *E*, *E*. These two axes are at right angles to each other, and intersect at the point where the needle rests on its pivot. Therefore, whatever position the supports, *E*, *E*, may have, the box, having its principal weight in the lower part, maintains its upright position, and the centre of the needle is not moved by the revolutions on the two axes.

On account of the dip, which increases with the distance from the equator, and is reversed by going from one hemisphere to the other, the needle needs to be loaded by a small adjustable weight, if it is to be used in extensive voyages to the north or south. In north latitudes the unmarked end must be heaviest; in south latitudes, the marked end.

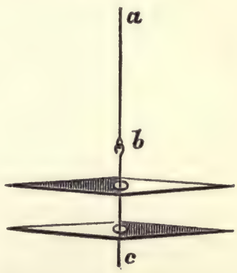
604. The Needle Rendered Astatic.—Though magnetic intensity increases at greater distances from the equator, yet the

directive power of the compass grows more feeble in approaching the poles of dip, because the horizontal component constantly diminishes, and at the poles becomes zero (Art. 590). A needle in such a situation, in which the earth's magnetism has no influence to give it direction, is called *astatic.* The compass needle is astatic at the north and south poles of dip. And the dipping needle may be rendered astatic at any place by setting its plane of rotation perpendicular to its line of dip at that place ; for then there will remain no component of the magnetic force in the only plane in which the needle is at liberty to move (Art. 578).

The needle may be rendered astatic by placing a magnet above or below it, with its axis in the magnetic meridian, its marked pole pointing north, and at such a distance from the needle as to cause it to take a position perpendicular to the plane of the magnetic meridian. Under such conditions the needle will be extremely sensitive.

A compound needle, consisting of two simple needles fixed upon a wire, as in Fig. 343, with their unlike poles opposed, may be suspended in any of the usual modes. If the needles are exactly equal in *all respects* the system will be perfectly astatic. The condition of perfect equality in all the conditions is never realized.

FIG. 343.



This method of liberating a magnetic needle from the earth's influence is of great use in electro-magnetism.

Needles may be balanced upon very sharp pivots, the needle cap being either of hard brass, or a jewel properly drilled ; this mode of mounting is adopted in declination and marine compasses.

For delicate experiments a better suspension is a filament of silk, or for light needles a spider's thread. This is called *unifilar* suspension ; if two fibres, parallel and very near together, are used, to determine the amount of torsion, we have *bifilar* suspension.

605. Theory of Magnetism.—The nature of the agency called magnetism is unknown. Much of the language employed by writers on the subject implies that there exist in iron, steel, &c., two imponderable *fluids,* called the *austral* and *boreal magnetisms ;* that these fluids attract each other, and are ordinarily mingled and neutralized, so that no magnetic phenomena appear ; and that in every magnet the two fluids have been separated by the inductive influence of the earth or of another magnet, one

fluid manifesting itself at one pole, and the other at the other pole. As science advances, however, these views seem more and more crude and unsatisfactory. Magnetism is now regarded by many as one of those modes of *molecular motion* which are so difficult of investigation. If it is a mode of motion, then it may manifest itself as a force, as we know it does. It will be seen in the discussion of Electro-magnetism that there is a most intimate connection between magnetism and electricity, so much so that the former is generally considered as only a particular form in which the latter is developed.

Magnetism differs from the other molecular agencies—electricity, light and heat—in producing no direct effect on any of our senses. We witness its direct effects only in the *motion* which it gives to certain kinds of matter, such as iron and steel.

PART VIII.

FRICTIONAL OR STATICAL ELECTRICITY.

CHAPTER I.

ELEMENTARY PHENOMENA.

606. Definition.—The name *Electricity*, from the Greek word for *amber*, is given to a peculiar agency, which causes mutual attractions or repulsions between light bodies, and which, under proper conditions, also produces heat, light, sound and chemical decomposition.

Lightning and thunder are familiar illustrations of the intense action of this agency.

607. Common Indications of Electricity.—If amber, sealing-wax, or any other resinous substance, be rubbed with dry woolen cloth, fur, or silk, and then brought near the face, the excited electricity disturbs the downy hairs upon the skin, and thus causes a sensation like that produced by a cobweb. When the tube is strongly excited, it gives off a spark to the finger held toward it, accompanied by a sharp snapping noise. A sheet of writing-paper, first dried by the fire, and then laid on a table and rubbed with India-rubber, becomes so much excited as to adhere to the wall of the room or any other surface to which it is applied. As the paper is pulled up slowly from the table by one edge, a number of small sparks may be seen and heard on the under side of the paper. In dry weather, the brushing of a garment causes the floating dust to fly back and cling to it.

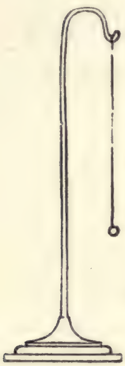
Such electricity is called *Frictional*, from the usual mode of its development, as distinguished from *Galvanic*, which results from chemical action. The former is often called *statical*, and the latter *dynamical* electricity.

Bodies are said to be electrically *excited* when they show signs of electricity in consequence of some mechanical action performed upon them, as in the experiments already described.

A body is *electrified* when it receives electricity, by communication, from another body already *excited* or *electrified.*

608. Pendulum Electroscope.—Attraction or repulsion is the most delicate test of the presence of electricity, and instruments prepared for showing these effects are called *electroscopes.* The word *electrometer,* though sometimes used in the same sense, is more properly defined to be an instrument for measuring the quantity of electricity.

FIG. 344.



The *pendulum electroscope* (Fig. 344) consists of a glass standard, supported by a base, and bent into a hook at the top, from which is suspended a pith ball by a fine silk thread.

When an electrified body is brought near the pith ball, its movement away from or toward the body indicates the presence of an electrical charge; if there is no motion of the ball the body is not charged.

Certain modifications are convenient for some purposes. In one a metallic wire with a gilt ball on the top has a pith ball suspended by the side of it; in another, two pith balls are suspended side by side, as in Fig. 353.

609. Nature of Electricity.—The real nature of electricity is unknown. Though it is in most treatises spoken of as a *fluid,* of exceeding rarity, and more rapid in its movements than light, yet the prevailing belief at the present day is, that it is a peculiar mode of *vibratory motion,* either in the luminiferous ether which is imagined to fill all space, or else in the ordinary matter constituting the bodies and media about us, or in both of these. Electricity is brought to view by friction, by heat, and by other agencies which are calculated to cause *movements* in matter, rather than to bring new kinds of matter to light. It is undoubtedly one of the forms of *force,* into which other forces may be transformed. But until a more definite *wave-theory* or *force-theory* can be constructed than exists at present, it is comparatively easy to give to the learner an intelligible description of electrical phenomena by using the language of the *two-fluid theory* of Du Fay. In trying to give a statement of observed facts without the use of these hypothetical terms, it is necessary to employ in their stead tedious circumlocutions, which only confuse the mind of the learner.

610. Du Fay's Theory.—According to this theory, the two fluids are imagined to inhere in all kinds of matter, combined with

each other and neutralized. In this condition, they afford no evidence of their existence. But they can in several ways be *separated* from each other; and when thus separated, they give rise to electrical phenomena.

611. The Two Electrical States.—After friction between two bodies, the electrical condition of each is *unlike that of the other*. These two electrical states are usually called the *positive* and the *negative*, terms which were employed by Franklin in his theory of *one* electric fluid, to indicate that the excited body has either more or less electricity than belongs to it in its common unexcited condition. Du Fay uses the words *vitreous* and *resinous* to distinguish the two electrical conditions, vitreous corresponding to the positive, and resinous to the negative. It is very common to use Du Fay's *theory*, and to apply Franklin's *terms*, positive and negative, to the two kinds of electricity.

The student must ever bear in mind that these terms are merely *convenient*, and their use must not lead to an acceptance of the *theory of a fluid.*

612. The Two States Developed Simultaneously.—If bodies are rubbed together, the two electricities are separated, and one body is electrified positively, the other negatively. For example, glass rubbed with silk is itself positive, and the silk is negative. But the same substance does not always show the same kind of electricity, since that depends frequently on the substance against which it is rubbed. Dry woolen cloth rubbed on smooth glass is negative, but on sulphur it is positive. The following table contains a few substances, arranged with reference to this. Any one of them, rubbed with one that follows it, is positively electrified itself, aud the other negatively:

1. Fur of a cat.	7. Silk.
2. Smooth glass.	8. Gum lac.
3. Flannel.	9. Resin.
4. Feathers.	10. Sulphur.
5. Wood.	11. India-rubber.
6. Paper.	12. Gutta-percha.

According to the above table, silk rubbed on smooth glass is negatively excited; but rubbed on sulphur, it is excited positively. It is sometimes found, however, that the previous electrical condition of one of the bodies will invert the order stated in the table. For example, if silk, having been rubbed on smooth glass, and therefore being negative, should then be rubbed on resin, it would probably retain its negative state, and the resin become positively electrified, contrary to the order of the table.

The mechanical condition of the surface sometimes changes the order of the two electricities. Thus, if glass is ground, so as to lose its polish, it is likely to be negative when rubbed with silk; but the excitation of rough glass is very feeble.

613. Mutual Action.—Bodies electrified *in different ways attract,* and *in the same way repel* each other. Thus, if an insulated pith ball, or a lock of cotton, be electrified by touching it with an excited glass tube, it will immediately recede from the tube, and from all other bodies which are charged with positive electricity, while it will be attracted by excited sealing-wax, and by all other bodies which are negatively electrified. If a lock of fine long hair be held at one end, and brushed with a dry brush, the separate hairs will become electrified, and will repel each other. In like manner, two insulated pith balls, or any other light bodies, will repel each other when they are electrified the same way, and attract each other when they are electrified in different ways.

Hence it is easy to determine whether the electricity developed in a given body is positive or negative; for, having charged the electroscope with excited glass, then all those bodies which, *when excited,* attract the ball, are negatively charged, while all those which repel it are positively charged.

614. Conduction.—Electricity passes through some bodies with the greatest facility; through others with difficulty, or scarcely at all; and others still have a conducting power intermediate between the two. As the conducting quality exists in different substances in all conceivable degrees, it is impossible to draw a dividing line between them, so as to arrange all good conductors on one side, and all poor conductors on the other. The following brief table contains some of the more important of the two classes, in the order of *conducting* power:

Good conductors.	Poor conductors (or insulators).
The metals,	Baked wood,
Charcoal,	Air, the gases,
Plumbago,	Paper,
Water, damp snow,	Silk, wool, hair, feathers,
Living vegetables,	Glass, precious stones,
Living animals,	Wax,
Smoke, steam,	Sulphur,
Moist earth, stones,	Paraffine,
Linen, cotton.	Lac, amber, the resins.

Good conductors are usually termed *conductors,* while poor conductors are called *insulators.* The less the conductivity of any substance the greater is its insulating power.

When air is rarefied, its insulating power is diminished, and the further the rarefaction proceeds, the more freely does electricity pass. Hence, we might expect that it would pass with perfect freedom through a complete vacuum. It is found, however, that in an absolute vacuum electricity cannot be transmitted at all.

Although under ordinary conditions the arrangement of the above table is correct, yet conductivity varies with changes of physical state. Glass becomes a much better conductor at red heat than it is when cold; resins become better conductors with rise of temperature; metals become poorer conductors with increase of temperature.

615. Modes of Insulating.—Solid insulating supports are usually made of glass; and, in order to improve their insulating power, they are sometimes covered with shell-lac varnish. Insulating threads for pith balls, or cords for suspending heavier bodies, are made of silk. The best insulator for suspending any very small weight is a single fiber of silk, a hair, or a fine thread of gum-lac. In order to perform electrical experiments, the air must be dry, or no care whatever relating to apparatus can insure success; and therefore, in a room occupied by an audience, especially if the weather is damp, it is necessary to dry the air artificially by fires, and to warm all glass insulators to drive off the film of moisture which condenses upon them. If the air were a good conductor, it is probable that no facts in this science would ever have been discovered.

If an *iron* or *brass* rod be held in the hand and rubbed with silk, the rod shows no sign of electricity, the electricity excited in the rod being conveyed away by the conducting quality of the metal and the human body; but if the metal rod be insulated by a glass or ebonite handle, as in Fig. 345, it, as well as the rubber, will give signs of electricity when properly tested.

FIG. 345.

616. Other Modes of Developing Electricity.—Cleavage of many minerals causes unlike electrical states in the separated laminæ; this is especially noticeable in the case of mica. Loaf sugar, broken in the dark, becomes slightly luminous.

Pressure of a crystal of Iceland spar will produce an electric charge.

Tourmaline when heated, or cooled, becomes electrically charged at two opposite ends of the crystal, owing to the *change* of temperature, and not to the particular temperature at any instant. One end of the crystal will be positive, and the other negative.

This development of electricity, in the case of tourmaline, seems to be confined to changes of temperature between 10° C. and 150° C.

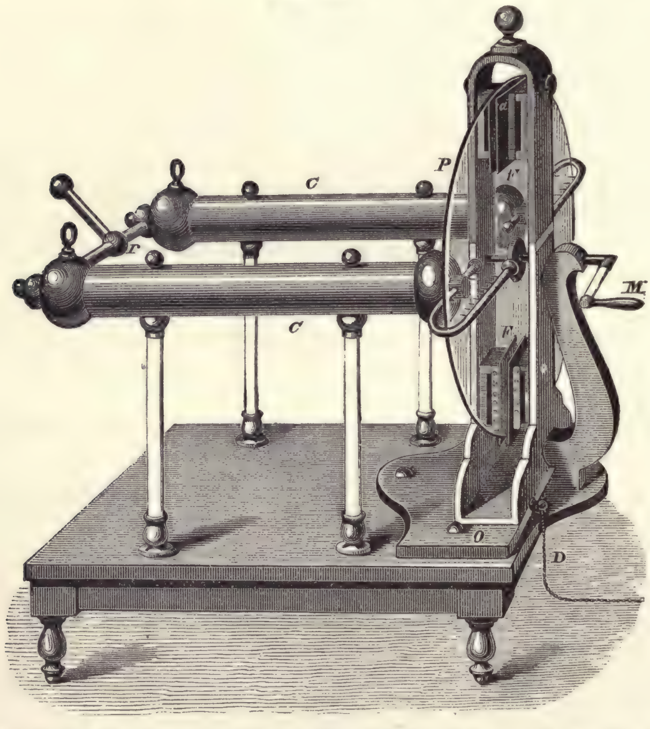
Pyroelectricity is the name given to the charge thus developed, and crystals so affected are said to be *pyroelectric*.

CHAPTER II.

ELECTRICAL MACHINES.—LAW OF FORCE.—MODE OF DISTRIBUTION.

617. The Plate Machine.—In order that glass may be conveniently subjected to friction for the development of electricity, it is made in the form of a circular *plate*, and mounted on an axis, which is supported by a wooden frame, and revolved by a crank, while *rubbers* press against its surface. Fig. 346 represents

FIG. 346.



one of the many forms which have been adopted. The crank, *M*, gives rotary motion to the plate, *P*, which is pressed by the rubbers, *F, F*; this pressure is equalized by their being placed at top and bottom, and on both sides of the glass. The prime conductor, *C C*, is made of hollow brass, and supported by glass pillars. The extremities terminate in two bows, which pass around the edges of the plate, and present to it a few sharp points, to facilitate the passage of electricity. But all other parts are carefully rounded in cylindrical and spherical forms, without edges or points, as they tend to dissipate the electricity. The glass, as it revolves from the rubbers to the points of the prime conductor, is protected by silk covers, to prevent the electricity from escaping into the air. The rubbers are made of soft leather, attached to a piece of wood or metal, and from time to time are rubbed over with an *amalgam* of 1 part zinc, 1 part tin, and two parts mercury, powdered and mixed with grease; or with the bi-sulphuret of tin, which is one of the best exciters on glass. The diameter of the plate varies from 1½ to 3 feet; but in some of the largest it is 6 feet, and two plates are sometimes mounted on one axis.

To give free passage of the negative electricity from the rubbers to the earth, a chain, *D*, may be attached to the wooden support, while its other end lies on the floor.

It is necessary to dry and warm all insulators of the machine to prevent escape of electricity by the supports; also to dry the surrounding air (Art. 615). As the electrical charge increases with the rapidity of the rotation, soon the accumulation will be so great as to cause some of the electricity developed upon the plate to overcome the low conductivity of the glass and recombine with the opposite electricity of the rubber, and a greater charge than this can not be maintained by any increase of velocity of rotation.

618. The Cylinder Machine.—In many electrical machines of the smaller sizes, a hollow cylinder is employed, having a length considerably exceeding its diameter. In the cylinder machine, the rubber is applied to one side, and the prime conductor receives the fluid from the opposite. The rubber is usually mounted on a glass pillar, so that it can be insulated, whenever it is desired.

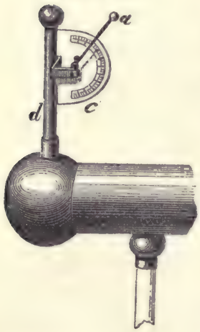
619. The Hydro-Electric Machine.—It was discovered in 1840 that a steam-boiler electrically insulated gave out sparks, and that the steam issuing from it was also electrified. Hence resulted the construction of *the hydro-electric machine.* It consists of a boiler mounted on glass pillars, and furnished with a row of jet-pipes and a metallic plate, against which the steam strikes. The prime conductor, to which the steam-plate is attached, is

electrified positively, and the boiler itself negatively. Faraday ascertained that the electricity in this case is developed, not by evaporation or condensation, but by the friction of watery particles in the jet-pipes, perfectly dry steam producing no charge. That the machine may act with energy, it was found necessary to make the interior of the jet-pipes angular, and quite irregular.

A machine of this kind, whose boiler was 6½ feet long and 3½ feet in diameter, gave sparks 22 inches in length. The great amount of moisture discharged, and the necessary and troublesome precautions in preparing the machine for use, render it valueless for any practical purpose.

620. The Quadrant Electrometer.—In order to measure the intensity of electricity in the prime conductor, there is set upon it, whenever desired, a *quadrant electrometer* (Fig. 347). This consists of a pillar, *d*, about six inches high, having a graduated semicircle, *c*, attached to one side, and a delicate rod and ball, *a*, suspended from the centre of the semicircle. As the conductor becomes electrified, the rod is repelled from the pillar, and the arc passed over indicates rudely the degree of electrical intensity.

FIG. 347.



621. First Phenomena of the Machine.—When an electrical machine is skillfully fitted up, and works well, there is first perceived, on turning it, a crackling sound; and then, on bringing the knuckles toward the prime conductor, a brilliant spark leaps across, causing a sharp pricking sensation. If the room be darkened, brushes of pale light are seen to dart off continually from the most slender parts of the prime conductor, with a hissing or fluttering noise, while circles of light snap along the glass between the rubbers and the edges of the covers. When electricity is escaping plentifully from the machine, a person standing near also perceives a peculiar odor, which is that of *ozone*, and which seems always to accompany the development of electricity.

Therefore, at least *four* of the senses are directly affected by this remarkable agency, while magnetism affects none of them.

The phenomena of repulsion of like and attraction of unlike electricities, are well shown by the machine. A skein of thread or a tuft of hair, suspended from the prime conductor, will, as soon as the plate is revolved, spread into as wide a space as possible, by the repellency of the fibers which are electrified alike. Melted sealing-wax is thrown off in fine threads, and dropping water is diverged into delicate filaments. Even air, on those parts

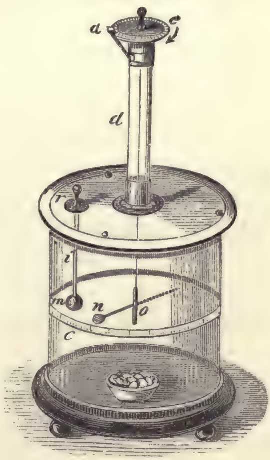
of the prime conductor which are most strongly charged, becomes so self-repellent as to fly off in a stream of wind, which is plainly felt.

On the other hand, light bodies, when brought toward the machine while in action, instantly fly to the prime conductor; for that is positive, but the nearer sides of the other bodies are made negative by induction.

The difference between substances as to their conducting quality is readily perceived by setting the quadrant electrometer on the prime conductor, raising the index by turning the plate, and then touching the prime conductor with the remote end of the body to be tried. If an iron rod, or even a fine iron wire, be thus applied, the index will fall instantly; a long dry wooden rod will cause it to descend slowly, while a glass rod will produce no effect at all. These experiments show that iron is a good conductor, wood an imperfect conductor, and glass a very poor conductor.

622. Coulomb's Torsion Balance.—When a long fine wire is stretched by a small weight, its elasticity of torsion is a very delicate force, which is successfully employed for the measurement of other small forces. When such a wire is twisted through different angles, the *force* of torsion is found to vary as the *angle* of torsion; it is therefore easy to measure the force which is in equilibrium with torsion. The torsion balance is represented in Fig. 348. The needle of lac, *n o*, is suspended by a very fine wire from a stem at the top of the tube *d*. The cap of the tube, *e*, is a graduated circle, whose exact position is marked by the index, *a*. The stem from which the wire hangs is held in place in the centre of the cap by friction, but can be turned round so as to place the needle in any direction desired. At the end of the lac needle is a small disk of brass-leaf, *n*, and by its side a gilt disk, *m*, connected with the handle, *r*, by the glass rod, *i*.

FIG. 348.



This apparatus is suspended in the glass cylinder, covered with a glass plate, on the centre of which the tube *d* is fastened. There is a graduated circle around the cylinder on the level of the needle.

623. Law of Electrical Force as to Distance.—Adjustment is now made by turning the stem so that, while the wire is

in its natural condition, the disk, n, touches the disk, m, and is at zero, and the index at top also at zero on the circle e. Let a minute charge of electricity be communicated to m, and it will repel n, and cause it, after a few oscillations, to settle at a certain distance—suppose for instance, at 36°. The circle e is now turned in the opposite direction, until the needle is brought within 18° of the ball m. In order to bring it thus near, the index has to be turned 126°, which added to the 18°, makes the whole torsion 144°, or *four times* as great as before. Therefore, at *one-half* the distance there is *four times* the repulsion. In like manner, it is found that at *one-third* the distance there is *nine times* the repulsion. Hence the law that for two given charges,

Electrical repulsion varies inversely as the square of the distance.

In a manner somewhat similar to the foregoing, it was conclusively proved by Coulomb that electrical *attraction* obeys the same law of distance, though there is more practical difficulty in performing the experiments. But if the electrified body m is placed outside of the circle described by n, so that the latter is allowed to vibrate both to the right and left, the square of the *number* of vibrations in a given time becomes a measure of the attractive force, as in the case of the pendulum (Art. 163).

If the charge upon m be doubled, that upon n remaining unchanged, the angle of torsion will be double that found above in each case; and if we now treble the charge upon n, the angle of torsion will be six times as great as at first. Hence we learn that the force of attraction or repulsion, for any given distance, is proportional to the product of the two charges. Calling the charges q and q' respectively, the distance between them d, and force exerted F, we have

$$F = \frac{q \times q'}{d^2},$$

an expression similar to that found in Art. 574.

624. Unit of Electricity.—The expression for the force of repulsion between two quantities, q and q', of like electricity, at the distance l from each other, was found above to be $f = \frac{qq'}{l^2}$; if we make $q = q'$ we have $f = \frac{q^2}{l^2}$. If l be taken equal to one unit of length in any system which we may choose to adopt, f equal to one unit of force, and the separating medium be air, then q will be the unit of electricity. In the C. G. S. system $l =$ one centimetre, and $f =$ one dyne; therefore *the unit of electricity* in this system *is that quantity of electricity which will repel an*

equal quantity, at the distance one centimetre, with a force of one dyne.

Suppose two spheres, very small as compared with their distance from each other, to be charged positively, one having a charge three times as great as that of the other, and that when placed five centimetres apart they repel with a force of three dynes, then we have $f \times l^2 = q \times 3\,q$, whence $3 \times 5^2 = 3\,q^2$ and $q = 5$; hence one sphere was charged with 5 units of electricity, and the other with 15 units.

625. Waste of Electricity from an Insulated Body.—In making accurate investigations like the foregoing, in which considerable time is necessarily occupied, a difficulty arises from the loss of the electrical charge. The *first* and most obvious source of waste is the moisture in the air, which conducts away the fluid; but this may be nearly avoided by setting into the cylinder a cup of dry lime, or other powerful absorbent of moisture, as represented in the figure. A *second* is the imperfect insulation afforded by even the most perfect non-conductors. A *third* is the mobility of the air, whose particles, when they have touched the electrified body, and become charged, are repelled, taking away with them the charge they have received. The loss in these ways is very slight, when the charge is small, and allowance can be made for it with a good degree of accuracy. But when bodies are highly charged, they lose their electricity at a rapid rate.

626. A Statical Electrical Charge Lies at the Surface.—This is proved in many ways. A *hollow* ball, no matter how thin, will receive as large a quantity of electricity as a *solid* one. Hence it is that the prime conductor of the electrical machine, and metallic articles of electrical apparatus generally, are made of sheet brass, for the sake of lightness.

Let a dish *a* (Fig. 349), be made of two brass rings and cambric sides and bottom, with an insulating handle, *b*, attached to the larger ring. If this vessel be charged with electricity, the charge is found on the outside; turn it over quickly, so as to throw it the other side out, and the charge is instantly found on the outside again, and none on the inside. It may be inverted several times with the same result, before the charge becomes too feeble to be perceived.

FIG. 349.

If an insulated hollow metallic sphere, or a cylinder of wire gauze, be charged and its interior surface be tested, no charge will be detected. To make this test, a *proof plane*, consisting of a gilt

disk insulated by a slender rod of lac, is touched to the interior surface, and is then applied to a delicate torsion balance; as no motion of the needle ensues, it is proof that no charge was present.

Another proof that the charge occupies only the outside surface is that the intensity diminishes as the *surface* is enlarged, while the *mass* of the conductor remains the same. A metallic ribbon rolled upon an insulated cylinder may be unrolled, and thus the surface enlarged to any extent. An electroscope standing on the instrument will fall as the ribbon is unrolled, and rise when it is again rolled up.

It will be shown in Art. 636, that a charge may be *induced* upon the interior surface of a hollow conductor, but it will be observed that this fact is no contradiction of those given above.

A charge *moving from one point to another* passes *through* the substance of the conductor and not over its surface merely. If a powerful charge be sent through a narrow strip of gold leaf it will melt, and perhaps vaporize, the metal; a similar charge passed through a gold wire whose surface equals that of the gold leaf will produce no such effect, owing to the vastly greater cross-section of the wire.

627. Potential.—The difference of electrical condition of two bodies which causes a transfer of electricity from one to the other, either through a good conductor or across a poorly-conducting medium, is called *difference* of *potential.* The body, A, from which electricity is transferred, is said to have a high potential relatively to the lower potential of the body, B, to which the transfer is made.

The measure of *difference of potential* is the amount of work which would be done in moving a unit of electricity from B to A, in a direction contrary to that in which a free unit would move. The potential of the earth is taken as the standard from which to measure these differences, and is called zero; hence the difference of potential between a body, or point, and the earth, is called the potential of the body, or point.

In Art. 36, we have *work* $=$ *force* $\times$ *distance;* hence, calling V_a and V_b the potentials at A and B respectively, $A\ B$ their distance apart, and f the *average* force of repulsion, we have

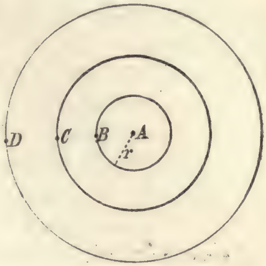
$$f \times A\ B = V_a - V_b, \text{ whence } f = \frac{V_a - V_b}{A\ B}.$$

When A and B are near together, the force of repulsion at all points of the line joining them will be practically constant.

628. Equipotential Surfaces.—Suppose A (Fig. 350) to

be an electrical charge, *concentrated at a point,* and let B represent a unit of electricity at a distance r. The difference of potential will be measured by the work required to move B, through the distance r, towards A; this work will be the same wherever B may be placed upon the sphere having r for its radius; hence the definition,

FIG. 350.



An equipotential surface, with respect to an electrified point A, is a surface such that the work required to transfer a unit charge from any point of it to the point A will be constant.

At any point of the sphere C, the potential will be lower than upon any point of the sphere B, since more work will be done in carrying the unit charge from C to A, than from B to A. If C be so far from B as to cause an expenditure of one unit of work in moving unit charge from C to B, there will be a *unit difference of potential* between these two equipotential surfaces.

The distances A B, B C, C D, &c., form an increasing series, since the force of repulsion decreases while the radii increase, and hence a unit charge must be moved through increasing distances to produce the unit of work.

An equipotential surface about two electrified points, near each other, would not be spherical; and if a number of electrified bodies, or points, constitute an electrical centre, the equipotential surface with respect to them would be irregular.

The unit of work, usually taken in this connection, is the work done in overcoming a resistance of one dyne through a distance of one centimetre, and is called an *erg.*

629. Capacity.—The number of units of electricity which must be imparted to a body to raise its potential from zero to unity measures the relative electrical capacity of the body.

A body that requires only a unit of electricity to raise its potential from 0 to 1, is said to have *unit capacity.* A unit of electricity communicated to a sphere of one centimetre radius, will raise its potential from 0 to 1; such a sphere has therefore *unit capacity.*

The capacities of spheres are proportional to their radii.

In the above definitions no external disturbing electrical forces are considered. The capacity of a conductor is increased by bringing near it a charge of opposite kind; for the potential of the charged conductor, in such case, is the difference of potential due to its own charge and that due to the charge of opposite kind;

hence a greater charge must be given the conductor to raise its potential from 0 to 1.

630. Distribution of a Charge on the Surface.—Statical electricity resides at the surface of a body, as we have seen, but is not uniformly diffused over it, except in the case of the sphere. In general, the more *prominent* the part, and the more *rapid its curvature,* the more intensely is the fluid accumulated there.

In a long slender rod the density is greatest at the ends, nearly the whole charge being collected at these points. On a sphere, not influenced by induction, the density is uniform, as illustrated in Fig. 351, the dotted line denoting by its constant distance from the surface the uniform distribution of the charge.

Fig. 352 represents the varying density upon an ellipsoid.

FIG. 351.

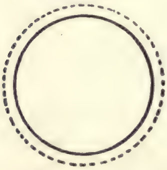


FIG. 352.

The two ellipsoids are similar, and the ellipsoidal shell included between them represents the densities at various points. In this case the densities at any two points of the ellipsoid are nearly proportional to the diameters through those points.

The student must remember that the charge does not form a layer upon the body in any sense whatever, and that the above figures are given merely to aid the memory in retaining the law of distribution.

631. Surface Density.—The number of units of electricity per unit area of surface is called the *surface density* at the point. Let Q be the quantity of electricity on any given area S, then the surface density, called p, will be expressed by the equation

$$p = \frac{Q}{S}.$$

In dry air the limit of charge is given as about 20 units per square centimetre. If the distribution of charge be uniform, as in the case of a sphere removed from the influence of electrical inductions, the density is found by dividing the total charge Q by the area of surface, $4\,\pi\,r^2$, giving $p = \frac{Q}{4\,\pi\,r^2}$; hence, if equal charges be given to two spheres of different radii, the densities will be inversely as the squares of the radii.

Two spheres, at some distance apart and connected by a fine wire, will divide a common charge imparted to them in the proportion of their capacities, that is to say, in the proportion of their radii. Suppose the radius of one to be r, and of the other $2r$, then their charges will be as r to $2r$. The densities will be found by dividing the charges by the respective surfaces, which are as r^2 to $(2r)^2$; hence we have $\frac{r}{r^2} : \frac{2r}{(2r)^2} :: 2r : r$, or the densities are inversely as the radii. If the diameter of the smaller sphere is made less and less, its capacity grows less also, and the potential grows greater and greater, till at the limit, when the sphere becomes a *point*, the potential is so great as to cause a total discharge, and prevent any accumulation of charge upon it.

632. The Charge Held on the Surface by Atmospheric Pressure.—The mutual repellency, which forces the electric charge to remain upon the surface of the conductor, tends to make it escape in all directions from that surface; and it is the air alone which prevents. For if one extremity of a charged and insulated conductor extends into the receiver of an air-pump, the charge is dissipated by degrees, as the receiver is exhausted; and when the exhaustion is as complete as possible, the most abundant supply from the machine fails to charge the conductor.

If the vacuum is made as perfect as possible no dissipation of charge occurs, since electricity will not pass through a vacuum (Art. 614).

This limit cannot be reached by the use of even the best air-pump alone, but is attainable in other ways.

As the atmospheric pressure is limited to about 15 lbs. per square inch, so the amount of charge is limited which can be retained on a conductor of given form. Hence the reason for the well-known fact that the prime conductor receives all the charge which it is capable of retaining in one or two turns of the machine. All that is gained over and above this, by continuing to turn, flies off through the air.

633. Rotation by Unbalanced Pressure.—As the electric charge on the surface of a body presses outward in all directions, wherever it escapes from a point, there the pressure is removed; consequently, on the opposite part there is *unbalanced* pressure. Therefore, if the body is delicately suspended, and one or more points are directed tangentially, the unbalanced pressure will cause rotation in the opposite direction, just as Barker's mill rotates by the unbalanced pressure of water. Electrical wheels and orreries are revolved in this way.

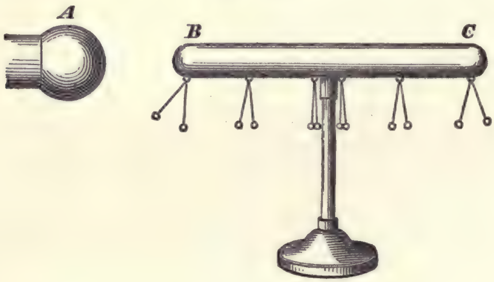
A windmill may also be revolved by the stream of air issuing from a stationary point attached to the prime conductor (Art. 621).

CHAPTER III.

ELECTRICITY BY INDUCTION.—LEYDEN JAR.

634. Elementary Experiment.—When an electrified body is placed near one which is unelectrified, but not near enough to cause discharge between them, the natural electricities of the latter are decomposed, one being attracted toward the former, the other repelled from it. Thus the ends become electrified by the *influence* of the first body, without *receiving* any electricity from it. Let *A* (Fig. 353) be charged with positive electricity, and let the

FIG. 353.



insulated conductor, *B C*, be furnished with several electroscopes, as represented. Those at the end *B* will diverge more widely than those which follow, in the direction *B C*, until a point is reached, somewhat less than half the distance from *B* to *C*, where there is no sign of electricity; passing this neutral point the electroscopes diverge more and more till the maximum effect is again reached at *C*. By taking off small quantities with the proof plane it is found that *B* is charged negatively and *C* positively, and also that the negative charge at *B* is of greater density than the positive at *C*, owing to the attraction of unlike electricities at *A* and *B* through the short distance *A B*, being greater than the repulsion between the like charges, at *A* and *C*, through the greater distance *A C*.

Remove the bodies to a distance from each other, and *B C* returns to its unelectrified condition; bring them near again, and it is electrified as before. As this electrical state is *induced* upon

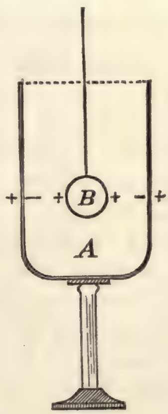
the conductor by the electrified body in its vicinity, without any *communication* of electricity, it is said to be electrified by *induction.* If A is first charged with the negative electricity, the two electricities of B C will be arranged in reversed order; the positive will be attracted to the nearest end, the negative repelled to the farthest.

Electrical induction is exactly analogous to *magnetic* induction ; the opposite kind is developed at the nearer end, and the like kind at the remote end.

635. Successive Actions and Reactions.—If A is itself an insulated conductor, the foregoing is not the entire effect; for a reflex influence is exerted by the electricity in the nearer end of the conductor. Let A have a positive charge, as at first. After the negative electricity is attracted to the nearer end of B C, it in turn attracts the positive charge of A, and accumulates it on the nearest side, leaving the remote side less strongly charged than before. This is shown by electroscopes attached to the opposite sides of A. The charge of A, being now nearer, will exert more power on B C, separating more of its original electricities, and thus making the nearest end more strongly negative and the remote end more strongly positive than before; and this new arrangement of fluids in B C causes a second reaction upon A, of the same kind as the first. Thus an indefinite diminishing series of adjustments takes place in a single moment of time.

636. Induced Charge within a Hollow Conductor.—Let A (Fig. 354), represent a section through an insulated, hollow, metallic cylinder upon an insulating stand. Introduce within it the charged ball B, suspended by a long silk thread for insulation. Connect the outer surface of the cylinder, by a wire conductor, with a delicate electroscope. If B is charged positively it will induce upon the inner surface of A an equal quantity of negative electricity, while the outer surface will become positively charged. When B is removed the cylinder will return to its neutral state. As B is lowered into A the electroscope will indicate a gradually increasing charge upon the outer surface, until the ball reaches a point two or three inches below the mouth of the cylinder, after passing which a further descent of the ball produces no change. If the ball be allowed to touch the bottom of A, no change in the indications of the electroscope will appear, although the whole charge of B will be com-

FIG. 354.



municated to that induced upon the inner surface of A, exactly neutralizing it, and on withdrawing B it will be found wholly discharged; this constancy of the indication of the electroscope shows that the induced charge upon the outside of A is exactly equal to the original charge upon B.

To vary the experiment, let A be charged positively, the whole of which charge will reside upon the outer surface.

If now the ball B, connected to earth by a suspending wire conductor, be lowered into it, B will be charged negatively by the inductive action of A, a portion of whose positive charge is thus transferred to the inner surface by the reciprocal action of the negative induced upon B.

637. Division of the Conductor.—Suppose that before the experiment (Art. 634) begins, B C is in two parts with ends in contact; the entire series of mutual actions takes place as already described. Now, while A remains in the vicinity, let the parts of B C be separated; then the negative electricity is secured in the nearest half, and the positive in the other. And if A is now removed, each charge diffuses over that half of the original conductor upon which it was induced.

Thus each kind of electricity can be completely separated from the other by means of induction.

Here we find a marked difference between magnetism and frictional electricity. The electricities may be secured in their separate state, one in one conductor, the other in another. In magnetism this is not possible; for when an iron bar is magnetized, and then broken, each kind of magnetism is found in each half of the bar (Art. 572); at the point of division both polarities exist, and as soon as the bar is broken, they manifest themselves there as strongly as at the extremities.

638. Effect of Lengthening the Conductor.—If the conductor, B C, is *lengthened*, the accumulation on the adjacent parts of the two bodies is somewhat increased. The positive electricity which, at the remote end of the shorter conductor, operated in some degree by its repulsion of the charge upon A and its attraction for that upon B to prevent accumulation on the nearest side of A, is now driven to a greater distance; and therefore a larger charge will come from the remote to the nearer side of A, which in turn attracts more negative to the nearer end of B C, and thus a new series of actions and reactions takes place in addition to the former. To obtain the greatest effect from this cause, C is connected with the earth; then the positive electricity is driven to the earth, and entirely disappears, and the negative is attracted to the nearer end. This experiment is performed by touching the

finger to the conductor, after it has become electrified by induction. The electroscope nearest to A instantly rises a little higher, and the distant ones collapse.

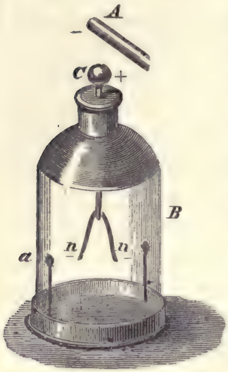
639. Disguised Electricity.—The electricity which occupies the surface of the prime conductor, or any other body electrified in the ordinary way, and which is kept from diffusing itself in every direction only by the pressure of the air (Art. 632), is called *free* electricity; for it will instantly spread over the surface of other conductors, when it touches them, and therefore will be lost in the earth, the moment a communication is made. But the electricity which is accumulated by the inductive influence is not free to diffuse itself; the same *attractive force* which has condensed it still holds it as near as possible to the original charge; and if we touch the electrified body with the hand, the electricity does not pass off; it is therefore called *disguised* electricity. In this respect the two fluids on the *contiguous* sides of A and B C are alike; either may be touched, or in any way connected with the earth, but, unless communication is made between them, or unless they are both allowed to pass to the earth, they hold each other in place by their mutual attraction, and show none of the phenomena of free electricity.

640. A Series of Conductors.—If another insulated conductor, D, is placed near to the remote end of B C (Art. 634), and A is charged positively, then that extremity of B C nearest to D is inductively charged with positive, as already stated. Hence, the electricities of D are separated, the negative approaching B C, and the positive withdrawing from it; there is therefore the same arrangement of fluids in both bodies, but a less intensity in D than in B C. For, on account of distance, the positive is not so intensely accumulated at the remote end of B C as in the original body A, and therefore a less force operates on D than on B C The same effects are produced in a less and less degree in an indefinite series of bodies; and the shorter they are, the more nearly equal will be the successive accumulations. The same facts were noticed in a series of magnets.

641. The Gold-Leaf Electroscope.—The gold-leaf electroscope consists of two narrow strips of gold-leaf, n, n (Fig. 355), suspended within a glass receiver, B, from a metallic rod which passes through the top and terminates in a ball, C. A metallic base is cemented to the receiver, and strips of tin-foil, a, are attached to the inside, reaching to the base, in order to discharge the gold leaves whenever they are caused to diverge excessively. Let a body positively electrified be brought within a few feet of the knob. It attracts the negative from the leaves into the

knob, and repels the positive from the knob into the leaves; they are thus electrified alike, and repel each other. If the charged body is brought so near that the leaves touch the conductors, which are placed on the sides of the cylinder, and discharge their induced electricity to them, then they collapse. After this, they will diverge again, whether the electrified body is brought still nearer, or withdrawn; if brought nearer, they diverge by means of a new portion of positive, repelled from the knob; if withdrawn, they diverge by the return of negative electricity from the knob, which is no longer neutralized by the positive, since the latter has been discharged to the earth.

FIG. 355.

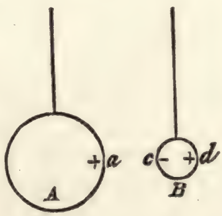


As there is danger of rupturing the gold leaves by too violent action, the method usually adopted is as follows: Let a charged body, positively charged for example, be brought near the knob, and the leaves will diverge, being charged positively by induction; if the knob be now touched by the finger, the inducing body remaining as before, the leaves will at once collapse, the repelled positive electricity being driven to earth. If now the finger be removed, and *afterward* the inducing body be withdrawn, the negative charge will diffuse, causing the leaves to diverge by mutual repulsion.

If now a charged body, suspended by an insulating silk thread, in order that the hand may be kept so far from the knob as to prevent its inductive influence, be brought near the knob, an increase of divergence would indicate a negative charge upon the body, while decrease of divergence would indicate a positive charge.

642. Mutual Attractions and Repulsions of Bodies.—That an electrified body attracts an unelectrified body, a fact which is among the first to be noticed in observing electrical phenomena (Art. 621), is explained by induction.

FIG. 356.



1. Let A (Fig. 356) be charged positively, and let B be unelectrified. A acting by induction will decompose the electricity of B, as shown in Fig. 356, attracting the negative to the nearest point c and repelling the positive to the furthest point d. As the distance $a\ c$ is less than $a\ d$, then, according

to the law given in Art. 623, the mutual attraction must exceed the repulsion, and the bodies will move toward each other.

2. Let B be a conductor charged negatively, then there will be mutual attraction.

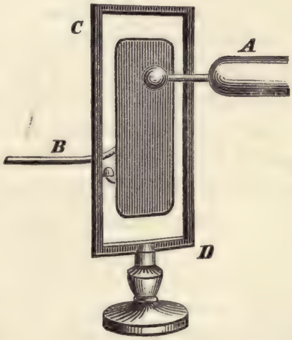
3. Let B be a conductor charged positively, then there may be repulsion or attraction according to the distance $a\ c$. In addition to the positive *charge* on B there is also some decomposed electricity. This, acted upon by A, is separated, the positive increasing the repelled positive charge at d, and the negative appearing at c. At a distance from A, B will be repelled; but if $a\ c$ be diminished the attractive force increases more rapidly than the repellent, and finally $a\ c$ may become so small that the attraction may exceed the repulsion.

4. If B be a poor conductor with negative charge, attraction will result; if with positive charge, repulsion ensues. If it be unelectrified there will be some inductive decomposition of its electricity, and attraction will be shown.

That the charge does not leave either body, but constrains each to move with it, is explained by Art. 632.

643. The Inductive Action Greatly Increased.—In the experiments as now described, the inductive influence is feeble, and the accumulation of electricities very small; for the bodies present toward each other only a limited extent of area, and they are necessarily as much as four or five inches distant, in order to prevent the fluid from passing across. By giving the bodies such a form that a large extent of surface may be equidistant, and then interposing a solid non-conductor, as glass, between them, so that the distance may be reduced to one-eighth of an inch or less, it is easy to increase the attracting and repelling forces many thousands of times (Art. 629). Let a glass plate, $C\ D$ (Fig. 357), supported on a base, have attached to the middle of each side a rectangular piece of tin-foil. This is called a Franklin plate. Let A be connected with one coating, and B with the other. If, now, A forms a part of the prime conductor of an electrical machine, and B has communication with the earth, there will be accumulation of great quantities of electricity, positive on one side and negative on the other. If the amount of surface and the thickness of glass are the same, the particular form of the instru-

FIG. 357.



ment is immaterial; but, for most purposes, a vessel or jar is more convenient than a pane of glass of equal surface, and is generally employed for electrical experiments.

644. The Condensing Electroscope.—Instruments called by this name are intended for the accumulation of electricity from some feeble source, until it may be rendered sensible. The most delicate is the gold-leaf condenser. Suppose the gold-leaf electroscope to have a disk, A, instead of a knob on the top (Fig. 358). Another disk, B, is furnished with an insulating handle, and between the disks is placed the thinnest possible non-conductor, as a film of varnish. Set the plate B upon A, and place the finger upon the upper surface of B. Then touch the under surface of A with the body feebly charged; the charge will diffuse itself over the upper surface of A and will induce an opposite charge on the near surface of B. Apply successive weak charges in like manner, all of which will accumulate upon the upper surface of A. Thus far the gold leaves have shown no indications of electricity, the charge upon A being condensed upon its surface by the opposite induced charge in B; but on taking up B by the insulating handle, the electricity condensed in A is set free, flows down to the leaves and repels them, thus rendering the accumulation perceptible.

Fig. 358.

645. Influence of the Interposed Non-Conductor.—If instead of a glass plate between the two metallic surfaces in the Franklin plate, a film of air of the same thickness be substituted, the inductive action will be about one-third as great as before, and if a plate of sulphur had been used instead of glass the effect would have been five-sixths as great as with glass. The effect of the interposed insulator, or *dielectric*, may be shown thus: Let the charged disk B (Fig. 358) be placed above A at a distance which will cause the leaves to diverge slightly, as in the figure. If now a plate of any dielectric, as rubber or paraffine, be interposed between the disks, the gold leaves will separate more widely, showing the inductive influence of the dielectric. The following table is taken from results given by Gordon:

Specific Inductive Capacities.		Specific Inductive Capacities.	
Air or any gas	1.00	Gutta-percha	2.46
Paraffine	1.99	Sulphur	2.58
Black India-rubber	2.22	Shellac	2.74
Ebonite	2.28	Glass	3.25

646. The Leyden Jar.—This article of electrical apparatus

consists of a glass jar (Fig. 359), coated on both sides with tin-foil, except a breadth of two or three inches near the top, which is sometimes varnished for more perfect insulation. Through the cork passes a brass rod, which is in metallic contact with the inner coating, and terminates in a ball at the top.

On presenting the knob of the jar near to the prime conductor

FIG. 359.

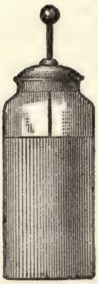
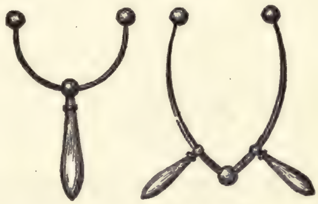


FIG. 360.



of an electrical machine, while the latter is in operation, a series of sparks passes between the conductor and the jar, which will gradually grow more and more feeble, until they cease altogether. The jar is then said to be *charged.* If now we take the *discharging-rod,* which is a curved wire, terminated at each end with a knob, and insulated by glass handles (Fig. 360), and apply one of the knobs to the outer coating of the jar, and bring the other to the knob of the jar, a flash of intense brightness, accompanied by a sharp report, immediately ensues. This is the *discharge* of the jar.

If, instead of the discharging-rod, a person applies one hand to the outside of the charged jar, and brings the other to the knob, a sudden *shock* is felt, convulsing the arms, and when the charge is heavy, causing pain through the body. The shock produced by electricity was first discovered accidentally by persons experimenting with a charged phial of water. This occurred in Leyden, and led to the construction and name of the Leyden jar.

647. Theory of the Leyden Jar.—This instrument accumulates and condenses great quantities of electricity on its surfaces, upon the principle of mutual attraction between unlike electricities, one of which is furnished by the machine, the other obtained from the earth by induction. First, suppose the outer coating insulated ; a spark of the positive electricity passes from the prime conductor to the inner coating, which tends to repel the positive from the outer coating ; but as the latter cannot escape, it remains to prevent, by its counter-repulsion, any addi-

tion to the charge of the inside, and thus the process stops. But now connect the outer coating with the earth, and immediately some of its positive electricity, repelled by the charge on the inside, passes off, while its negative is attracted close upon the glass. The negative upon the outside, by its attraction, condenses the positive of the inner coating, and a second spark passes in from the prime conductor. This produces the same effect as the first, and a second addition of negative is made to the outer coating, the latter being obtained from the earth as before. These actions and reactions go on in a diminishing series, till there is a great accumulation of the two electricities, held by mutual attraction as near each other as possible, on opposite sides of the glass. The jar in this condition is said to be charged.

If the positive electricity is on the inner coating, the jar is said to be *positively* charged; if on the outside, *negatively* charged.

648. The Spontaneous Discharge.—This occurs when the quantities accumulated are so great that their attraction will cause them to fly together with a flash and report over the edge of the jar. If the glass is soiled or damp, the fluids may pass over and mingle with only a hissing noise, in which case it is impossible for the jar to be highly charged.

If the glass is clean and dry, and especially if varnished with gum lac, a charge may not wholly disappear for days, or even weeks.

649. Series of Jars.—The same amount of electricity from the prime conductor which is required to charge one jar will charge an indefinite series, the strength of the charge being less and less from the first to the last. This case is analogous to the series of conductors (Art. 640). Insulate a series of jars, *A*, *B*, *C*, &c., and connect the inner coating of *A* with the prime conductor, and its outer coating with the inner coating of *B*, the outer of *B* with the inner of *C*, and so on. Then, as *A* is charged, the positive electricity of its outer coating, instead of passing to the earth, goes to the inside of *B*, and that on the outside of *B* to the inside of *C*, &c., while that on the outside of the last in the series passes to the earth. Thus each jar is charged positively by the inductive influence of the preceding, just as a series of magnets is formed with poles in the same order by a succession of magnetic inductions. No gain results from this arrangement however, for the total charge, or sum of the separate charges, is only equal to that which a single jar would have taken under the given conditions.

650. Division of a Charge in any Given Ratio.—If one

of two jars be charged, and the other not, and if the inner coatings be brought into communication, and also the outer coatings, the charge of the first jar is instantly diffused over the two, with a report like that of a discharge. In this way a charge may be halved, or divided in any other ratio, according to the relative capacities of the jars.

The capacity of a jar with *equal* coatings is proportional to the area of either coating, inversely proportional to the thickness of the dielectric, and proportional to the specific inductive capacity of the dielectric. Thus two jars of the same kind of glass, of equal thickness and having equal coatings, will be of equal capacity.

The self-repellency of each charge tends to diffuse it over a greater surface, and they will be thus diffused if allowed to remain within each other's attracting influence; but *one* of the charges will not spread over the coatings of another jar, unless opportunity is given for *both* to do so.

An experiment somewhat resembling the foregoing is this: charge two equal jars, one positively, the other negatively, and insulate them both. If the two knobs be connected by a conductor, the electricities, notwithstanding their strong attraction, will not unite; for each is held disguised by that on the other side of the glass. But if the outer coatings are first connected, then, on joining the knobs, the jars are both discharged at once.

651. Use of the Coatings.—If a jar is made with a wide open top, and the coatings movable, then, after charging the jar and removing the coatings, very little of the electricities adheres to the latter, but nearly the whole remains on the glass. The same mutual attraction which condensed them at first still holds them there after the coatings are removed. When they come to be replaced, the jar can be discharged as usual. But the coatings are necessary in charging, to diffuse the electricity over those parts of the glass which they cover, and also in discharging, to conduct off the whole charge at once.

652. The Free Portion of an Electrical Charge.—Either kind of electricity is said to be *free* when it remains on a body only because held by the pressure of the air; but if held by the attraction of the opposite kind, it is said to be disguised (Art. 639). Nearly all the electricity of a charged jar is disguised, but not the whole.

The moment after a jar is charged there is a small quantity of free electricity on the coating to which the fluid was furnished in charging, but not on the other. If the charged jar be upon an insulating stand, and the finger brought to this coating, a slight

spark is taken off; if it be touched again immediately, there is no spark, for the free electricity all escaped by the first contact. Let the finger now be brought to the other coating, and a spark flies from that. Immediately afterward a second spark can be taken from the first coating, and so on alternately for hundreds of times usually before the charge wholly disappears. What is removed at each contact is the free part of the charge, which always appears *alternately* on the two coatings. If a small electroscope be connected with each coating, the fluid alternately set free is indicated to the sight. The electroscope on the coating which is touched instantly falls, and the other rises.

653. Explanation of this Phenomenon.—The positive electricity which is conveyed to the inner coating, in charging a jar, attracts to the outer coating from the earth a quantity of the negative fluid which is a little *less* than itself. This is because of the thickness of the glass. If it were infinitely thin, the negative would be just equal to the positive, and they would neutralize each other, and both be perfectly disguised. But as the glass has some thickness, the positive exceeds the negative, and disguises it. Now if the jar, after being charged, is insulated, it is obvious that the negative charge on the outer coating cannot disguise all the positive (which is more than itself), but only a quantity a little less than itself. Hence there must be a little of the positive on the inner coating in a free state. By touching the knob, we allow this free portion to pass off, and there is left less of the positive in the inner coating than there is of the negative in the outer. Therefore, *all* the negative cannot now be disguised, but a slight quantity is liberated and ready to pass off as soon as touched. And thus, by alternate contacts, the process of discharge goes on, the series being longer as the glass is thinner, because then the two charges are more nearly equal, and less electricity is set free after the alternate contacts.

654. Electrical Vibrations.—If two jars be charged in opposite ways, and a figure made of pith be suspended between the knobs by a long thread, it will be attracted by that knob whose action on it happens to be greatest. As soon as it touches, it is charged with electricity and repelled, and of course attracted by the other knob, which is in the opposite state; thus it vibrates between them, causing a very slow discharge of both jars. In this case, the outside of each jar, which must not be insulated, discharges the free electricity, resulting from the contact of the figure with the knob of its inner coating, directly to earth, thus setting free upon its knob a new portion of the inner charge as explained in Art. 653.

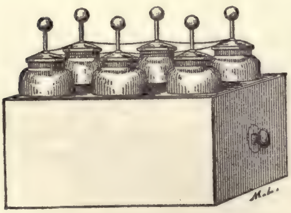
The electricity of the prime conductor will also cause vibrations, without the use of a jar. Suspend from it a metallic disk horizontally a few inches above another which is connected with the earth; then if a glass cylinder surround the two disks so as to prevent escape, a number of pith balls between the disks will continue to vibrate up and down so long as the machine is in action. Each ball lying on the lower disk, being electrified by induction, springs up to the upper disk, and then, being charged in the same way, is repelled.

In a similar manner a chime of bells may be rung.

655. Residuary Charge.—If a jar stand charged a few minutes, and after the discharge remain some minutes more, then a *second*, and possibly a *third*, discharge can be made; but these are usually very slight. The electricity remaining after the first discharge is called the *residuary charge*. The larger the jar, and the greater the density of the charge, the larger is this residuum. The outer and inner charges not only attract each other to the surfaces of the intervening dielectric, but also penetrate its substance sensibly. When the jar is discharged the portion of the charge thus absorbed can not pass off instantly because of the poor conductivity of the dielectric. After some moments a new distribution of the *residual* electricity has taken place, much of it having returned to the surface of the coatings, and a second discharge is possible. It is probable that there is also some diffusion of the charge above the edges of the coatings, the gradual return of which adds to the intensity of the residual charge.

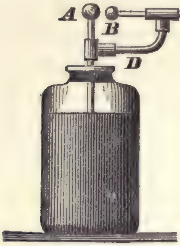
656. The Electric Battery.—Leyden jars are made of various sizes, from a half-pint to one or two gallons. But when a great amount of surface is needed, it is more convenient, and, in case of fracture by violent discharge, more economical, to connect several jars, so that they may be used as one. Four, nine, twelve, or even a greater number of jars, are set in a box (Fig. 361), whose interior is lined with tinfoil, so as to connect all the outer coatings together. Their inner coatings are also connected, by wires joining all the knobs, or by a chain passing round all the stems. Care is necessary in discharging batteries, that the circuit is not too short and too perfect, since the violence of discharge is liable to perforate the jars. A chain, three or four feet long in the circuit, will generally prevent the accident.

FIG. 361.



657. Discharging Electrometers.—These are instruments contrived for measuring the charge in the act of discharging the jar. Fig. 362 represents Lane's discharging electrometer. *D* is a rod of solid glass, which holds a sliding metallic rod with balls *B* and *C*.

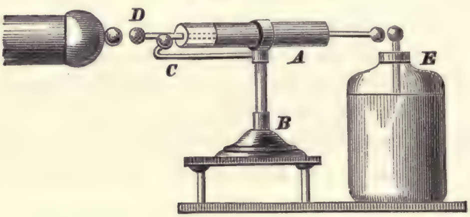
FIG. 362.



If the circuit through which the charge is to be sent is made to begin at the knob *C*, and end with the outer coating of the electrometer, and if the knob *A* be connected with the prime conductor of a machine, the inner coating of the electrometer will acquire a high potential which will cause a discharge across the intervening air between *A* and *B*. The distance between *A* and *B* may be made great or small at will, and hence a succession of charges of greater or less intensity may be caused to traverse the circuit.

The *unit jar* is used to measure the charge of another jar, by conveying to it successive equal charges of its own. *A B* (Fig. 363) is the instrument, consisting of a small open jar, placed

FIG. 363.

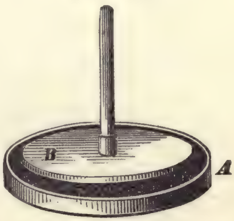


horizontally on an insulating stand, *B*. From the metallic part of the support, the bent rod and ball, *C*, come near to *D*, the rod of the inner coating, and can be turned so as to increase or diminish its distance. Let the knob, *D*, be near the prime conductor, and that of the outer coating near the top of the jar, *E*, which is to be charged, the outside of the latter being in communication with the earth. While *A* is charging, the positive electricity of its outer coating goes to the inner coating of the large jar, and partially charges it, the negative being held by induction upon the outer coating of *A*. When the difference of potential is sufficiently great, a discharge takes place between *D* and *C*, and the outer and inner coatings of *A* become neutral again. A second charge of positive is now repelled into the jar *E*, by the continued working of the electric machine, as before, and so on till the required number of unit charges have passed.

658. The Effect of a Point presented to an Electrified Body.—It has been noticed (Art. 631) that a pointed wire wastes the charge very quickly, because of the accumulation at the point. A prime conductor loses its charge just as quickly by presenting a pointed rod *toward* it. For the induced electricity of the rod and person holding it is in like manner accumulated at the point, and readily escapes to mingle with and neutralize its opposite in the prime conductor. Thus the charge disappears at once. In a similar way is to be explained the use of the points on the prime conductor presented to the glass plate. When the two electricities are separated at the surface of contact between the plate and rubbers, the plate is positively electrified. This positive charge acts inductively on the prime conductor, attracting the negative kind to the points, where it passes off and neutralizes what is on the plate, and leaves a positive charge on the prime conductor, thus charging it, not by giving to it *positive* electricity, but by robbing it of its *negative*, and thus leaving its positive upon it.

659. The Electrophorus.—This is a very simple *electrical machine* for giving the spark. It consists of a circular cake of resin in a metallic base, A (Fig. 364), and a metallic disk, B, having a glass handle. Excite the resin by fur or flannel; set the disk B upon it, and touch the back of the disk with the finger. Having withdrawn the finger again, lift the disk B by its insulating handle, and it will be found charged positively, and a brilliant spark may be drawn from it. Again place the disk upon the resin, touch it with the finger as before, and then remove it, and a second charge may be taken from it, and so on indefinitely.

Fig. 364.

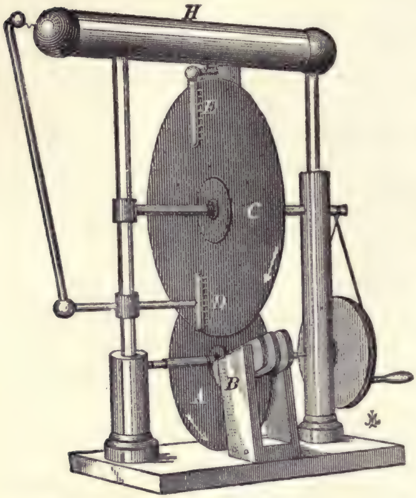


The negative electricity of the resin, the result of the friction by the fur, does not reside wholly upon the surface (Art. 655), but penetrates the dielectric. This, by induction, produces a positive charge in the metallic plate A, and the two charges are held in place by their mutual attractions. When B is set upon A a positive charge is drawn to its under surface by the inductive action of the negative charge, while the negative of B is repelled to earth through the finger. Upon removing the finger and lifting the plate B, the positive charge, no longer disguised, rises in potential (Art. 629), and may be drawn off in a spark.

660. The Dielectric Machine.—In this machine the lower

revolving vulcanite disk A (Fig. 365) is excited by the clamped rubbers at B, as it is made to revolve as indicated by the arrow. The upper plate C, also of vulcanite, is caused to revolve in the opposite direction, in close proximity to the combs at D and E. The supports of the prime conductor H are of glass or other insulating material.

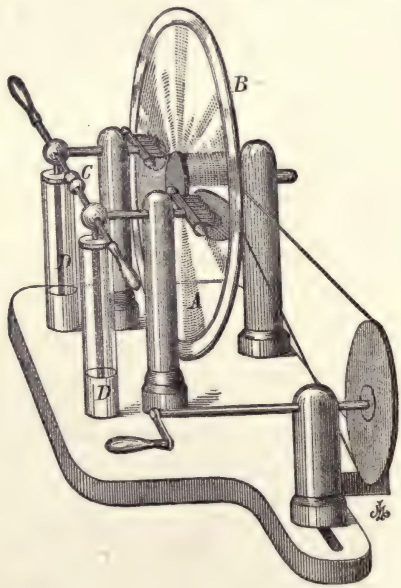
FIG. 365.



Suppose that negative electricity is developed upon the disk A by the friction of the rubbers at B; as this charge passes behind the revolving disk C it *induces* a positive charge in the *comb D*, which is discharged upon the front of C, the negative of D and its connections being repelled through the knob and rod to the knob near the prime conductor H. The plate C carries the positive charge it has received around to the *comb E*, inducing in it negative electricity, which is discharged upon the plate, restoring it to its neutral state, while the positive of E is repelled into the prime conductor H.

Sparks from seven to twelve inches long may be made to pass between the conductor H and the knob of the adjustable rod.

FIG. 366.

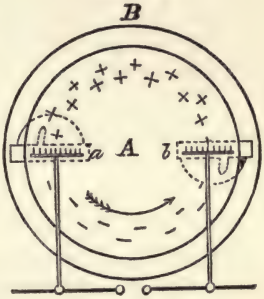


661. The Holtz Machine.—This machine, being named from its inventor and illustrated in Fig. 366, consists of a revolving glass disk A and a stationary glass disk B, both well coated with shellac to guard against moisture. In front of A and close

to it, as shown in the figure, are two combs, as in the dielectric machine, connected with the discharging knobs at *C*. The inner coatings of two Leyden jars *D* are connected with their respective knobs and combs, the outer coatings of the jars being connected together. On the back of the disk *B*, opposite to the combs, are two paper sectors, a paper tongue or point from each projecting, through an opening in the stationary disk, towards the revolving plate. If a plate of vulcanite be excited, and then be laid against one of the paper sectors, while the disk *A* is rapidly rotated, the discharging knobs being in contact, electrical decomposition will ensue, and after a few moments the knobs may be *gradually* separated until they give sparks twelve or twenty inches long, according to the size of the machine.

To explain, in a very general way, the action of the machine, let *A* (Fig. 367), represent the revolving plate and *B* the stationary plate behind it, carrying the paper sectors *a* and *b*. Imagine the combs to be in front of *A*, and call them *a* and *b* also, remembering that the sectors are on the plate *B*, behind *A*. If now a positive charge be communicated to the *sector a* it will act by induction, as in the dielectric machine, and will cause the *comb a* to discharge negative electricity upon the front of the plate *A*, the positive of the comb being repelled. This repelled positive charge passes, the knobs being in contact, to the *comb b*, and induces a negative charge in the *sector b*, the positive electricity of the *sector b* passing off by means of the pointed tongue attached to it. As the plate *A* is now made to revolve the constant discharge of the *comb a* electrifies its lower half negatively, while the discharge of the *comb b*, under the influence of the *sector b*, charges the upper half of the plate positively. The machine differs from the preceding in the application of some device, such as the pointed slips of paper attached to each paper sector, by which the repelled electricities of the sectors may escape, thus strengthening their inductive action until a great difference of potential is established between the two discharging knobs.

Fig. 367.



The Leyden jars increase the capacity of the discharging knobs.

CHAPTER IV.

EFFECTS OF ELECTRICAL DISCHARGES.

662. Variety of Effects.—Some of the effects of electrical discharges have been incidentally noticed in the foregoing chapters. The bright light, the sharp sound, and the great suddenness of the transmission, are remarkable phenomena in every discharge of a Leyden jar or battery. The various effects may be classified as *luminous, mechanical, chemical, magnetic,* and *physiological.*

663. Luminous Effects.—Light is seen only when charges of electricity, differing greatly in potential, are discharged through an obstructing medium. Hence, no light is perceived when it flows through a good conductor, unless of very small diameter. But if there is the least interruption, or if the conductor is reduced to a very slender form, then light appears at the interruption, and at those parts which are too small to convey the electricity. Thus, the discharge of a battery through a chain gives a brilliant scintillation at every point of contact between the links.

664. Modifications of the Light.—The length, color, and form of the electric spark vary with the nature and form of the conductors between which it passes, and with the quality of the medium interposed between them.

Electrical sparks are more brilliant in proportion as the substances *between* which they occur are better conductors. A spark received from the prime conductor upon a large metallic ball is short, straight, and white ; on a small ball it is longer, and crooked ; received on the knuckle, a less perfect conductor, the middle part is purplish ; on wood, ice, a wet plant, or water, it is red.

From a point positively electrified, the electricity passes in the form of a faint brush or pencil of rays ; a point connected with the negative side exhibits a luminous star.

When electricity passes through rarefied air, the light becomes faint, and is generally changed in color. The electrical spark, which in common air is interrupted, narrow, and white, becomes, as the rarefaction proceeds, continuous, diffused, and of a violet color, which tint it retains as long as it can be seen. If a battery is discharged through a tube several feet long, nearly exhausted of air, the whole space is filled with a rich purple light. The

sparks from the machine, conveyed through the same tube, exhibit flashings and tints exceedingly resembling the Aurora Borealis.

The *Geissler tubes* are tubes of complex forms, and containing a slight trace of some gas or vapor, which show various colors and intensities of electric light, according to the kind of gas, the diameter of the parts, and the quality of the glass. The electricity is conveyéd into the tubes by platinum wires sealed into their extremities.

Various colors are obtained by sending charges through dif ferent substances. An egg is bright crimson ; the pith of cornstalk, orange ; fluor-spar, green ; and loaf-sugar, white and phosphorescent.

If the spark be examined by means of the electroscope it will be found that the spectrum indicates that the substance of the conductors has been volatilized, and that the air or gas, through which the discharge has taken place, is also incandescent, which facts explain the differences of color noted above.

665. The Leichtenberg Figures.—When a spark of electricity is laid upon a non-conductor, it will, by its own self-repellency, extend itself a little distance along the surface. The *Leichtenberg figures* furnish a visible illustration of this fact, and also show that the two fluids diffuse themselves in very different forms. Lay down sparks of positive electricity from the knob of the Leyden jar upon a plate of resin, and near them some sparks of negative electricity. Then blow upon the plate the mingled powders of sulphur and red lead. The sulphur, by the agitation of passing through the air, will be electrified negatively, and attracted therefore by the positive sparks; the red-lead, positively electrified, will be attracted by the negative. Thus the spots on which the electricities are placed will appear in their exact forms by means of the colored powders attached to them. The positive resemble stars, or rather a group of crystals shooting out from a nucleus; the negative spots are circles with smooth edges; and the size of the electrified spots in each case depends on the quantity of electricity in the spark.

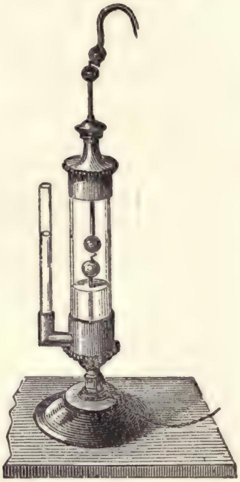
666. Luminous Figures.—Metallic conductors, if of sufficient size, transmit electricity without any luminous appearance, provided they are perfectly continuous ; but if they are separated in the slightest degree, a spark will occur at every separation. On this principle, various devices are formed, by pasting a narrow band of tinfoil on glass, in the required form, and cutting it across with a penknife, where we wish sparks to appear. If an interrupted conductor of this kind be pasted round a glass tube in a spiral direction, and one end of the tube be held in the hand,

and the other be presented to an electrified conductor, a coil of brilliant points surrounds the tube. Words, flowers, and other complicated forms, are also produced nearly in the same manner, by a suitable arrangement of interruptions in a narrow line of tin-foil, running back and forth on a plate of glass.

667. Mechanical Effects.—Powerful electric discharges through imperfect conductors produce certain mechanical effects, such as perforating, tearing, or breaking in pieces, which are all due to the sudden and violent repulsion between the electrified particles.

A discharge through the air is supposed to perforate it. If the air through which the spark is passed lies partially inclosed between two bodies which are easily moved, the force by which the air is rent will drive them asunder. Thus, a little block may be driven out from the foundation of a miniature building, and the whole be toppled down. But this enlargement of inclosed air is best seen in Kinnersley's *air thermometer* (Fig. 368). As the spark passes between the knobs in the large tube, the air confined in it is suddenly driven asunder, so as to press the water which occupies the lower part two or three inches up the tube, as represented. As soon as the discharge has occurred, the water quietly returns to its level. The sharp sound which is produced by the discharge of a Leyden jar is due to the sudden compression of the air, and also to the collapse which immediately succeeds.

FIG. 368.



The path of the electric spark through the air, when short, is straight; but if more than about four inches long, is usually *crinkled.* This is supposed to arise from the condensation of the air before it, by which it is continually turned aside.

When the charge is passed through a thick card, or the cover of a book, a hole is torn through it, which presents the rough appearance of a burr on each side. By means of the battery, a quire of strong paper may be perforated in the same manner; and such is the velocity with which the fluid moves, that if the paper be freely suspended, not the least motion is communicated to it. Pieces of hard wood, of loaf-sugar, and brittle mineral substances, are split in two, or shivered to pieces, by an intense charge of a battery. But good conductors of much breadth are not thus

affected. The charge, as it is transmitted, passes over the whole body, instead of being concentrated in any one line. But if liquids which are good conductors are closely confined on every side, they show that a violent expansion is produced by a discharge. Thus, when a charge is sent through water confined in a small glass tube or ball, the glass is shattered to pieces; and mercury in a thick capillary tube is expanded with a force sufficient to splinter the glass.

668. Different Routes of Discharge.—If two or more circuits are opened at once between the two coatings of a charged jar or battery, the discharge will take one or another, or divide between them, according to circumstances. If the circuits are alike except in length, the discharge will follow the *shorter*. If they differ only in conducting quality, the electricities will take the *best conductor*. If the circuits are interrupted, and in all respects alike, except that the conductors of one are pointed at the interruptions, and of the others not pointed, the discharge will follow the line which has *pointed conductors*. If the circuits are *very attenuated* (as very fine wire, or threads of gold-leaf), the charge is liable to divide among them.

669. Chemical Effects.—These are various: combustion of inflammable bodies; oxydation, fusion, and combustion of metals; separation of compounds into their elements; reunion of elements into compounds.

Ether and alcohol may be inflamed by passing the electric spark through them; phosphorus, resin, and other solid combustible bodies, may be set on fire by the same means; gunpowder and the fulminating powders may be exploded, and a candle may be lighted. Gold-leaf and fine iron wire may be burned, by a charge from the battery. Wires of lead, tin, zinc, copper, platinum, silver, and gold, when subjected to the charge of a very large battery, are burned, and converted into oxides.

The same agent is also capable of restoring these oxides to their simple forms. Water is decomposed into its gaseous elements, and these elements may again be reunited to form water. By passing a great number of electric charges through a confined portion of air, the oxygen and nitrogen are converted into nitric acid. The ozone which is almost always perceived in connection with electrical experiments is to be considered as one of the chemical effects of electricity.

Galvanic electricity is a form of this agent much better adapted than frictional electricity to produce chemical as well as magnetic effects.

670. Magnetic Effects.—It is difficult to cause deflections

of a magnetic needle on account of the high potential of frictional electricity, which renders it necessary to take special pains to render the insulation of the helix of the galvanometer very perfect. A galvanometer of 400 or 500 turns of fine wire very perfectly insulated by layers of insulating substance of appreciable thickness, will give deflections of the needle when connected with the two discharging knobs of a Holtz machine.

A steel needle may be magnetized by passing the electricity through a properly insulated helix.

The galvanometer and its use will be explained in Part IX.

671. Physiological Effects.—The shock experienced by the animal system, when the charge of a jar passes through it, has been already mentioned.

A slight charge of the Leyden jar, passed through the body from one hand to the other, affects only the fingers or the wrists; a stronger charge convulses the large muscles of the arms; a still greater charge is felt in the breast, and becomes somewhat painful. The charge of a large battery is sufficient to destroy life, if it be sent through the vital organs. By connecting the chains which are attached to the jar with insulating handles, it is easy to pass shocks through any particular joint, muscle, or other part of the body, as is frequently done for medical purposes.

The charge may be passed through a great number of persons at the same time, in which case those at the centre of the chain will receive a less severe shock than those near the ends. Hundreds of individuals, by joining hands, have received the shock at once, though there is more difficulty in passing a given charge as the number is increased.

If the spark is taken by a person from the prime conductor, the quantity is not sufficient, unless the conductor is of extraordinary size, to produce what is called the shock; a pricking sensation in the flesh where the spark strikes, and a slight spasm of the muscle, is all that is noticeable. A person may make his own body a part of the prime conductor by standing on an *insulating stool*—that is, a stool having glass legs, and touching the conductor of the machine. This occasions no sensation at all, except what arises from the movement of the hair, in yielding to the repellency of the fluid. If another person takes the spark from him, the prick is more pungent, as the quantity is larger than in the prime conductor alone.

672. Velocity of Electricity.—This is so great that no appreciable time is occupied in any case of discharge. It has been determined that the duration of the lightning flash is less than $\frac{1}{1000}$ of a second. Wheatstone found the duration of the spark

of a Leyden jar to be $\frac{1}{24000}$ of a second. The duration of the spark is greater as the resistance of the medium is greater, as the *striking distance* is greater, and as the charge is increased.

Wheatstone devised an ingenious method of measuring the time in which electricity passes over a wire only half a mile long. The wire was so arranged that three interruptions, one near each end, and one in the middle of the wire, were brought side by side. When the discharge of a jar was transmitted, the sparks at these interruptions were seen by reflection in a swiftly revolving mirror. An exceedingly small difference of time between the passage of those interruptions could be easily perceived by the *displacement* of the sparks as seen in the whirling mirror. The amount of observed displacement and the known rate of revolution of the mirror, would furnish the interval of time occupied by the electricity in passing from one interruption to the next. By a series of experiments, Wheatstone arrived at the conclusion *that, on copper wire, one-fifteenth of an inch in diameter, electricity moves at the rate of 288,000 miles per second,* a velocity much greater than that of light.

Galvanic electricity moves very much slower. Its rate on iron wire, of the size usually employed for telegraph lines, is about 16,000 miles per second.

No definite velocity can be assigned to electricity, except under specified conditions. As we cannot say what will be the velocity of sound, or of light, without specifying the medium traversed, so in the case of electrical currents we must consider the material of the conductor, its cross-section, its length, and the surrounding inductive influences.

CHAPTER V.

ATMOSPHERIC ELECTRICITY—THUNDER-STORMS.

673. Electricity in the Air.—The atmosphere is always more or less electrified, sometimes positively, sometimes negatively. This fact is ascertained by several different forms of apparatus. For the lower strata, it is sufficient to elevate a *metallic rod* a few feet in length, pointed at the top, and insulated at the bottom. With the lower extremity is connected an electroscope, which indicates the presence and intensity of the electricity. For experiments on the electricity of higher portions, a kite is employed,

with the string of which is intertwined a fine metallic wire. The lower end of the string is insulated by fastening it to a support of glass, or by a cord of silk. If a cloud is near the kite, the quantity of electricity conveyed by the string may be greatly increased, and even become dangerous. Cavallo received a large number of severe shocks in handling the kite-string; and Richmann, of St. Petersburgh, was killed by a discharge of electricity which came down the rod which he had arranged for his experiments, but which was not provided with a conductor near by it, for taking off extra charges.

The electricity of the atmosphere is most developed when hot dry weather succeeds a series of rainy days, or the reverse; and during a single day, the air is most electrical when dew is beginning to form before sunset, or when it begins to exhale after sunrise. In clear, steady weather, the electricity is generally positive; but in falling or stormy weather, it is frequently changing from positive to negative, and from negative to positive.

674. Thunder-Storms.—Thunder-clouds are, of all atmospheric bodies, the most highly charged with electricity; but all single, detached, or insulated clouds are electrified in greater or less degrees, sometimes positively and sometimes negatively. When, however, the sky is completely overcast with a uniform stratum of clouds, the electricity is much feebler than in the single detached masses before mentioned. And, since fogs are only clouds near the surface of the earth, they are subject to the same conditions: a driving fog, of limited extent, is often highly electrified.

Thunder-storms occur chiefly in the hottest season of the year, and after midday, and are more frequent and violent in warm than in cold countries. They never occur beyond 75° of latitude—seldom beyond 65°. In the New England States they usually come from the west, or some westerly quarter.

The storm itself, including everything except the electrical appearances, is supposed to be produced in the same manner as other storms of wind and rain; and the electricity is developed by the rapid condensation of watery vapor, and by friction. Electricity is not to be regarded as the *cause*, but as a *consequence* or *concomitant* of the storm. But the precipitation of vapor must be sudden and copious, since when the process is slow, too much of the electricity evolved would escape to allow of the requisite accumulation. Also, if a storm-cloud is of great extent, it is not likely to be highly electrified, because the opposite electricities, which may be developed in different parts of it, have opportunity to mingle and neutralize; and points of communication with the earth will here and there occur. Clouds of rapid formation,

violent motion, and limited extent, are therefore most likely to be thunder-clouds.

675. Lightning.—When a cloud is highly charged, it operates inductively on other bodies near it, such as other clouds, or the earth. Hence, discharges will occur between them. Lightning passes frequently between two clouds, or even between two parts of the same cloud, in which opposite electricities are so rapidly developed that they cannot mingle by conduction. But, in general, the discharges of lightning take place between the electrified cloud and the earth, whose nearer part is thrown into the opposite electrical state by induction. It is supposed that, in some instances, a discharge occurs between two distant clouds by means of the earth, which constitutes an interrupted circuit between them. The crinkled form of the path of lightning is explained in the same way as that of the spark from the machine, and the thunder is caused by the simultaneous rupture and collapse of air in all parts of the line of discharge. The words *chain-lightning, sheet-lightning,* and *heat-lightning,* are supposed not to indicate any real differences in the lightning itself, but only in the circumstances of the person who observes it. If the crinkled line of discharge is seen, it is *chain* or *fork* lightning; if only the light which proceeds from it is noticed, it is *sheet*-lightning; if, in the evening, the thunder-storm is so far distant that the cloud cannot be seen, nor the thunder heard, but only the light of its discharges can be discerned in the horizon, it is frequently called *heat*-lightning.

676. Identity of Lightning and Electrical Discharges.—Franklin was the first to point out the resemblances between the phenomena of lightning and those of frictional electricity. He was also the first to propose the performance of electrical experiments by means of electricity drawn from the clouds. The points of resemblance named by Franklin were these: 1. The crinkled form of the path. 2. Both take the most prominent points. 3. Both follow the same materials as conductors. 4. Both inflame combustible substances. 5. They melt metals in attenuated forms. 6. They fracture brittle bodies. 7. Both have produced blindness. 8. Both destroy animal life. 9. Both affect the magnetic needle in the same manner. In 1752, he obtained electricity from a thunder-cloud by a kite, and charged jars with it, and performed the usual electrical experiments.

677. Lightning-Rods.—Franklin had no sooner satisfied himself of the identity of electricity and lightning than, with his usual sagacity, he conceived the idea of applying the knowledge

acquired of the properties of the electric fluid so as to provide against the dangers of thunder-storms. The conducting power of metals, and the influence of pointed bodies to transmit the fluid, naturally suggested the structure of the lightning-rod. The experiment was tried, and has proved completely successful; and probably no single application of scientific knowledge ever secured more celebrity to its author.

678. Rules for the Protection of Buildings.—The following rules are derived from the "Report of the Lightning Rod Conference," consisting of delegates from the following British societies: Meteorological Society, the Royal Institute of British Architects, the Society of Telegraph Engineers and Electricians, and the Physical Society:

Points.—The point of the upper terminal should be a cone whose height is equal to the radius of its base. A foot below this cone a copper ring should be *screwed* and *soldered* to the rod, in which ring should be soldered three or four sharp copper points, about six inches long, gilded or nickel-plated.

Number of Upper Terminals.—There can be no rule as to the proper number of terminals. The space protected is assumed to be a conical space, the radius of whose base is equal to the height of the terminal. All portions of a building within such cone will be reasonably protected. Chimneys should always be protected by terminals on account of the conducting power of the ascending heated gases and vapors.

Attachment.—The rod should never be insulated from the building. It should be supported by metal straps, or fastenings, of the same material as the rod, of such form as not to compress or distort the rod, and in such manner as to allow for expansion and contraction of the rod from changes of temperature.

Ornamental Metal Work.—All vanes, finials, ridge-ironwork, metal roofing, and other metal masses upon a building should be connected with the conductor.

Material for Rod.—Copper is recommended as eventually cheaper than iron, as being lighter, and as being more durable. It is more likely to be stolen, in places where buildings are not occupied during the whole year.

Cross-section of Conductor.—The minimum dimensions authorized by the Report are:

Material.	Form.	Section. Sq. in.	Weight per ft.
Copper	Rope,	.10	6 oz.
"	Rod,	.11	7 oz.
"	Tape ($\frac{3}{4} \times \frac{1}{8}$),	.09	6 oz.
Iron	Rod,	.64	35 oz.

The advantages of *rods* are their durability, and rigidity when used for terminals. The disadvantages are the numerous joints required, and the difficulty of bending to conform to the outlines of the building.

Tapes are good since they are flexible, and the joints may be made very perfect by riveting and then soldering. No sharp bends should be allowed. The copper used, in any form, must have a conductivity of not less then 90 per cent. that of *pure* copper.

Ropes should be made of wires of not less than No. 12 B. W. G., about $\frac{11}{100}$ of an inch diameter. Iron should only be used in the form of rod, either square or round.

Joints.—Every joint must have bright metallic surfaces in contact, and after being screwed, or riveted, should be entirely covered with solder.

Curves.—The rod should not be bent abruptly. In no case should the length, measured along a curve, be more than half as long again as the chord subtending such curve.

Metal Pipes.—The conductor should be connected with all pipes and other large masses of metal within the building, *except gas pipes.* An electrically defective joint in a gas pipe might result in an explosion.

Earth Connection.—The lower end of the conductor must be placed in permanent moisture. The earth connection should not be poorer than that afforded by a copper plate 3 feet square and $\frac{1}{16}$ inch thick, buried in permanently wet earth, and surrounded by coke or charcoal. If iron is used for the rod, a galvanized iron plate of similar dimensions should be used in like manner.

Examination.—The protective system of rods should be regularly inspected and tested for conductivity, as breaks may occur, either above ground or below, which would not only render the rod useless as a protection, but positively a source of additional danger.

Painting.—The rods should be protected by paint, except where the terminals require a bright metallic surface to be exposed.

679. In what way Lightning-Rods Afford Protection. —Lightning-rods are of service, not so much in receiving a discharge when it comes, as in diminishing the number of discharges in their vicinity. The sharp points upon the copper ring continually carry on a silent communication between the two electricities, which are attracting each other, one in the cloud, the other in the earth; so that a village well furnished with rods has few discharges of lightning in it. All tall pointed objects, like spires

of churches and masts of ships, exert a similar influence, though in a less degree, because not so good conductors.

During a thunder-storm, or immediately after it, if a person can be near the top of a high rod, he will sometimes hear the hissing sound of electricity escaping from it, as from a point attached to the prime conductor of a machine. In the same circumstances, if it were quite dark, he would probably see the brushes or stars of light on the points. The statement of Cæsar in his Commentaries, "that the points of the soldiers' darts shone with light in the night of a severe storm," probably refers to the visible escape of electricity from the weapons as from lightning-rods.

680. Protection of the Person.—Silk dresses are sometimes worn with the view of protection, by means of the insulation they afford. They cannot, however, be deemed effectual unless they completely envelop the person; for if the head and the extremities of the limbs are exposed, they will furnish so many avenues as to render the insulation of the other parts of the system of little avail. The same remark applies to the supposed security that is obtained by sleeping on a feather bed. Were the person situated *within* the bed, so as to be entirely enveloped by the feathers, they would afford some protection; but if the person be extended on the surface of the bed, in the usual posture, with the head and feet nearly in contact with the bedstead, he would rather lose than gain by the non-conducting properties of the bed, since, being a better conductor than the bed, the charge would pass through him in preference to that. If the bedstead were of iron, its conducting quality would probably be a better protection than the insulating property of the feathers, since, by taking the charge itself, it would keep it away from the person. So, a man's garments soaked with rain have been known to save his life, being a better conductor than his body. Animals under trees are peculiarly exposed, because the trees by their prominence are liable to be the channels of communication for the electric discharge, and the animal body, so far as it reaches, is a better conductor than the tree. Tall trees, however, situated near a dwelling-house, furnish a partial protection to the building, being both better conductors than the materials of the house, and having the advantage of greater elevation.

681. How Lightning Causes Damage.—The word *strike*, which is used with reference to lightning, conveys no correct idea of the nature of the movement of electricity, or of the injury which it causes. One kind of electricity, developed in a cloud, causes the other to be accumulated by induction in the part of the earth nearest to it. These electricities strongly attract each other;

consequently, that in the earth presses upward into all prominent conducting bodies toward the other; and, if those bodies are numerous, high, the best of conductors, and terminated by points, the electricity will flow off from them abundantly, and mingle with its opposite in the air above ; and thus discharges are in a great degree prevented. But if these channels for silent communication are not furnished, the quantity of electricity will increase, till the strength of attraction becomes so great that the fluid will break its way through the air, usually from some prominent object, as a building or tree, and thus the union of the two electricities takes place. The building or tree in this case is said to be *struck by lightning ;* it is rent, or otherwise injured, by the great quantity of electricity which passes violently through it, in an inconceivably short space of time. The effects produced are exactly like those caused by discharges of the electrical battery, on a greatly enlarged scale. The charge of a large battery, taken through the body in the usual way, would prostrate a person by the violence of the shock ; but the same charge, if allowed to occupy a few seconds in passing by means of a point, would not be felt at all.

Fulgurites are tubes of silicious matter formed in the ground, where lightning has struck in sandy soil, and melted the sand around its path towards the conducting moist earth below.

PART IX.

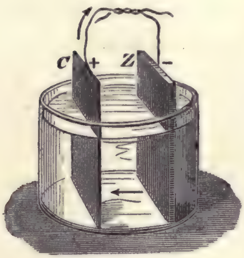
DYNAMICAL ELECTRICITY.

CHAPTER I.

THE GALVANIC CURRENT, AND APPARATUS FOR PRODUCING IT.

682. Electricity Developed by Chemical Action.—In a glass vessel (Fig. 369) containing a mixture of one part of sulphuric acid and seven or eight parts of water, put two plates, one of copper, *C*, and the other of zinc, *Z*, to each of which is soldered a copper wire. On bringing the extreme ends of the wires together, a feeble flow of electricity will take place through the wires, the plates, and the liquid, as may be shown by the peculiar taste or sensation which is perceived on placing one terminal wire upon and the other beneath the tongue, or better still, by deflections of the magnetic needle arranged as described hereafter. Electricity thus produced is called *galvanic* or *voltaic*, from Galvani and Volta, two Italian philosophers, who made the first discoveries of importance in this branch of science. It is also called *dynamical* electricity because of its constant flow through the conductor.

FIG. 369.



In using the terms *flow* and *current* the student must keep in mind that there is no transfer of any substance, no flow nor current in the sense in which we use the terms when applied to water; such words are convenient to express the transmission of the molecular disturbances, vibrations, or tensions, due to electric action, just as we may speak of the flow of heat through a diathermal substance.

683. Definitions.—An *element* or *cell* is a jar containing any arrangement of substances for the purpose of producing a current

of electricity. A *battery* is a number of elements properly connected with each other.

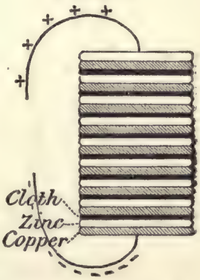
The *poles* or *electrodes* of a cell or battery are the extremities of the wires where the electricities appear.

The *circuit* is the path or conductor provided for the flow of the current—that is, the liquid, the plates, and the wires. The circuit is said to be *closed* when the wires are joined, so that there is a flow of the current; when they are separated, the current ceases, and the circuit is said to be *broken*, or to be *open*.

When the circuit is broken it will be found that the outer end of the wire attached to the copper plate will be of higher potential than the outer end of the wire attached to the zinc. The current is said to flow from the copper plate, through the wire conductor to the zinc plate, and from the zinc plate through the the liquid to the copper plate.

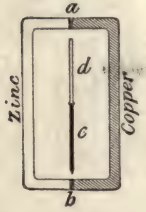
684. Volta's Pile.—In this form of current generator, the origin of the ordinary battery of cells, a large number of alternate disks of copper, zinc, and cloth moistened with very dilute acid, are used, as shown in Fig. 370. If this pile be set upon an insulating plate it will be found, upon testing with the proof-plane and electroscope, that the opposite ends are charged with opposite electricities, the potential increasing from the middle, or neutral point, towards each end. If the terminal disk of copper be connected with the terminal zinc by a conducting wire, a current will flow as in the battery of cells, the moistened cloth disks corresponding in this case to the liquid in the ordinary cell.

FIG. 370.



685. Electricity Due to Contact.—If a strip of copper be soldered to a strip of zinc the copper will become negatively and the zinc positively charged, as may be shown by touching the lower plate of a condensing electroscope with the copper end of the couple, the zinc being held in the hand. If a rectangle be formed of copper and zinc, or other dissimilar metals, as in Fig. 371, and a needle, one half *c* made of gilt paper and the other half *d* of lac, be properly suspended within, then upon charging *c* with positive electricity, it will turn towards the copper side of the junction *b*, while a negative charge will turn it towards the zinc side of *b*. From these, and similar experiments, it is concluded that electric decomposition is the result of contact of dissimilar substances.

FIG. 371.



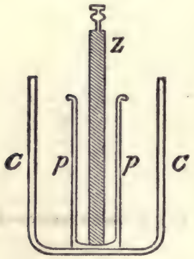
686. The Cell of Two Fluids.—An element of copper, zinc, and dilute acid, already described, soon loses its efficiency, because of the deposit of hydrogen upon the copper plate, the result of the chemical decomposition due to electric action, which deposit not only acts as a film of low conductivity, but also tends to cause a reverse current through the circuit.

In improved batteries, by which a constant flow of electricity may be maintained for a considerable length of time, two liquids are employed, and generally some other substance than copper for one of the metals. The liquids must be separated by some porous substance, which shall prevent them from mingling, and at the same time, being saturated by the liquid, shall not interrupt the necessary moist communication between the metals.

687. Constant Batteries.—Batteries composed of cells containing two liquids are called *constant*, because their action continues for so long a time without sensible abatement.

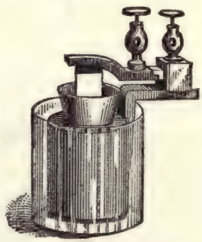
Daniell's Cell.—One of the most constant of all the forms of cell yet devised is that of Daniell. This consists of a copper cup *c* (Fig. 372), within which is a smaller cylinder of porous earthenware *p* containing a rod of zinc *z*. The outer copper vessel is filled with a saturated solution of cupric sulphate, and the porous cup is charged with dilute sulphuric acid. As the strength of the cupric sulphate decreases gradually, when the current flows, crystals must be added from time to time.

FIG. 372.



Grove's Battery.—One element is shown in Fig. 373, which represents a glass jar containing a hollow cylinder of zinc, which has a narrow opening on one side from top to bottom, that the liquid in which it is placed may circulate freely within it. Within the zinc is a cylindrical cup of porous earthenware, and within that is suspended a lamina of platinum. One of the circuit wires is in metallic communication with the zinc, and the other with the platinum, by means of the binding screws at the top. The earthen cup is now filled with strong nitric acid, while the space outside of it, in which the zinc is placed, contains dilute sulphuric acid.

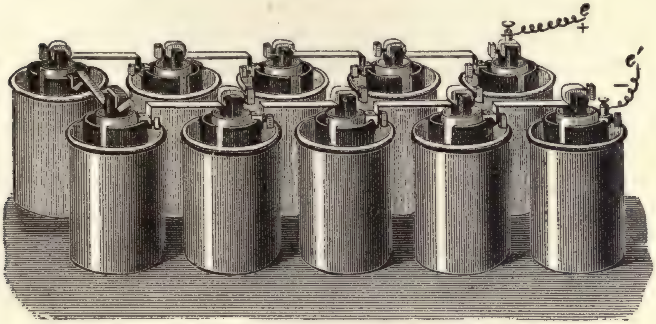
FIG. 373.



Bunsen's Battery is the same as Grove's, except that in it a cylinder of carbon is used instead of a leaf of platinum, on account

of the expense of the latter. It is very generally employed in telegraphy. Fig. 374 is a Bunsen battery of ten cells.

FIG. 374.



Gravity Batteries.—As the porous cups are gradually cracked and impaired by crystallization within their substance, or by a deposit of copper in cupric sulphate cells, the difference in density of different solutions has been relied upon to maintain separation of the liquids. In one form of *gravity cell* the copper plate is a horizontal disk at the bottom of a jar ; upon this is a saturated solution of cupric sulphate, above which is a solution of zinc sulphate, and immersed in this, at the top of the jar, is a horizontal disk or grating of zinc. Such batteries gradually become impaired by diffusion of the two liquids, unless special devices be employed to prevent such result.

It would extend the subject beyond our limits to attempt a description of the numberless forms of cells in use, or which have been proposed; it is also unnecessary here to enter upon a discussion of the chemical decompositions and exchanges which result when a current flows.

688. Amalgamation of Zinc.—Pure zinc is not dissolved by dilute sulphuric acid, while commercial zinc is rapidly acted upon.

When a commercial zinc plate is used in a cell the impurities present, either lead or iron, give rise to local currents from one part of the plate to another, and not only cause a rapid solution of the plate, but by such currents impair the efficiency of the cell.

If the zinc plate be well amalgamated, by first cleaning with dilute sulphuric or hydrochloric acid, and then applying a little mercury to the bright surfaces, a film of zinc dissolved in mercury spreads over the whole plate, acting as a surface of pure zinc when in contact with the liquid of the cell.

689. Electromotive Force.—The force which causes a current to traverse a conducting medium, such as the conducting wire of the battery circuit or the liquids within the cells, is termed the *electromotive force;* it is this force which causes the differences of potential upon contact of heterogeneous substances.

Electromotive force is independent of the amount of surface of contact. A Daniell cell with plates of one square inch each has the same electromotive force as a similar cell having plates 1000 square inches each ; for the electromotive force is measured by the differences of potential, and the difference of potential being the same at every elementary unit of surface of contact of like substances, the electromotive force must remain constant, whatever change may be made in the areas of these surfaces.

The practical unit of electromotive force is the *volt,* and is very nearly equal to that of a Daniell cell, which is usually referred to as the standard.

The following table gives the electromotive force of the different cells mentioned, amalgamated zinc being used in each :

Daniell.	Sul. Acid, $\frac{1}{12}$,	Saturated Cupric Sulphate.	Copper.	1.079 volts.
"	" " "	Saturated Cupric Nitrate.	"	1.000 "
Grove..	Sul. Acid, $\frac{2}{15}$,	Nitric Acid (Fuming).	Platinum.	1.956 "
Bunsen.	" " "	" " "	Carbon.	1.734 "

690. Resistance.—The consideration of electromotive force directs attention at once to the resistances which that force overcomes in urging the current through a medium. The laws governing the resistance of a conductor are :

1. *The resistance of a conductor depends upon the material of which it is made.*

Among metals silver and copper offer the least resistance, while the resistance of liquids is very great, being many million times that of pure silver.

2. *The resistance varies inversely as the cross-section of the conductor.* A large wire conducts better than a small one, and the larger the wire the less the resistance.

3. *The resistance varies directly as the length.* A wire one mile long offers twice the resistance that would be offered by a similar wire half a mile long.

The unit of resistance is called an *Ohm,* and is very nearly that of a pure copper wire, $\frac{1}{20}$ inch in diameter and 250 feet long, or that of 330 feet of ordinary iron wire, $\frac{155}{1000}$ of an inch in diameter.

The *Siemen's Unit* is the resistance of a column of pure mercury 1 metre long and of 1 square millimetre section, at 0° Centigrade, and is .9705 British Association Units, or ohms. The

resistance of ordinary telegraph wire No. 8 is about 13 ohms per mile.

At a mean temperature of 20° C., an increase of 1° C. in temperature produces an increased resistance in

Pure solid metals of about		$\frac{4}{10}$	of 1 per cent.	
German silver	"	$\frac{4}{100}$	"	"
Mercury	"	$\frac{8}{100}$	"	"

The *specific resistance* of any substance is determined for a cube of that substance measuring one centimetre upon the edge. Since resistance varies as length divided by cross-section, we can determine the resistance of any conductor by the formula,

$$R = \frac{L}{A} \times r,$$

in which $R =$ the whole resistance, $L =$ length in centimetres, $A =$ area of cross-section in sq. cm., and $r =$ the specific resistance as given in the following table :

	Specific Resistance in Ohms.		Specific Resistance in Ohms.
Silver	.000001609	Pure Water at 22° C.	71.8
Copper	.000001642	Dilute Sul. Acid, $\frac{1}{12}$	3.32
Iron	.000009827	Dilute Sul. Acid, $\frac{1}{3}$	1.26
Lead	.000019850	Glass at 200° C.	22700000
German Silver	.000021170	Gutta Percha at 20° C.	35×10^{13}
Mercury	.000096190		

The prefixes *mega* and *micro* are used to denote a million, or one millionth ; thus, one million ohms = one megohm, and one millionth of a volt = one microvolt. These prefixes are applied to the various electric units.

691. Strength of Current.—Knowing the electromotive force and the resistances, we next seek to determine the strength of the current, or the quantity which flows through the conductor in a given time.

The practical unit of strength of current is called one "*Weber per second*," or more commonly one "*Weber*," and is that which flows, in one second, through a resistance of one *ohm* under an electromotive force of one *volt*. It has been proposed that this unit of current, in future, shall be called an *Ampère*.

692. Ohm's Law.—It is evident that as the electromotive force is made greater, the resistance being supposed unchanged, the quantity of electricity which flows per second will be greater also ; and as the resistance is made greater, the electromotive force being unchanged, the current will be less per second. The relations of strength of current, resistance and electromotive force first stated by Ohm, and therefore called *Ohm's Law*, are expressed

thus: *The strength of the current varies directly as the electromotive force and inversely as the resistance.*

Expressing this as a working formula we have

$$C = \frac{E}{R},$$

in which $C =$ the number of *webers per second* of current, $E =$ the number of *volts* of electromotive force, and $R =$ the number of *ohms* of resistance in the entire circuit, within the cells as well as in the external conductor.

693. Manner of Connecting the Elements of a Battery.—The electromotive force of cells connected "in series," that is, having the zinc of the first connected with the copper of the second, the zinc of the second with the copper of the third, and so on to the last whose zinc is joined by the external circuit with the copper of the first, is equal to the sum of the electromotive forces of the cells taken separately; and, since the current traverses the cells in succession, the *internal resistance* is equal to the sum of the internal resistances.

Let us call the electromotive force of each cell E, the external resistance of the conducting wire R, and the internal resistance of each cell, due to the liquid which the current must traverse, r; then in a battery of n cells we have strength of current,

$$C = \frac{n\,E}{n\,r + R}, \text{ according to Ohm's Law.}$$

Suppose we have a number of Daniell cells of electromotive force 1 volt each, and of internal resistance of 3 ohms (Art. 690), and that we wish to send a current of $\frac{1}{16}$ of 1 *weber per second* through a circuit wire 10 miles long and offering a resistance of 13 ohms per mile (Art. 690).

If we use one cell we have,

$$C = \frac{1 \text{ volt}}{3 \text{ ohms} + 130 \text{ ohms}} = \frac{1}{133} \text{ weber per second.}$$

If we join two cells in series we have,

$$C = \frac{2 \text{ volts}}{6 \text{ ohms} + 130 \text{ ohms}} = \frac{1}{68} \text{ of a weber per second.}$$

If we use ten cells we find,

$$C = \frac{10 \text{ volts}}{30 \text{ ohms} + 130 \text{ ohms}} = \frac{1}{16} \text{ weber per second.}$$

Had we joined our cells in "multiple arc," that is, joined all the zincs together to form one electrode and all the coppers together for the other electrode, the effect would have been the same as that produced by a single cell having plates 10 times the

area of the plates of one cell of the battery; there would be no increase of electromotive force, but the internal resistance would be only $\frac{1}{10}$ that of a single cell, or $\frac{3}{10}$ of an ohm; for the current now divides and traverses the liquid in all the cells at the same time, instead of successively, and the total cross-section of liquid is the sum of the cross-sections of the single cells, and as the resistance is inversely as the cross-section, we have the above result.

Suppose we desire a strong current through an external circuit of six ohms resistance, and have eight cells like those above; if we connect them "in series" we have

$$C = \frac{8}{3 \times 8 + 6} = \tfrac{8}{30} \text{ weber per second.}$$

If we join them in "multiple arc" we get

$$C = \frac{1}{\frac{3}{8} + 6} = \tfrac{8}{51} \text{ weber per second.}$$

If we now join them in "multiple arc" in groups of two each, and then join the four groups in "series" we have

$$C = \frac{4}{\frac{3}{2} \times 4 + 6} = \tfrac{1}{3} \text{ weber per second,}$$

the best combination of the three which we have tried.

The maximum current is obtained when the internal resistance equals the external resistance.

Let $N =$ number of cells, $r =$ internal resistance of each cell, $R =$ external resistance of the circuit, $g =$ number of cells in a group joined in "multiple arc," and $S =$ number of groups joined in "series;" then

$$C = \frac{S\,E}{\frac{r}{g}S + R}; \text{ but } N = g \times S, \text{ whence } g = \frac{N}{S},$$

therefore $C = \dfrac{S\,E}{\frac{S^2}{N}r + R}$. If now the total internal resistance, $\dfrac{S^2}{N}r$, equals the external resistance R, we have

$$C = \frac{S\,E}{2\,R}, \text{ a maximum value.}$$

For, if the value of C be not greatest when the internal resistance equals the external, let us increase or decrease the former, which we may do by increasing or decreasing the number of groups.

For S substitute $S \pm a$; then $g = \dfrac{N}{S \pm a}$, which values sub-

stituted in the formula for C give $C' = \dfrac{(S \pm a)\,E}{\dfrac{(S \pm a)^2\,r}{N} + R}$. But $\dfrac{S^2\,r}{N} = R$, by our hypothesis, and hence $r = \dfrac{N\,R}{S^2}$, which values give $C' = \dfrac{(S \pm a)\,E}{2\,R \pm \dfrac{2\,a\,R}{S} + \dfrac{a^2\,R}{S^2}}$; comparing this value with the former we have

$$C - C' = \frac{S\,E}{2\,R} - \frac{(S \pm a)\,E}{2\,R \pm \dfrac{2\,a\,R}{S} + \dfrac{a^2\,R}{S^2}},$$

which, reducing to common denominator and cancelling, gives $C - C' = \dfrac{a^2\,E\,S}{R\,[\,(2\,S \pm a)^2 + a^2]}$, a remainder which is positive; hence C must be greater than C', and therefore that combination which makes the *internal* resistance equal to the *external* gives the greatest current.

694. Galvanic and Frictional Electricity Compared.— The electricities furnished by chemical action and by friction are undoubtedly the same in kind. But they differ in that the former is produced in greater *quantity*, while the latter is in a state of greater *intensity*. This will be understood by referring to heat. The *quantity* of heat in a warm room is vastly greater than that in the flame of a lamp; yet the former is agreeable, while the latter, if touched, causes severe pain by its greater intensity. In a similar manner, a quantity of galvanic electricity may pass through the body without harm, which, if it possessed the intensity of frictional electricity, would instantly destroy life.

The word *tension*, or *intensity*, expresses the degree of force exerted by electricity in overcoming a given obstacle, as a break in a circuit.

(1) From this difference in quantity and intensity results a very great difference in *continuance of action*. This is indicated by the terms *dynamical* and *statical*. Galvanic electricity, being produced in prodigious quantities and with very feeble electromotive force, may flow in a steady, gentle stream for many hours, and is hence called *dynamical*, while frictional electricity, being small in quantity and having great electromotive force, darts through an opposing medium instantaneously, and with great violence. What motion it has is therefore merely incidental to its passage from one state of *rest* to another. Hence the propriety of the term *statical*. The Holtz machine has been made to give a continuous current like that of a battery of cells, the strength

of the current being nearly proportional to the velocity of rotation. The electromotive force of the machine was about 53,000 volts, at all speeds. The resistance, at 120 revolutions per minute, was 2,810 megohms (Art. 690), and at 450 revolutions per minute only 646 megohms.

(2) Again, owing to its low tension, galvanic electricity will traverse many thousands of feet of wire rather than pass through the thin covering of silk with which the wire is insulated, and which would be but a slight obstacle in the path of frictional electricity.

But by joining cells in series, the current approaches more nearly to the character of the frictional current. A battery of 3,520 zinc and copper cells, in series, gave a succession of sparks between poles separated $\frac{1}{60}$ of an inch.

(3) Comparisons have been made of the actual quantities of electricity obtained by chemical action and by friction. Faraday has shown that to decompose one grain of water into its constituent elements, oxygen and hydrogen, requires an amount of frictional electricity equal to the charge of a Leyden battery with a metallic surface of *thirty-two acres,* equal to a very powerful flash of lightning ; but by a galvanic current, furnished by three or four Grove cells, the same result is accomplished in a few minutes.

From this some idea may be formed of the vast quantity of electricity produced during the steady flow for several hours of a Grove or Bunsen battery.

The deflection of a magnetic needle (as explained hereafter) depends solely upon the quantity of the current flowing around it. For this reason the current from a frictional or induction machine, being of small quantity through having great electromotive force, will not deflect the needle unless special arrangements for multiplying the effects are made ; on the other hand Faraday immersed a zinc and a platinum wire, each $\frac{1}{18}$ inch in diameter, in acidulated water to the depth of $\frac{5}{8}$ of an inch, during $\frac{3}{20}$ of a second, and the current thus generated produced a greater effect upon the needle than was produced by 28 turns of the large electric machine of the Royal Institution.

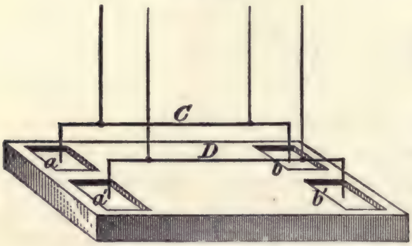
CHAPTER II.

ELECTRO-MAGNETISM.

695. Mutual Action of Currents.—

1. If galvanic currents flow through parallel wires in the *same direction,* they *attract* each other; if in *opposite directions,* they *repel* each other. These effects are shown by suspending wires, bent as in Fig. 375, so that their lower ends may dip into four separate mercury cups *a, b, a', b',* by means of which connection between the wires *C* and *D* and the battery may be readily made. The suspending threads should be two or three feet long, and the mercury cups should be large enough to allow considerable lateral movement of the wires. If simultaneous currents be sent through the two wires *C* and *D,* in the same direction, the wires will move towards each other; if currents be sent through the wires in opposite directions at the same time, they will separate more widely.

FIG. 375.



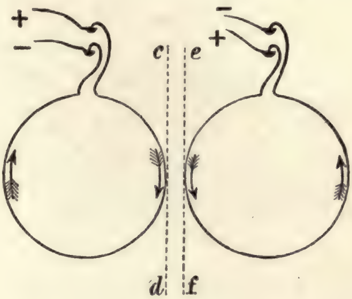
Hence, when a current flows through a loose and flexible helix, each turn of the coil attracts the next, since the current moves in the same direction through them all. In this way, a coil suspended above a cup of mercury, so as to just dip into the fluid, will vibrate up and down as long as a current is supplied. The weight of the helix causes its extremity to dip into the mercury below it; this closes the circuit, the current flows through it, the spirals attract each other, and lift the end out of the mercury; this breaks the circuit, and it falls again, and thus the movement is continued.

2. If currents flow through two wires near each other, which are free to change their directions, the wires tend to become parallel to each other, with the currents flowing in the same direction. Thus, two circular wires, free to revolve about vertical axes, when currents flow through them, place themselves by mutual attrac-

tions in parallel planes, as in Fig. 376, or in the same plane, as in Fig. 377. In the latter case, we must consider the parts of the two circuits which are nearest to each other as small portions of the dotted straight lines, *c d* and *e f*

FIG. 376.

FIG. 377.



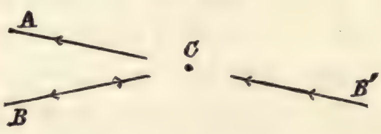
It appears, therefore, that *galvanic currents, by mutual attractions and repulsions, tend to place themselves parallel to each other in such a manner that the flow is in the same direction.*

696. Currents not Parallel.—*Currents, both of which flow towards a common point, or both of which flow away from a common point, attract each other.*

If one of two currents flows towards, and the other away from a common point, the two currents repel each other.

These cases are evident deductions from the preceding paragraph. Suppose the two currents (Fig. 378) to flow in A and B as though they came from C, then the tendency of the wires A and B is towards parallelism, and as we suppose the currents to flow from the direction C, the wires must tend to move towards each other in order to become parallel. The same effect would be produced if the currents in A and B were to flow towards C. But if the current in A flows *from* the direction C, and that in B towards the point C, then the tendency of the wires to become parallel, with the currents flowing in the same direction, causes B to revolve about C as a centre till it reaches the position B', and then the condition that the currents shall flow in the same direction will be fulfilled. It is not necessary that we should regard A and B as lying in the same plane.

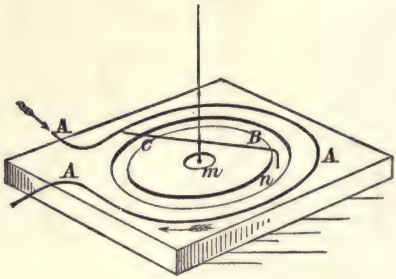
FIG. 378.



697. Continuous Rotation Produced by Mutual Ac-

tion of Currents.—Suppose a continuous current to flow through a wire A, as indicated in Fig. 379, and that a wire B, so bent as to dip into the mercury cup m at one end, and into the annular mercury trough n at the other, be suspended at the middle, a counterpoise C keeping it balanced.

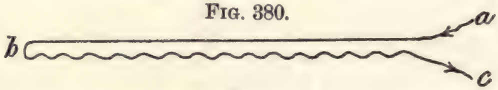
Fig. 379.



If now a current be made to flow from the cup m, through B, and thence out again by means of the mercury contact in n, the wire B will rotate in a direction opposite to that of the current in A; for the current in B, and that in the part of A to the right of n, are both flowing towards n and hence attract, while the current in B and that part of the current in A immediately to the left of n are flowing in directions to cause repulsion.

A sinuous current produces the same effect as a straight current having the same general direction and length. If a conductor, having one portion sinuous and the other straight, be

Fig. 380.



bent as in Fig. 380, so that the current may flow from a to b through the straight part, and from b to c through the sinuous part, the two portions of the current thus flowing close together in opposite directions, the joint effect upon a movable conductor parallel to $a\ b$ will be inappreciable.

698. Helices.—A wire bent in a spiral, as in Fig. 381, is called a *coil* or *helix*. If the wire is coiled in the direction of the thread of a common or right-hand screw (Art. 134), it is called a *right-hand helix;* if in the direction of the thread of a left-hand screw, it is called a *left-hand helix.* Without referring to the screw, the distinction between the right and left hand helix may be described thus: When a person looks at a helix in the direction of its length, if the wire, as it is traced *from* him,

Fig. 381.

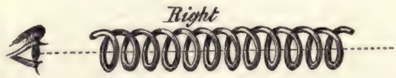
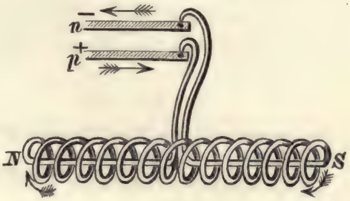


Fig. 382.

winds from the left *over* to the right, it is a right-hand helix (Fig. 381); if from the right over to the left, a left-hand helix (Fig. 382).

699. The Solenoid.—Let a helix be constructed as in Fig. 383, in which the ends are turned back through the coil, metallic contact being avoided throughout; this is called a *solenoid*—that is, a *tubular* or *channel-shaped* magnet. Next, let the electrodes p and n of a battery be furnished with sockets, one vertically above the other, in which the two ends of the helix wire are placed. The solenoid is then free to turn nearly a whole revolution around a vertical axis, at the same time that a current is passing through it. The helix is supposed to be a left-hand one, and is so connected with the battery that the current passes through it from N to S, and therefore around it from right over to left.

Fig. 383.



The direction of the spirally coiled wire, at any point, may be resolved into two other directions, one parallel to the axis of the helix, and the other in a plane perpendicular to this axis; the effect of the former components is equal and contrary to that of the straight returned portion N S, and is thus neutralized, while the latter components act as a series of circular parallel currents in planes at right angles to the axis.

While the current flows, the following phenomena may be observed:

1. If a magnet be brought near it, N will be attracted by the south pole, and S by the north pole. If, instead of a magnet, another solenoid be presented to it, whose corresponding extremities are N' and S', N and S' will attract each other, as also S and N'.

2. If not disturbed, the coil will place itself lengthwise in the direction of the magnetic meridian, with the extremity N toward the north, and S toward the south.

3. If a bar of iron be placed within it, the bar will become a magnet, having its north pole at N, and its south pole at S.

If a right-hand helix had been employed, all these phenomena would have been reversed.

700. Ampere's Theory of Magnetism.—In these experiments a coil is found to act the same as a magnet whose north and south poles are at N and S respectively. We therefore deduce the following:

1. A helix traversed by a galvanic current is a magnet the position of whose poles depends on the direction of the current.

2. Conversely, a magnet, like a coil, *may* be conceived to owe its magnetic properties to currents of electricity which traverse it.

This is the theory of Ampère, and is the one generally received, notwithstanding some objections to it.

In the helix a single current is present. But in a magnet we must conceive of an infinite number of currents, the circuit of each being confined to an individual molecule. Fig. 384 represents a magnet according to this theory, and *N* and *S* (Fig. 385) show the extremities of the north and south poles on a larger scale. The arrows on the convex surface show the general direction of all the currents—that is, of those portions of them nearest the surface, where magnetism is in fact developed—and may therefore represent them all.

FIG. 384.

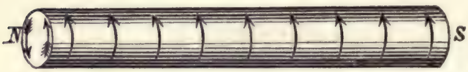
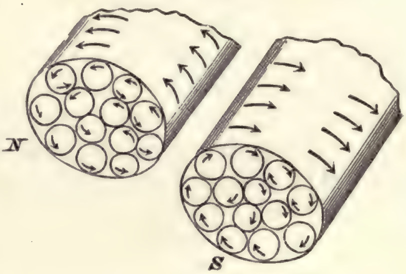


FIG. 385.



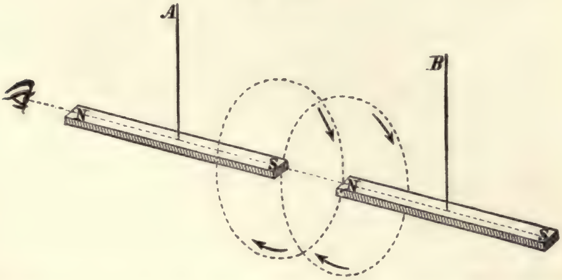
Since *S* is the south pole of the magnet, as supposed to be seen by an observer looking at it in the direction of its axis, it follows that when a magnet is in its normal position, that is, with its *north pole pointing northward,* its currents circulate from *west over to east,* and therefore from left over to right if the observer is also looking northward.

These supposed currents of the magnet are so small that we cannot take cognizance of them directly. But on the basis of Ampère's theory, we may substitute for them the large and manageable current of a helix. Then, by determining experimentally the causes of magnetic phenomena in the case of the latter, we may assign the same causes to like phenomena of the magnet.

701. Relations of Currents and Magnets to Each Other.—It should be constantly borne in mind that when we hold a magnet before us, the north seeking end farthest from us, so that we are *looking along its length from S to N,* the currents circulate around the magnet from left over to right, *or in the direction in which we would turn an ordinary screw when driving it into wood.*

1. When two solenoids, suspended as in Fig. 383, or when a solenoid and a magnet, or two magnets, are brought near each other, poles of different names attract, and those of the same name repel. For, when the magnets suspended from A and B (Fig. 386)

Fig. 386.



are in the same line, it is seen that the currents are parallel and flow in the same direction in all the corresponding parts; and in Fig. 387, where they hang side by side, the nearer parts of the

Fig. 387.

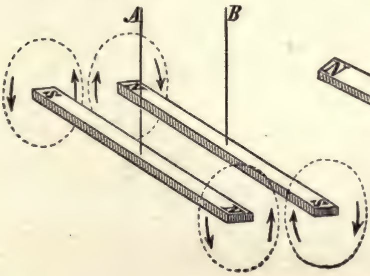
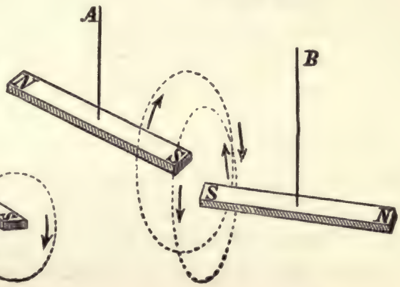


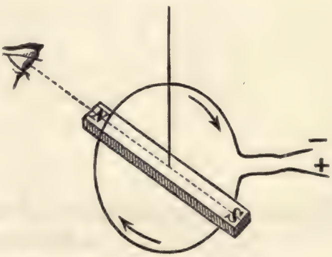
Fig. 388.



currents are parallel and flow in the same direction. While in Fig. 388, where like poles are contiguous, the corresponding parts of the currents flow in opposite directions.

2. When a magnet is suspended within a loop through which a current flows, if free to move it will place itself at right angles to the plane of the circuit, with the north pole pointing toward a person, when the current passes from his right over to his left (Fig. 389). Therefore if the circuit is in a horizontal plane, the magnet turns its north pole downward, if the current flows as in Fig. 390, or upward if the current is reversed.

Fig. 389.



3. When a magnet is brought near a closed circuit wire, as + — (Fig. 391), it will place itself tangentially to a circle, $x\ y\ z$,

Fig. 390.

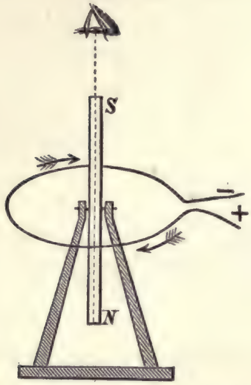
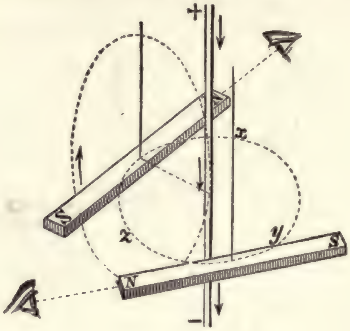


Fig. 391.



whose centre is in the wire, and its plane perpendicular to it. The part of the wire nearest to the magnet may be considered as a small portion of a loop around it, as in Fig. 389. This tangential relation is maintained on all sides of the circuit, it being everywhere true that when the south pole is directed to a person, the current *descends on the right,* as if it had passed from the left over to the right.

Comparing Figs. 390 and 391, it is evident that the current and the magnet may change places without disturbing their relative directions, it being understood that the *current flows* in the same direction in which the *north pole points.*

702. Magnet Used to Measure Current.—If a mounted magnetized needle be allowed to come to rest it will place itself in the magnetic meridian with its marked end toward the north as in Fig. 392. If a wire be held above it and parallel to it, (Fig. 392), and a current be sent through the wire from A to B, the

Fig. 392.

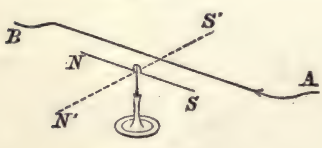
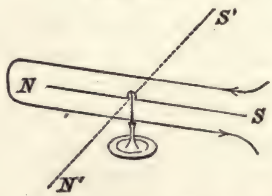


Fig. 393.



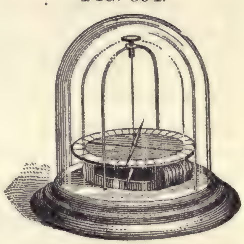
needle will be deflected, the marked end moving to the west (Art. 701). If the current be passed *below* the needle from South to North, the marked end would move to the East. If a con-

tinuous current be sent around the needle in the plane of the magnetic meridian (Fig 393), from South to North above and from North to South below the needle, each of the two branches of the encircling wire tends to move the marked end of the needle to the West, and the two together produce double the deflection due to either alone. The discovery of these effects was made by Oersted in 1820.

703. Galvanometer.—The action of the current upon the needle is proportional to *the quantity of electricity which flows around it in a unit of time.*

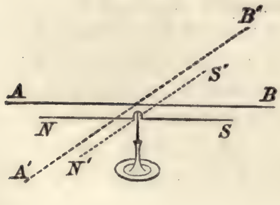
When the coil consists of many convolutions of wire, a very feeble current passing through will deflect the needle from its north and south direction, and the amount of deflection, not exceeding 20°, serves as a measure of the galvanic force. Hence the name of the instrument. To render it still more sensitive, a second smaller needle, with poles reversed, attached to the same vertical wire, makes the first nearly astatic with relation to the earth. In making such a coil, the wire must be carefully insulated. This is generally done by winding it with silk thread. In Fig. 394 the galvanometer is represented as covered by a bell-glass. The coil is seen beneath the graduated circle. The two needles of the astatic combination are shown, the first above the graduated scale and the other just protruding from the coil, beneath the first, and parallel to it. The character of the coil should depend upon the current to be measured; for currents which have already traversed great resistances, many hundred turns of fine wire are needed, while for thermo-electric currents (Art. 706) a few turns of thick wire are most suitable. By means of properly insulated coils of fine wire, of 3000 or more turns, the current of the Holtz machine may be made to produce an effect upon the needle.

FIG. 394.



704. Sine Galvanometer.—It was stated, Art. 703, that up to about 20° the deflections of the needle were proportional to the strength of the current, the conducting wire remaining in the plane of the magnetic meridian. Now suppose that the conducting wire be made to follow the needle, always remaining parallel to it, as in Fig. 395, in which the wire A B has followed the needle, till a position of rest N' S' has been found,

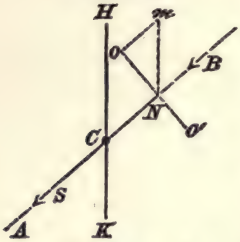
FIG. 395.



in which position the deflecting force of the current in $B'\ A'$ just balances the directive force of the earth's magnetism.

Let $H\ K$ (Fig. 396) represent the magnetic meridian, and $B\ A$ the direction of the current, parallel to and above the needle. Let $N\ m$ represent, in intensity and direction, the directive force of the earth's magnetism at the place. Now $N\ m$ may be resolved into two components; $o\ m$, in the direction of the needle, has no tendency to produce deflection, while $N\ o$, at right angles to the needle, equals and opposes the deviating effect, represented by $N\ o'$, of the current in $B\ A$. From the triangle $N\ o\ m$ we have $N\ o = N\ m \times \text{sine}\ N\ m\ o = N\ m \times \text{sine}\ N\ C\ H$. Hence, since the factor $N\ m$ is constant, it being the directive force of the earth's magnetism at the place, the deviating effect of the current is proportional to the sine of the angle of deflection. Any *sensitive* galvanometer may be used as a *Sine Galvanometer* by turning the coil, while the current is flowing, till it is parallel to the needle in its position of rest; if now the circuit be broken, the needle will return to the plane of the magnetic meridian, and the sine of the number of degrees passed over will be the relative strength of the current.

FIG. 396.



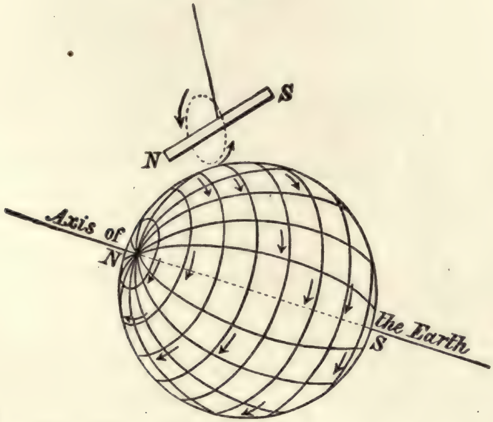
If the coil be of great diameter as compared with the length of the inclosed needle, the strength of the current is proportional to the *Tangent* of the angle of deflection, the plane of the coil being coincident with the plane of the magnetic meridian. An instrument the diameter of whose coil is ten to fifteen times the length of the included needle is called a *Tangent Galvanometer*.

705. Polarity with Respect to the Earth.—It is believed that currents of electricity are constantly traversing the earth's crust, passing around it from east to west, and making the earth itself a magnet, with boreal magnetism developed at the north pole, and austral at the south pole. Thus the earth may be taken as the standard magnet, and both it and the currents around it control the polarity of the needle. For, as in Fig. 397, in order that the current of the magnet may be parallel with the adjacent terrestrial current, and in the same direction with it, since the latter passes from east to west, the lower side of the former must also pass from east to west. But in order that this may be the case, the north pole of the magnet must point northward, and this it does when free to obey the directive influence of the earth.

At first view, the earth currents from east to west seem to be in the wrong direction; for that is from left over to right, to a

person to whom the north pole points. This, however, is explained by recollecting that the magnetism of the north pole of the earth is the same as that of the south pole of a magnet (Art.

FIG. 397.

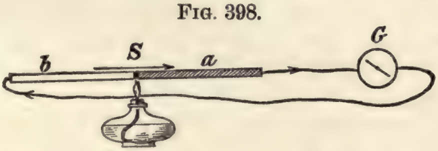


595). For convenience, that end of a needle which *points* north is called the north pole; but by the law of attraction between opposite poles, it must be unlike the north pole of the earth. Therefore, the rule for the direction of currents around a magnet must be reversed when applied to the earth.

The existence of currents traversing the earth's crust has been variously accounted for. The strong analogy between them and those of thermo-electricity points to the heat of the sun as at least a very probable cause.

706. Thermo-Electricity.—Let two bars of bismuth (*b*) and antimony (*a*) be soldered together as in Fig. 398. If now the

FIG. 398.



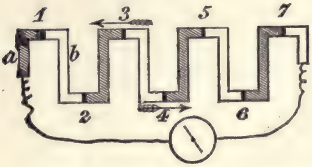
joint *s* be heated by a lamp a current will flow across the heated junction from the bismuth to the antimony, as will be shown by the galvanometer *G*.

The electromotive force of the current depends upon the metals in contact at the heated junction. If any one of the metals given below be joined with any one following it in the list, upon applying heat the current will flow across the junction from the former to the latter: Bismuth, platinum, lead, tin, copper,

silver, zinc, iron, antimony. In some cases a continued increase of temperature at a junction finally reverses the current.

707. Thermo-Electric Pile.—If a series of bars of bismuth and antimony be arranged, as in Fig. 399, and the junctions marked *3* and *4* be equally heated, no current will be indicated by the galvanometer; for the flow at *3* would be from the bismuth to the antimony as indicated by the arrow, while at *4* it would also be from *b* to *a*, as shown, and these two currents would neutralize each other. But if we heat only one set of junctions, the odd-numbered for instance, then a current flows whose electromotive force is proportional to the number of heated junctions.

Fig. 399.



A set of twenty or thirty pairs, conveniently arranged so that the alternate junctions may be simultaneously subjected to heating or cooling effects, is called a *thermo-pile*, and has been an important instrument in investigations upon heat. The electromotive force of the best thermo-electric piles is comparatively very feeble.

We may suppose that the terrestrial current may be caused, in part at least, by the unequal heating of the heterogeneous substances composing the earth's crust, as the sun's heat is alternately poured upon and withdrawn from them once in every diurnal revolution.

708. Magnetic Induction by Currents. — Ampère accounted for the phenomena of magnetic induction by supposing that galvanic currents circulate through the molecules of all bodies, but in different directions, so that they mutually neutralize each other. That in a few substances, such as steel and iron, it is possible to control these currents and cause them all to flow in the same direction; and that when this is done, the phenomena of polarity ensue.

Supposing this to be the correct explanation, the effect of a galvanic current (and in fact of any method of magnetizing) is simply, by repulsion and attraction, to produce uniformity of direction among these magnetic currents.

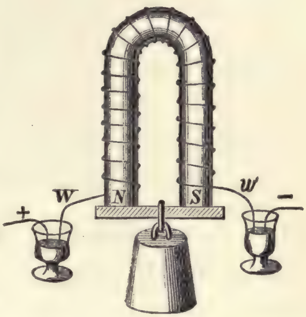
709. The Permanent and Temporary Magnet.—When a current of sufficient strength is passed around a bar of well-tempered steel, a *permanent* magnet of considerable power may be obtained.

With *soft* iron, the result is a *temporary* magnet, which retains

its magnetic properties only while the current is in motion. In either case the poles are always in the position which those of a needle would voluntarily assume if placed in the same relation to the current.

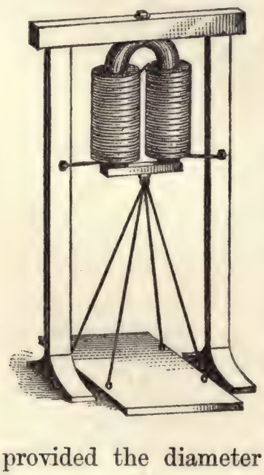
710. The U-Magnet.—Let a piece of soft iron, in the form of a horseshoe or the letter U (Fig. 400), be wound with a coil of insulated copper wire whose extremities, *W* and *w*, are dipped in cups of mercury, in which are also dipped the electrodes + and — of a battery. When all the wires are in metallic communication, the circuit is closed, and the current passing around the iron makes it a magnet; and since to a person looking along the length of the helix the current passes from right over to left, the north pole is at *N*, and the south pole at *S*. As soon as the circuit is broken by lifting out of the mercury any one of the wires, the weight which was previously sustained will fall, showing that the iron is no longer a magnet.

FIG. 400.



711. Helices.—The form of coil or helix generally employed is shown in Fig. 401. Many hundreds or even thousands of feet of insulated wire are wound around two bobbins, and through the centre of each passes a branch of the U-shaped iron; or, more frequently the central cores of iron are separate pieces, joined by a third one across two of the ends, and thus a U-magnet of modified form is obtained. By employing a fine wire coiled many times around the bobbins, a magnet of very great power may be formed, considering the weakness of the battery which furnishes the current. A magnet formed by the use of a small Bunsen cell has been known to lift five hundred pounds, and with twenty Grove cells can be made to sustain a weight of three tons.

FIG. 401.



The strength of the magnet increases nearly with the strength of the current, and with the number of coils of wire, provided the diameter of the bobbin does not become so great

as to remove the outer coils too far for their due effect. This law is general, but is true only within certain limits, determined for each bar. If the wire be increased in length, the resistance is also increased, and consequently less current flows than before, so that there is a certain relation between the length of the coil and the electromotive force which will give the maximum magnetic effect. The rule in practical telegraphy is to make the resistance of the wire on the bobbin equal to that of the circuit including that of the battery itself.

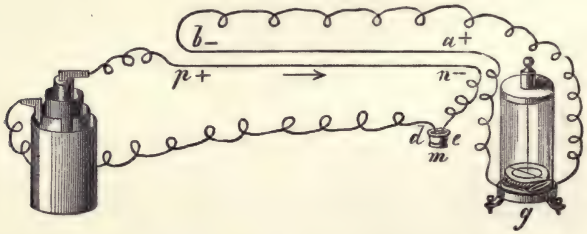
CHAPTER III.

INDUCED CURRENTS.

I. Currents Induced by Currents.

712. Experiments.—Arrange a straight portion of a broken galvanic circuit, *p n* (Fig. 402), parallel to a part *b a* of a galvanometer circuit. Suppose the needle of the galvanometer to stand at zero. If now the battery circuit be closed, by dipping the wires into the mercury cup *m*, at the instant of closing the needle will

Fig. 402.



be deflected to that side which indicates a current flowing in the galvanometer circuit in a direction opposite to that of the battery current. This deflection of the needle is but momentary, and it returns speedily to zero, and remains at rest, although the battery circuit remains closed. At the instant of breaking the circuit, the needle is again deflected, but in a direction indicating a flow of current in the galvanometer circuit in the same direction as that of the battery. After a few oscillations the needle comes to rest at zero again.

These two currents in $a\ b$ are called *induced currents;* and the one in $p\ n$, to which they owe their origin, is called the *inducing current.* The presence, direction, and duration of the induced currents are indicated by the galvanometer g.

713. Characteristics of Induced Currents.—It is obvious that induced currents differ materially from the current of a battery which is uniform in direction and constant in intensity for an appreciable length of time. The following are the distinctive features of induced currents:

(1) Induced currents are *instantaneous.*

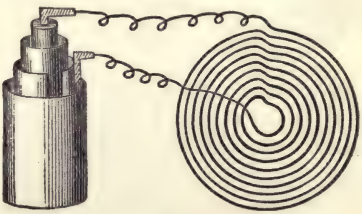
(2) On *closing* the circuit, the direction of the resulting induced current is *opposite* to that of the inducing current.

(3) On *breaking* the circuit, the induced and inducing currents are in the *same* direction.

714. Inducing and Induced Currents in one Wire.— We have thus far considered only the inductive influence of the current on a wire exterior to its circuit.

In order to produce the preceding results with a single wire, let the circuit-wire be coiled as in Fig. 403. Each spire is now acted upon inductively by the galvanic current passing through the adjacent spires in the manner already described for separate wires.

FIG. 403.



The result of these several inductive actions is that when the circuit is closed and broken, regular induced currents are generated in it. *And since these coexist for an instant of time with the inducing current, and pass through the same electrodes with it,* it follows—

(1) That when the circuit is *closed,* the inducing current is partially neutralized, and has its intensity diminished by the induced current which flows in a direction contrary to its own; and

(2) That when it is *opened,* the induced current having now the same direction as the inducing current, reinforces it and augments its intensity.

715. Mode of Naming Circuits and Currents.—The phenomena of induced currents were discovered by Faraday in 1832, and to him we owe the foregoing description of them. The following terms now in use were also introduced by him:

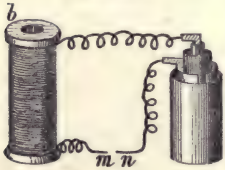
The *inducing* current is called the *primary current,* and the wire it traverses the *primary wire.* Currents *induced* in the primary wire are called *extra currents;* the one obtained on *closing* the circuit is the *inverse extra current;* the one on *opening* it is the *direct extra current* (Art. 713).

A wire exterior to the primary, as *a b* in Fig. 402, is a *secondary* wire, and the currents induced in it are *secondary currents.*

716. Currents Induced in Coils.—Instead of straight wires or loose spirals, compact coils of carefully insulated wire are employed. Thus all parts of the wire are brought much nearer to each other, and the inductive influence is far more energetic. Indeed, without a coil, the presence of induced currents can generally be detected only with a delicate galvanometer. The following experiments show the effects of coils:

(1) Around a hollow wooden bobbin, *b* (Fig. 404), coil about 100 feet of No. 16 insulated copper wire. Let this be made a part of the circuit of a battery, as shown in the figure. This circuit is of course closed when *m* and *n* touch each other. Now if *m* and *n* be held in contact, one in each hand, and then be separated, the body of the operator becomes a part of the circuit, and the primary current, not having sufficient intensity to pass through it, ceases. But the direct extra current passes through, producing a shock. When the wires are brought together again, the primary and inverse extra currents pass through the metallic circuit, and no shock is felt.

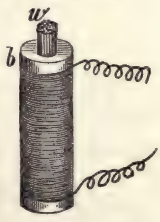
FIG. 404.



The more rapid the rate at which *m* and *n* are brought together and separated, the more decided are the results obtained. To produce the most marked effect, attach a coarse file to one end, as *m,* and hold it in one hand while *n* is drawn rapidly over the ridges of its surface with the other.

(2) Fig. 405 represents the same coil as Fig. 404, with the addition of a bundle of soft iron wires, *w*, inserted in the hollow bobbin.

FIG. 405.



When the circuit is closed, these wires are magnetized—that is, the Ampèrean currents supposed to reside in them are made to circulate in the same direction as the battery current (Art. 708).

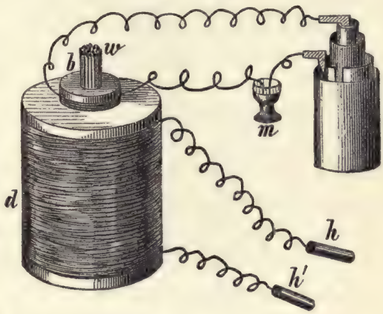
And since the appearance and disappearance of these magnetic currents are simultaneous with the appearance and disappearance

of the primary current, they augment the effects of the latter, and the resulting extra currents are of greater intensity.

The effect of soft iron in the primary coil is an observed fact, and the above is the way in which Faraday accounted for it on the basis of Ampère's theory.

(3) Let the primary coil and bundle of wires of the preceding figure be placed within a secondary coil, *d* (Fig. 406), from which it is carefully insulated. This secondary coil should be made of wire much greater in length and smaller in diameter than that of which the primary coil is made. For instance, let it consist of 1500 feet of No. 35 insulated copper wire. When the ends of this wire, *h*, *h'*, are held one in each hand, every time the primary circuit is interrupted, a secondary current traverses the secondary circuit of which the person forms a part. The resulting shocks will be quite appreciable, though the primary current be produced by only a single small cell.

FIG. 406.

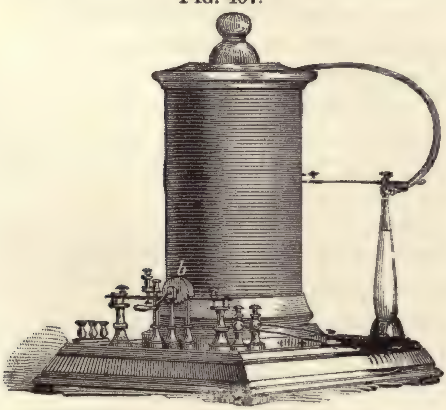


As in the first experiment, the effect on the person will become more marked as the interruptions increase in frequency.

In the third experiment, the magnetic currents of the iron core add their inductive influence, as already explained, to that of the primary current, thus increasing the intensity of the secondary currents.

The effect of the *extra currents* also should not be overlooked. As these traverse the primary coil, alternating with each other in direction, they materially modify the effects of the primary and magnetic currents, which are uniform in direction.

FIG. 407.



717. Ruhmkorff's Coil.—The celebrated Ruhmkorff coil (Fig. 407) is not essentially different from the one

just described, except in having (1) an arrangement for producing a continued and rapid succession of interruptions in the primary current, called a contact breaker; this is a coarsely-toothed wheel, shown at *b*, the teeth of which, on its being turned, lift a lever and so interrupt the circuit. The wires from the battery to the primary coil are attached to the binding posts as shown, and thence the current flows to the contact breaker.

(2.) A *condenser*, consisting of sheets of tin-foil separated by oiled silk or other insulator. The first, third, fifth, &c., sheets are connected together, and also with one end of the primary coil, while the even-numbered sheets are in like manner joined to the other end of the same coil. When the circuit is broken the *direct extra* current in the primary wire spends its force in charging the two sets of sheets, one set positively and the other negatively, and this condenser instantly discharging again sends a reverse current through the primary, demagnetizing the core of wires and preparing the apparatus for the next break of circuit.

The capacity of a condenser which holds 1 *weber* (or *coulomb*, as it is proposed to call it) when charged to a potential of 1 volt is called a *farad.* This unit is too large for practical use and hence the *microfarad* is taken. A condenser of one microfarad capacity would require nearly 1000 sq. inches of tin-foil, or ten sheets 10 inches square.

718. Power of the Ruhmkorff Coil.—The efficiency of a Ruhmkorff coil depends largely on *complete insulation;* and, in different coils, varies greatly with the *length and fineness* of the *secondary wire.*

To secure insulation, the wires are (as usual) wound with silk thread, then each individual coil around the axis is separated from the succeeding one by a layer of melted shellac, and lastly a cylinder of glass is placed between the primary and secondary coils.

With regard to the secondary coil, one of the largest size contains 280 miles of the finest copper wire. With such an apparatus and with a primary current from thirty quart Grove cells a spark of 42½ inches has been obtained; indeed, all the tension effects of a large electrical machine, as well as the quantity effects of a powerful galvanic battery, may be reproduced.

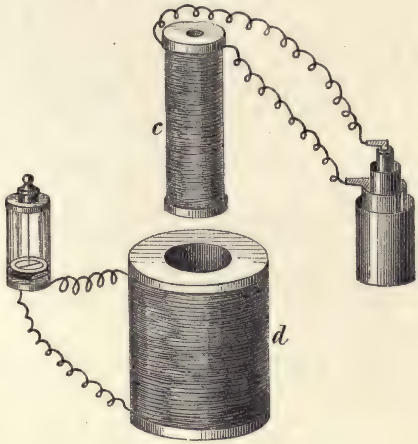
Great care should be taken in handling a large induction coil, for the shock resulting from its discharge through the body would be dangerous, and might possibly prove fatal.

719. One Coil moved into, and out of, another.—In all that has preceded, the interruptions of the primary current have been supposed to take place *instantaneously.* If these interrup-

tions are *gradual,* the resulting induced currents remain the same in direction as before, but vary in intensity and duration.

Thus, if the primary coil *c* (Fig. 408) be made to fit loosely in the secondary coil *d,* and then be moved up and down (the primary circuit remaining closed), it will be found—

FIG. 408.



(1.) That each insertion and removal of it corresponds, the one to a gradual *closing,* the other to a gradual *opening* of the primary circuit—the result of the former being an *inverse* secondary current, of the latter a *direct* secondary current.

And since a continuous motion of the primary coil produces a continuous series of instantaneous secondary currents with no appreciable interval between them, it will be found—

(2.) That the secondary currents are *continuous* in effect as long as the motion of the primary coil is continuous;

(3.) That their intensity varies with the rate of motion of the primary coil, diminishing or increasing as that is moved slowly or rapidly; from which it follows—

(4.) That they cease whenever the primary coil is brought to a state of rest in any position.

720. Changes of Intensity in the Primary Current.—All the results just mentioned may be obtained if, instead of changing the position of the primary coil, as above, it remain at rest while a corresponding series of variations be produced in the primary *current,*—an *increase* of intensity in that corresponding to an *insertion* of the coil, and a *decrease* to a *removal* of it.

If two flat spirals, one of coarse wire and one of fine, carefully insulated, be placed opposite each other and near together, and the ends of the coarse or primary wire be connected with the coatings of a Leyden jar, a current will be induced in the secondary spiral at the instant of discharge, thus adding another to the many evidences of the identity of frictional and dynamical electricity.

721. Lenz's Law.—*If two conductors, A and B, in one of which, A, a current is flowing, be made to change their relative positions, then a current will be induced in B in a direction which*

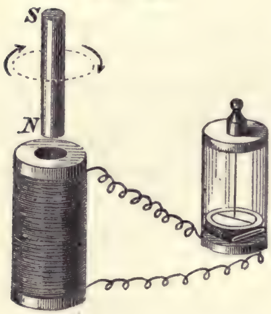
will cause a mutual action in the two conductors tending to oppose their motion. Thus, if A and B be brought nearer together an inverse current will flow in B, and currents flowing in opposite directions repel each other (Art. 695); and if A and B be caused to move apart, then a direct secondary current will flow in B, and, according to Art. 695, currents flowing in the same directions attract each other. This statement of the results of experiments will aid the memory in regard to the directions of the primary or secondary currents.

II. Currents Induced by Magnets.

722. Magneto-Electricity.—Faraday reasoned that if currents could induce magnetism, a magnet ought to induce currents. This he found to be the case, and thus discovered a new branch of physical science, to which he gave the name of magneto-electricity.

If a magnet be used instead of the primary coil in Fig. 408, all the phenomena mentioned in Art. 719 may be reproduced.

Fig. 409.



Thus, with the coil and magnet in Fig. 409, we obtain the following results:

(1.) When the magnet is alternately inserted in and withdrawn from the coil, the latter is traversed by induced currents alternating with each other in direction.

(2.) These currents are continuous while the magnet is in motion.

(3.) Their intensity diminishes or increases as the magnet moves slowly or rapidly.

(4.) They cease when the motion of the magnet ceases.

723. Explanation of the Foregoing Phenomena.—On the basis of Ampère's theory, the correspondence of these phenomena with those of Art. 719 can readily be accounted for. For the magnet may be considered a true primary coil, its magnetic currents corresponding to the primary current in Fig. 408. With regard to them, the induced currents are regular inverse and direct secondaries; for in any given case they will be found to have the same direction as those induced by a primary current whose direction corresponds with the *supposed* direction of the magnetic currents.

It will be seen at once what a strong argument is here furnished in favor of Ampère's theory.

724. An Iron Core Changing its Magnetic Intensity.—Replace the magnet (Fig. 409) by a bar of soft iron inserted in the coil, and let a magnet be alternately brought near this, and removed from it, as in Fig. 410. The same results will be obtained as in the preceding series of experiments. The proximity of the magnet induces magnetism in the soft iron, and its motions to and fro produce variations in this induced magnetism corresponding precisely with the varying intensity of the primary current mentioned in Art. 720, and, as might be expected, the results are the same.

Fig. 410.

In this experiment, it is obviously immaterial whether the coil be at rest and the magnet be moved, or the magnet be at rest and the coil be moved. The latter method is adopted in some magneto-electric machines.

If the *core* had been magnetic and a soft iron disk had been moved toward and away from its pole, the mutual induction would have varied the strength of the magnet and would thus have produced currents in the coil; if these currents had been led away to a second coil surrounding a magnet, near whose pole a second disk was held, the variations of the strength of this magnet would produce in this second disk motions corresponding to those of the first. Thus in the telephone (Art. 338), the vibrations of the transmitting disk are reproduced in the disk of the receiving instrument.

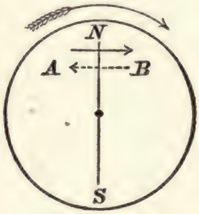
725. Arago's Rotations.—In 1824 Arago observed that the oscillations of a magnetic needle were reduced in number by suspending a copper plate above it. This observed phenomenon soon led him to the discovery that if a horizontal copper disk be made to rotate rapidly, a magnetic needle suspended above it would rotate also. This effect may also be produced with other metals though in less degree.

If a disk of copper be set spinning on an axis, between the poles of a powerful electro-magnet whose circuit is broken, the axis of the disk being parallel to the lines of force, the rotation continues with slight loss of velocity for a long time ; but if the circuit be suddenly closed the rotation is at once checked, or

possibly stopped. If such a disk be kept in rapid rotation by a suitable band and pulley, after the circuit is closed, the disk will be heated by the action of the magnet.

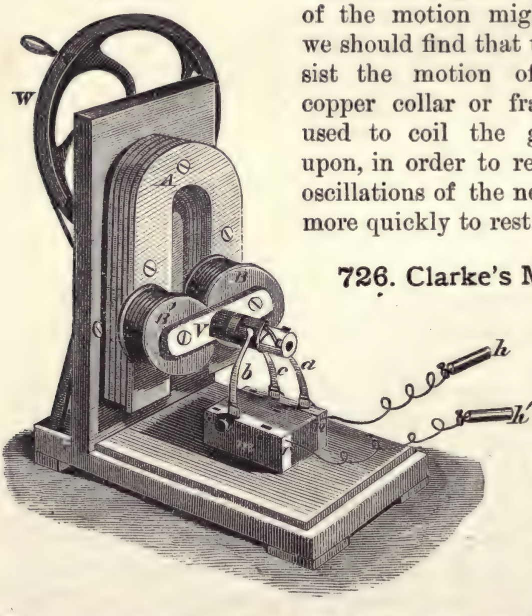
These effects were explained by Faraday as being due to currents induced in the mass of metal. Thus let a needle *N S* (Fig. 411) be suspended above a metal disk *A B*. The magnetic currents flow around the needle as indicated in the figure, the currents below the needle from right to left as shown by the dotted arrow, and those above from left to right, as shown by the full arrow (Art. 700). Now suppose the disk to be rotated in the direction from *A* to *B*; the portions of the currents around *N S* which are nearest to the disk will induce in that part of the disk towards *A* currents whose directions are such as to resist the motion of the disk, according to Lenz's law (Art. 721), that is to say, currents will flow in the disk from left to right; while in that part of the disk towards *B*, which is moving away from *N*, the induced currents are from right to left, and so resist the motion of *B away* from *N*.

FIG. 411.



If the needle had been moved, the disk remaining fixed, the same analysis of the motion might be made, and we should find that the disk would resist the motion of the needle. A copper collar or frame is sometimes used to coil the galvanometer wire upon, in order to reduce or *damp* the oscillations of the needle, and bring it more quickly to rest.

FIG. 412.



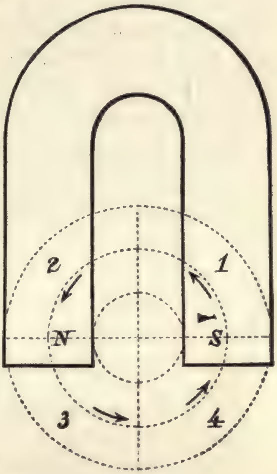
726. Clarke's Magneto-electric Machine. — In front of the poles of the U-magnet, *A* (Fig. 412), is revolved the *armature,* consisting of the two *bobbins, B, B',* which are coils of fine wire with cores of soft iron. These cores are joined to each other and to the axis of rotation

by the bar of soft iron, *V*, and motion is communicated by the multiplying wheel and band at *W*.

As one of the bobbins passes before a *north* pole while the other is passing before a *south* pole, the resulting induced currents are *relatively* of contrary directions; but as one of the coils is always right-handed and the other left-handed, the currents passing through them at any given instant have the same *absolute* direction, so that the two coils act as one.

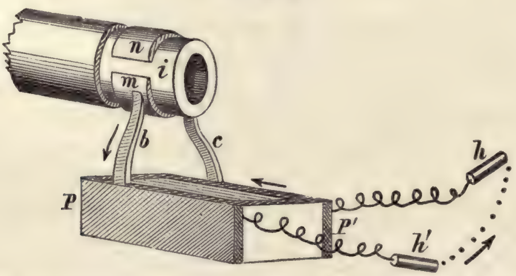
Fig. 413 shows the poles of the fixed magnet, and the direction in which the armature revolves. The maximum magnetization of the soft iron cores occurs when the bobbins are directly in front of *N* and *S*. While they move through the first and third quadrants they are *losing* their magnetism, and while moving through the second and fourth they are *acquiring* that of the *contrary kind.* The resulting induced currents will thus be direct and inverse *to contrary kinds of magnetism,* and will therefore have *the same absolute direction.* But as the bobbins pass from the second quadrant to the third, and from the fourth to the first, they lose the magnetism just acquired, and the induced currents change from inverse to direct *with reference to the same kind of magnetism,* and therefore become *reversed* in absolute direction.

FIG. 413.



Hence the semi-revolutions of the armature on opposite sides of a line joining the poles of the permanent magnet produce currents of contrary directions.

FIG. 414.



727. The Commutator.—Fig. 414 is an enlarged view of the outer end of the axis (Fig. 412), and shows the *commutator*, or

arrangement by means of which the contrary currents just mentioned are made to furnish one—or rather a series of currents—flowing in the same direction.

Two pieces of brass, *m* and *n*, fastened to the axis and revolving with it, are insulated from each other by being fastened to an ivory ring, *i*, around the axis, and are connected with the ends of the wires of the coils; that is, they are made the *poles* of the two coils acting as one. Against *m* and *n* press two springs, *b* and *c*, which are attached to the plates P and P', insulated from each other, from which plates the wires are led to the handles h h'. If the handles h and h' be joined, the circuit will be complete when *m* and *n* in their revolutions press against *b* and *c*. Let us suppose that the induced current is passing in the direction indicated by the arrows. When the armature has revolved through 180° from its present position, *m* and *n* will have changed places—*but they will also have changed polarities* (Art. 726). Therefore *n* presents to *b* the same polarity which *m* did, and hence there is no change in the direction of the current through *b* and *c*.

728. Effects of Rapid Revolution.—The intensity of the induced currents of this machine, as also the rapidity with which they succeed each other, is regulated by the rate of revolution of the armature. When this is rapidly revolved, they produce all the effects of a single voltaic current, so that the apparatus may be used as a galvanic battery with h and h' for its electrodes. At the same time its physiological effects are most remarkable, the shocks becoming unendurable when it is revolved with great rapidity.

729. Large Machines.—In large magneto-electric machines of this kind, increased efficiency is obtained in two ways:

First, by multiplication of magnets. In Nollet's machine, constructed in 1850, 192 magnetized steel plates are so combined as to make 40 powerful U-magnets. These are arranged in eight rows around the circumference of a large iron frame inside of which revolve sixty-four bobbins.

Second, by multiplication of currents. In Wild's machine, constructed on this plan, the induced currents first obtained, instead of being directly utilized, are passed through the coils of a large electro-magnet. Before the poles of this a second armature revolves, and the resulting induced currents are far more powerful than the first. These may in turn be made to magnetize a second electro-magnet before the poles of which a third armature revolves, &c.

Currents of very great intensity may be obtained from either of these machines. The motive power employed is generally a steam-engine of from one to fifteen-horse power.

730. Motion of a Straight Conductor through Lines of Force.—A magnetic field has been defined (Art. 577) and the direction of the lines of force has been studied. If now a conducting wire, perpendicular to the lines of force, be moved in a direction perpendicular to its own length, so as to cut new lines of force at each instant, a current will be generated in the moving wire, provided the circuit be closed outside the magnetic field ; if the circuit be not complete, then a difference of potential at the two ends of the wire will result. If the magnetic field be moved while the wire remains at rest, the same effects will follow. If the wire and the field be both at rest, but if the strength of the field be increased, a current will flow, since the increase of the magnetic field adds new lines of force which, crowding together as it were, are cut by the wire; a decrease of the strength of the field would cause a loss of lines of force, and a separating of those left, and so a movement of the lines across the wire, resulting in a reverse current. The strength of the current generated depends on the number of lines of force cut in a unit of time ; hence, for a given field it varies with the velocity of motion of the conductor, and increases as the length of the part of the conductor acted upon by the field increases; for a given velocity and length of conductor, it varies with the strength of the field ; that is, it varies in proportion to the closeness of the lines of force.

Let E represent the electromotive force, H the strength of the magnetic field, l the length of the conductor acted upon, L the distance moved, and T the time, and we have

$$E = H\frac{L}{T}l.$$

But $\frac{L}{T}$ equals the velocity of the conductor, therefore *the electromotive force, per unit of length of conductor, is equal to the strength of the field multiplied by the velocity of motion of the conductor.*

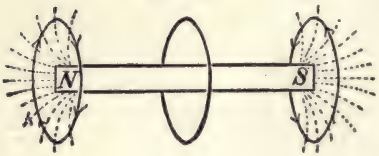
If the conductor be moved in the direction of the lines of force no current will result.

By applying Lenz's law the direction of the induced current may be determined. Let the student suppose himself to represent any line of force, his feet being upon the marked pole of a magnet ; then, holding a wire conductor in his outstretched hands, upon moving the wire *forward* through the magnetic field, in the direction in which he faces, a current will flow from his left hand to his right. A wire moved through the earth's magnetic field at right angles to the direction of the dipping needle will deflect the galvanometer needle.

731. Motion of a Coiled Conductor.—If instead of a

straight conductor a coil of wire be used the effect is as follows: Suppose a coil to be moved towards the north pole of a magnet, as in Fig. 415, the lines of force being represented by dotted lines.

FIG. 415.

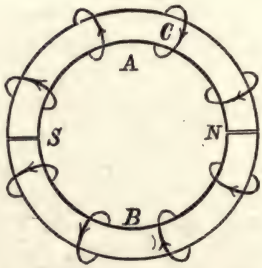


As the coil approaches the pole it is seen that new lines of force are cut at each instant, a greater number thus being included within it; and regarding any limited portion of the coil as straight, it follows from what precedes that a current will flow as indicated by the arrows. After passing the middle of the magnet negative lines of force will be cut, and the current will be reversed, as shown. If the motion of the coil had been away from the north pole in the first instance fewer and fewer positive lines of force would have been included within it, and a reverse current would have resulted.

732. Dynamo-Electric Machines.—These machines differ from the magneto-electric machines previously described, in that the whole, or a part, of the current from the revolving ring armature is made to pass through the coils of the field electro-magnets, the slight residual magnetism of which is sufficient to start a feeble current in the armature, which current instantly strengthens the magnetic field, and the machine is thus soon worked up to its full efficiency.

The *Gramme Machine* may be taken as a type, and the explanation of its working will lead to the understanding of other applications of the principle involved. Let two half-ring magnets be placed with their like poles in contact, as in Fig. 416, and suppose the coil *C* to be moved from the neutral point *A* towards the poles at *N*. From the last article it will be seen that a current will be induced in the direction of the arrow, and during the progress of the coil from *A* through *N* to *B* the current will not change direction, but will increase in strength by being subject to the same inductive influence

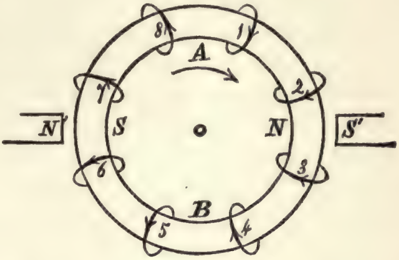
FIG. 416.



during the whole progress of the coil. On passing the neutral point *B*, the current will be reversed, as shown by the arrows, and will flow in the inverse direction during the movement of the coil from *B* through *S* to *A*. Now, as it is not convenient to move coils around a ring, let us move the poles *N* and *S* around within

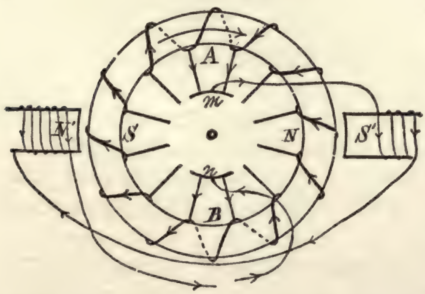
a set of coils placed on an iron ring at the points 1, 2, 3, &c. This we may accomplish by causing the iron ring, with its coils, to rotate between the poles of a magnet, in the direction of the arrow (Fig. 417). The *instantaneous pole N* remaining always opposite the pole S' of the inducing magnet, while 2 and 1 are made to approach it in succession by the rotation of the ring, causes currents precisely similar to those obtained by moving the coil.

FIG. 417.



Now let us examine the effect of the stationary pole S' upon the moving coils. It was shown in the last article that a coil approaching a S pole so as to include more and more lines of force would have a current flowing in it from right over to left, as one faces the direction of motion, or contrary to the motion of the hands of a watch, and if it were moved away from the pole so as to inclose fewer lines of force the current would flow from left over to right. Now as the coil moves from 1 to 2 its plane becomes more nearly parallel to the the lines of force, and therefore the number of these inclosed *decreases,* until at S' its plane coincides with their direction and none are inclosed. The current due to S' is therefore in the direction of the arrows as before. After passing S' the coil incloses more and more lines of force, and therefore the direction of the current should be from left over to right as in the figure. Thus it is seen that the stationary pole S' and the constantly-shifting pole N both conspire to produce currents of the same kind in the coils from A to B; and also that N' and S produce reverse currents in the coils from B to A.

FIG. 418.

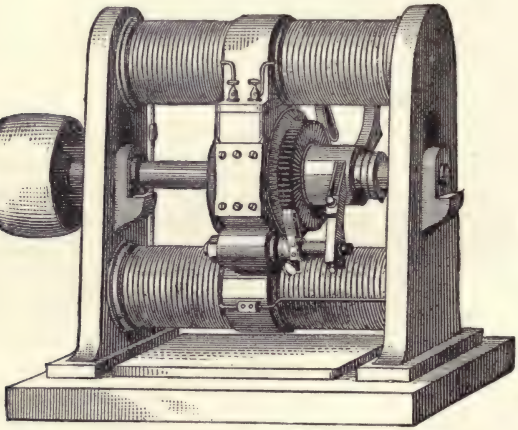


Now suppose a continuous wire to be wound around the ring armature, as in Fig. 418. It is evident, if the student has followed the explanation thus far, that a current flows from A through N to B, and from A through S to B; hence if we place stationary metal strips m and n not attached to the revolving ring, so that each may be in contact with two of the short wires soldered

to the inner points of the spiral at any given moment, a continuous current will flow through a wire of which these metal strips are the terminals, and this current may be carried around the field magnets before being led off for use, as shown in the figure. In the Gramme machine the wire is coiled in sections, and these are united into one continuous coil by attaching the end of one coil, and the beginning of the succeeding one, to a copper slip fastened upon a cylinder of insulating material and parallel to its axis. These copper terminals, in number equal to the number of coiled sections, are insulated from each other, and perform the office of the short wires used for illustration in Fig. 418. Two slips of copper press upon the revolving cylinder of terminals, one above and one below, and lead off the current.

Fig. 419 represents one form of Gramme machine. When

FIG. 419.



great electromotive force is required, as in arc electric lighting, it is obtained by increasing the number of turns of wire in the rotating coils, and by increasing the velocity of rotation within due limits. When currents of feeble electro-motive force, but of considerable strength are required, as in electro-plating, the internal resistance must be small, and hence the revolving coil must be a few turns of stout wire.

The continuous current dynamo-machines become *electro-motors* when driven by a current from a battery, or from another dynamo.

CHAPTER IV.

PRACTICAL APPLICATIONS.

733. Classification.—The applications of Galvanic electricity in the arts and sciences, as well as in the affairs of every day life, are eminently *practical.* They may be classified according to the way in which are utilized those molecular forces whose resultant is known as *the current.*

It may be stated in general that these applications are made either *within the circuit,* or *exterior to it.*

Within the circuit. Here the electrical force is employed (I) *directly,* as in chemical decomposition, or (II) by being first made to produce the effects *light* and *heat,* which are then applied as desired.

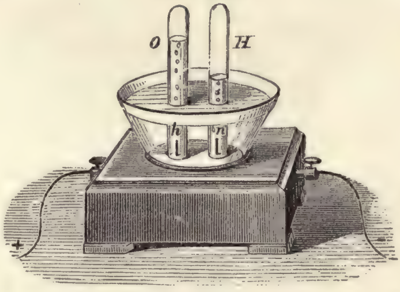
Without the circuit. Here it is employed *indirectly* (III) by being made to reappear as mechanical motion through the intervention of the kindred force, magnetism.

Examples will be given of each.

I.—Direct Applications of the Current.

734. Electrolysis.—When a current is passed through a binary compound (*i. e.,* one containing two elements), the compound is decomposed, one of its elements appearing at the positive electrode, the other at the negative.

Fig. 420.



For instance, water, consisting of the two gases oxygen and hydrogen, is thus decomposed.

In the bottom of the dish *D* (Fig. 420), partly filled with water, are fastened *p and n,* the platinum electrodes of a battery. Over these are placed two tubes, *O* and *H,* full of water. On closing the circuit, oxygen rises from *p* into *O,* and hydrogen from *n* into *H.*

Electrolysis is of the utmost importance in chemistry. Thus, the preceding experiment gives a correct analysis of water, and if oxygen had been previously unknown, would have been the means of its discovery. In this way were discovered several of the metallic elements.

735. Voltameter.—*The quantity of any compound decomposed in a given time is proportional to the strength of the current;* and hence by measuring the quantity of water decomposed in a given time, and reducing the resulting volume of mixed gases to a standard temperature and pressure, the strength of the current may be determined. An instrument for this purpose is called a *Voltameter.*

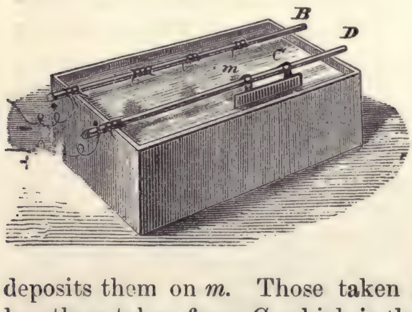
The objections to its use are that it gives the total of current during the time, but not the strength at any given instant, and hence the current might be very variable during the time of decomposition, and yet the instrument would give no indication of this fact; it cannot measure currents too weak to effect decomposition; and its results are affected by the acidity of the water, the size of the electrodes, and the distance between them.

736. Electro-plating.—Electrolysis is very important in its applications in the arts.

When a solution of a metallic salt is subjected to the action of the current, it is decomposed and a permanent film of the metal is deposited on any suitable material placed so as to receive it. The process is then called *electro-metallurgy,* or *electro-plating.*

The bath (Fig. 421) contains a saturated solution of blue vitriol (sulphate of copper). In this is suspended by wires from the metallic rod D a plate of copper C, and from B (also metallic) the cast of a medal m, which is to be coated with copper. Connect D with the positive electrode of a battery and B with the negative. The current passing through the solution removes from it particles of copper, and deposits them on m. Those taken from the liquid are replaced by others taken from C, which is thus gradually wasted away, and the solution is kept saturated.

FIG. 421



If the bath contains a solution of gold, and C is replaced by a piece of gold and m by a silver cup, the cup will be *electro-gilded. Electro-silvering* is an analogous process.

To produce in any case a firm and even coating, the process must be allowed to proceed *slowly* by the employment of a weak current. On a small scale a single cell is sufficient. In large establishments a magneto-electric machine turned by steam has been successfully and economically used.

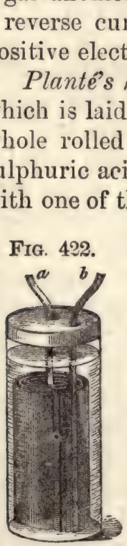
737. Electrotyping.—By taking proper precautions, the copper film deposited on *m* may be *removed,* and its surface will be found to present an exact fac-simile of the medal of which *m* is an impression.

Therefore if *m* is an impression, in wax or paper pulp, of the *type* from which a page is printed, the mould or impression having been coated with fine plumbago to render it a good conductor, the deposited copper may be removed, and having been stiffened by melted lead (or some alloy) poured over its under surface, it may be used in the printing-press instead of the type. It is then called an *electro-type plate,* and when not in use may be preserved indefinitely for succeeding editions, while the type of which it is a copy can be distributed and used for other purposes.

738. Secondary Battery.—In Art. 686 it was stated that single fluid cells become polarized by the accumulated hydrogen, and a reverse current is set up. If the platinum electrodes of a voltameter be disconnected from the battery, and connected with a galvanometer, the deflection of the needle indicates the flow of a reverse current, due to the condensation of oxygen upon the positive electrode and hydrogen upon the negative.

Planté's Secondary Cell consists of two sheets of lead, between which is laid a perforated sheet of non-conducting substance, the whole rolled up loosely, and placed in a jar containing dilute sulphuric acid as in Fig. 422. The strip of copper, *a,* is connected with one of the lead sheets and *b* with the other. If the poles of a battery of several Grove cells in series be connected with *a* and *b,* the sheet connected with the positive pole becomes coated with peroxide of lead while the surface of the other is deoxidized or reduced to the metallic state. In this polarized condition the cell is capable of giving a reverse current, which will flow until the surfaces of the lead sheets are reduced to identical chemical condition again. The electromotive force of such cells is about 2.5 volts, and the cells will retain their charge many days with but slight loss. As the Planté cell requires many chargings and reversals to reduce the lead surfaces to a spongy condition and produce its maximum efficiency, a modified form devised by Faure is more generally

FIG. 422.



used. In this cell the lead plates of the accumulator are coated with red lead at the outset, and cells thus prepared need but little "forming," as the process of charging and reversals is called.

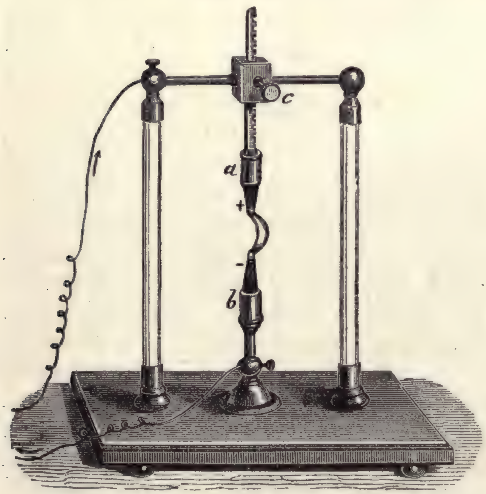
739. Medicinal Applications.—The shocks produced by the passage of interrupted currents through the system have already been alluded to. In certain ailments these shocks, when properly applied, have been known to produce beneficial results. On the other hand, great injury has resulted from their misapplication. Hence they should be employed as remedies only under the direction of a reliable physician.

The familiar medical magneto-electrical machine, which comes compactly stored in a box ten inches long and about four inches square at the end, does not differ essentially from Clarke's (Art. 726), except in lacking the *commutator*, so that its currents pass through the body alternating with each other in direction.

II. Applications of Electric Light and Heat.

740. Light by the Electric Current.—The electric light may be advantageously employed for brilliant illumination; also where a strong penetrating light is needed, as in light-houses, or for signals between ships. For exhibiting to an audience magnified images of small objects (as with the projecting microscope) it has no superior; and to the physical experimenter the various colors it assumes on passing through highly rarefied gases of different kinds are of great interest.

Fig 423.



To obtain the most brilliant effects carbon electrodes must be employed, and as these are constantly changing in length they must be kept at a uniform distance apart by machinery. The flame is not straight, but curved, as in Fig. 423, and is called the *voltaic arc*. To obtain it, the electrodes must first be made to

touch each other. With 92 Bunsen elements the light has been found to possess more than one-third the intensity of direct sunlight.

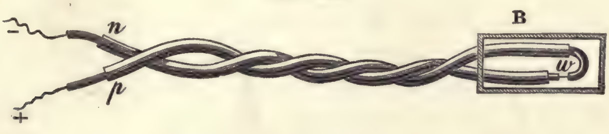
In the modern applications of electric lighting, two systems are advocated ; in one the voltaic arc is employed and in the other an incandescent conductor. In the *Jablochkoff candle* two rods of carbon placed parallel are separated by a layer of a fusible non-conductor, usually Kaolin, which melts at the point where the voltaic arc is maintained ; by occasionally reversing the current the waste of the carbons is equalized.

An *incandescent lamp* consists of a filament of carbon inclosed in a glass vacuum bulb. When the current traverses the filament it glows with great brilliancy. The current for lights of either system is now furnished by large dynamo machines, driven by steam-power, many lamps being included in the same circuit.

741. Heat by the Electric Current.—The heat as well as the light of the voltaic arc is intense. In the laboratory it is employed to deflagrate and volatilize refractory substances. When the lower electrode is hollowed out in the form of a cup, a piece of platinum (one of the least fusible of metals) placed in it is melted like wax in a candle, and a diamond, the hardest of known substances, is burnt to a black cinder. To produce either of these results, a battery of great power must be used.

Metals may also be deflagrated by being made part of the circuit in the form of very fine wires. They are thus employed to spring mines in time of war, or in blasting rocks. In Fig. 424, *B* is a box full of fulminating powder, and *w* is a very fine

FIG. 424.



platinum wire, about $\frac{3}{8}$ in. in length, fastened to *p* and *n*, which are insulated copper wires extending to a battery situated at any convenient distance. *B* and its contents are the fuse which is inserted in the powder to be fired. When a moderately strong current passes through *w*, it is heated sufficiently to ignite the fuse, and the powder explodes.

III. Mechanical Applications.

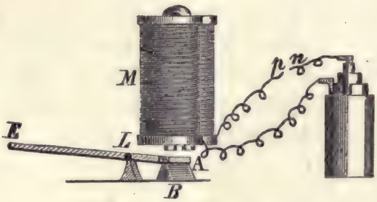
742. Made through the Medium of Induced Magnetism.—As has been seen in preceding experiments, magnetism

may be induced in a piece of steel or iron by the current passing through a circuit which is near it. Hence induced magnetism and its applications are results obtained *outside of the circuit.*

This magnetism may be utilized *directly.* For compass and galvanometer, needles are ordinarily made by placing a steel needle in a helix through which a current is sent (Art. 709).

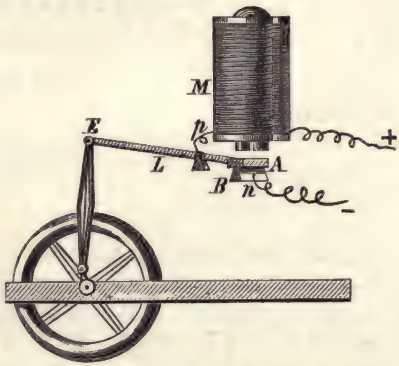
But its most numerous and important applications are in the way of *mechanical movements.* All these are modifications of the simple rising and falling of the armature of a U-magnet, mentioned in Art. 710. Thus, the armature *A* (Fig. 425) is limited in its fall by the metallic base *B*, so that it is within the influence of *M* the next time that it becomes a magnet. Hence, when the circuit is closed and broken at *n p*, the end *A* of the lever *L* rises and falls. It is evident that the corresponding motions of the end *E* may be applied in a variety of ways. A few of these are described in the following articles.

FIG. 425.



743. Electro-magnetic Engine.—*E* may be attached to a vertical arm, and that to the crank of a fly-wheel (Fig. 426), and the interruptions of the current may be made automatic by connecting *p* with *L*, and *n* with *B*. When *A* rests on *B* the circuit is closed; *M* becomes a magnet, and *A* is attracted by it; but as soon as *A* rises from *B* the circuit is opened, *M* no longer attracts it, and it falls back, only to close the circuit again and repeat the same movements as before. The tendency of this is to produce a rotary motion in the fly-wheel, and the apparatus involves the principle of a single-acting engine. With a second electro-magnet, and a somewhat different arrangement of parts, an actual double-acting engine may be constructed.

FIG. 426.



In another form of engine, a magnet is drawn into a hollow coil, and then, upon reversal of the current, repelled again, thus

acting like the piston of a steam-engine. In still another, a number of armatures are arranged upon a revolving wheel, like the floats upon a water-wheel, and these are attracted in succession by a series of electro-magnets arranged radially within the revolving armatures. This last form is practically applied, as in Edison's pen, to produce a very rapid reciprocating motion when but little power is required.

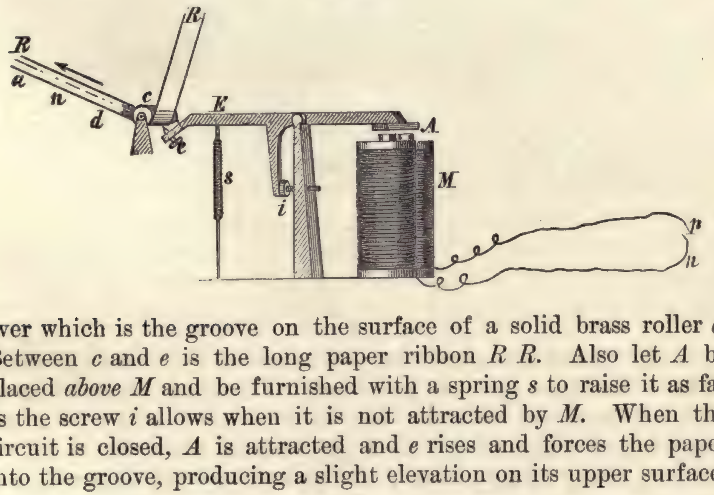
Various other forms of this engine have been coustructed and exhibited as curiosities, or used where expense was not regarded. But it cannot compete with the steam-engine as long as zinc and acids cost so much more than coal, unless a practical method shall be devised of transmitting a current generated by water-power, or other natural force, to long distances to be used in dynamo motors.

744. Electro-magnetic Telegraph.—To our countryman, Prof. S. F. B. Morse, is due the credit of the erection of the first telegraph line in the United States. It extended from Baltimore to Washington, and went into operation in 1844.

Communication in various ways by means of electricity between places a few miles apart was not unknown in Europe before that time, and several ingenious systems have appeared since; but the Morse system has been very generally preferred on account of its greater simplicity and efficiency, and it is now widely used in the United States and on the continent of Europe, where it is known as the *American system*. The principle of its operation is as follows:

Let *E* of Fig. 425 be furnished with a *style e* (Fig. 427) directly

FIG. 427.



over which is the groove on the surface of a solid brass roller *c*. Between *c* and *e* is the long paper ribbon *R R*. Also let *A* be placed *above M* and be furnished with a spring *s* to raise it as far as the screw *i* allows when it is not attracted by *M*. When the circuit is closed, *A* is attracted and *e* rises and forces the paper into the groove, producing a slight elevation on its upper surface.

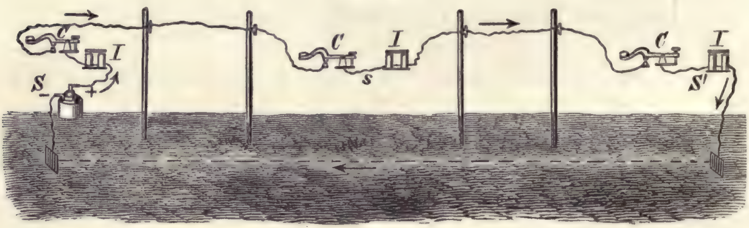
The ribbon is pulled along at a uniform rate in the direction of the arrow by clockwork (not shown in the figure), so that when the circuit remains closed for a little time, a *dash* is marked on the paper by *e*; when it is closed and instantly opened, the result is a *dot*—or rather a *very short dash.* *Spaces* are left between these whenever the circuit is opened. Combinations of these *dots, dashes,* and *spaces,* all carefully regulated in length, compose the letters of the alphabet. Spaces are also left between the letters, and longer ones between words.

By lengthening the circuit wire, it is evident that the person who sends the message at *n p,* and the one who receives it at *E,* may be miles apart, and the transmission will be almost instantaneous owing to the rapid passage of the current.

The essential parts of this system, or indeed of any system, are a *communicator* at *n p,* an *indicator* at *E,* and a *wire* extending from one to the other.

745. The Connecting Wire.—It was at first supposed that a complete metallic circuit was necessary, hence a return wire was employed. But this was rejected when it was found that the earth could be used as a part of the circuit, as shown in Fig. 428, in which the dotted line and arrow beneath the surface are not

FIG. 428.

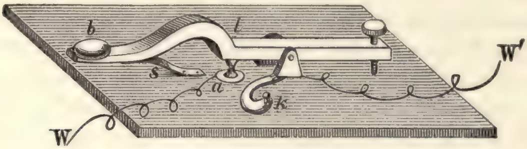


intended to convey the idea that a current actually flows from one earth plate to the other, but that a complete circuit is formed, the earth acting the part of an infinite reservoir of electricity. *S* and *S'* are the terminal stations, and *s* is one of the way stations which may occur anywhere along the line. At every station both a communicator, *C,* and indicator, *I,* are introduced into the circuit, so that messages can be both sent and received.

746. The Communicator.—This consists of a *lever, l* (Fig. 429), and *anvil, a,* both of brass, and insulated from each other. The anvil connects with the line wire *W,* and *l* with the rest of the circuit through *W',* and *W' W''* of the next figure. (See also Fig. 428.) The end of *l* is depressed by the finger of the

operator on the insulating button *b*, and is raised by the spring *s* when the pressure is removed. The former movement closes

FIG. 429.



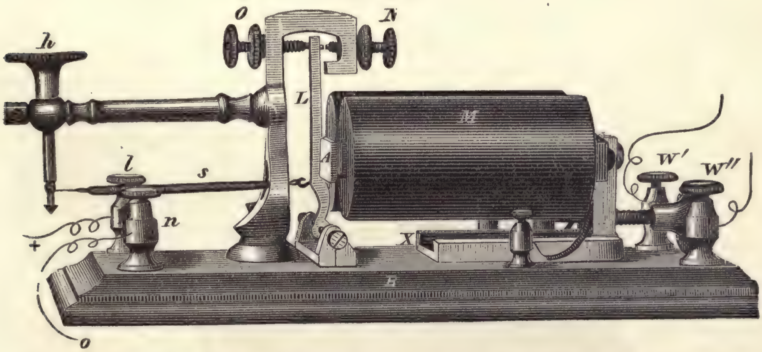
the circuit, the latter opens it, and by a succession of these the message is sent.

When the communicator is not in use, the brass bar *k* hinged to the base of *l* is pressed into contact with *a*. This closes the circuit for other stations on the line, and hence *k* is called the *circuit closer*. The whole apparatus is called *the key*.

747. The Indicator.—This consists of two parts, (1st) the *relay*, and (2d) either the *register*, or the *sounder*.

(1.) The *relay*, or the *relay magnet* (Fig. 430), consists of an *electro-magnet*, an *armature*, a *lever*, and a *spring*, the same as in Fig. 427, except that the electro-magnet is horizontal, and the other parts correspond in position. The tension of the spring *s*

FIG. 430.



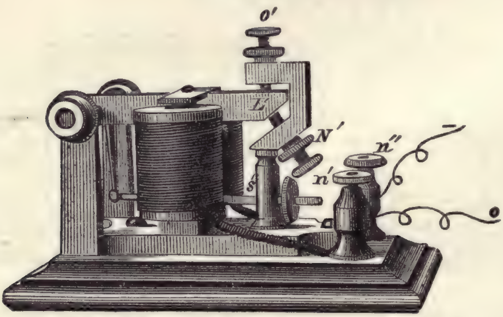
is regulated by the screw and milled head *h*, and *M* is adjusted by a similar screw (between *W′* and *W″* in the figure), which slides it along the grooved way *X*. One end of the coil wire passes out through *W′* to *W′* of Fig. 429. The other end connects at *W″* with one pole of the battery if it is at *S* (Fig. 428), with the earth if it is at *S′*, or with the line wire to the next station if it is at *s*.

The reason for introducing the relay is this: The current from the preceding station has become too feeble to cause inden-

tation of the paper by the style, and thus make a visible record, or even to produce a distinct sound of the armature upon the magnet for reading messages by the ear. The relay is therefore contrived for employing this feeble current to close and open the circuit of a *local battery*, whose current is powerful enough to deliver messages in either form, or even in both forms at once. All which the weak current of the distant battery has to do is to cause the armature A to move toward the magnet M till the top of L *touches* the screw N, and thus closes the circuit of the local battery. When the current ceases, a delicate spring, s, draws L back from contact with N, and breaks the circuit of the local battery. By the adjustments above described, the distance through which L moves, and the force of the spring s, may be made as small as the operator pleases.

(2.) The *second* part of the indicator is either a *register*, or a *sounder*, according as messages are to be addressed to the eye or to the ear. The register (Fig. 427) has been already described; the current of the local battery close at hand has force enough to cause visible indentations in the paper whenever the lever is drawn to the magnet; and this record can be read at any time subsequently. Within a few years a modified form of the register has come into use, and is called the sounder. In this the end of the lever L' (Fig. 431), instead of being furnished with a style, is made to strike against the two screws, N', O'. The downward

FIG. 431.



click is a little louder than the upward one, and so the beginning and end of each *dot or dash* are distinguished from each other. Many operators learn from the first to *read by the ear*, and have never used a register.

Whether a register or a sounder is employed, its coil wire is entirely distinct from the line wire, except on very short circuits, and belongs only to the local battery. The circuit of this battery may be traced (Figs. 430, 431) from the positive pole through

l, L, N, n, $o \ldots o$, n', the coil of the sounder, and n'', to the negative pole. The binding screws, l, n, n', n'', are connected with their respective levers, or contact screws, by insulated wires concealed in the bases. When A is attracted by M, L touches N, and the circuit is closed; when it is withdrawn by s, the circuit is opened, because O is insulated. Hence the motions of L and L' are simultaneous.

Since the relay is always in the main circuit, it communicates, by means of the local current, to the operator at whose station it is, all the messages sent between any two stations on the line, including those which he himself sends. Hence, if his own indicator does not operate while he is at work, he knows that his message is not passing over the line, owing to some break in the circuit.

748. Repeaters.—On a well insulated wire the weakness of the current at the distance of a few miles from the battery is mainly due to the *resistance* of the wire. The nature of this resistance is unknown, but it is subject to the same law as the *friction* of a fluid along the interior of a tube; it varies *directly* as the *length*, and *inversely* as the *diameter*. Hence telegraph wires of considerable thickness are employed, and even then, after a certain number of miles, varying according to strength of battery, insulation, etc., the current will not work even a relay.

A battery equivalent to 25 Daniell cells will work a relay through not more than about 100 miles of ordinary telegraph wire; before reaching that point, therefore, the wire is allowed to pass into the ground, and so complete the circuit.

To pass a message beyond the place at which the current will only work a relay, N' and L' are made parts of a new circuit called a *repeater*, which is closed and opened simultaneously with the preceding one by the motions of L', just as a local circuit is worked by L (Art. 747).

On the 28th of February, 1868, signals were sent through from Cambridge, Mass., to San Francisco, by the employment of thirteen repeaters. The time occupied by the signals in going and returning (making about 7,000 miles) was three-tenths of a second—allowance being made for the coil wires of the electro-magnets through which the current passed.

749. Resistance Coils.—In order to explain, in a very general way, the principle of *Duplex Telegraphy* it is necessary first to consider the practical measurement and distribution of resistances. The practical unit of resistance has been defined to be the ohm (Art. 690), and wire coils of resistance of a definite

number of ohms are furnished by makers of electrical apparatus. Fine German-silver wire is generally used because of its constancy under changes of temperature (Art. 690); and to prevent induction in the coil itself the insulated wire is doubled before being wound so that the two parts of the wire carry opposite currents, thus neutralizing each other's effect.

In Fig. 432 are represented, first in elevation and second in plan, two such coils of a set. *E*, *H*, and *M* are brass plates, each insulated from all others by being bedded in a plate of vulcanite, *A*, *B*. One end of the first coil is attached to *E* at *a*, the other end of the same coil being attached to *H* at *b*. The attachments of the second coil are evident from the figure. Two brass plugs, 1 and 2, may be inserted in the holes at the ends of the brass plates, thus making electrical connection between the plates *E*, *H*, and *M*. Let one wire of a battery be attached to *M* and the other, after passing through a galvanometer, be attached to *E*, then if the plugs be removed the current will flow from *M* through the coil to *H*, and thence through the connecting coil to *E*, and so through the galvanometer to the battery again, producing a certain deflection of the needle; if now the plugs be inserted the current will flow from *M* through the plug 2 to *H* and thence through plug 1 to *E*, and the lessened resistance, due to the removal of the two coils from the circuit, will be manifested by the greater deflection of the needle.

FIG. 432.

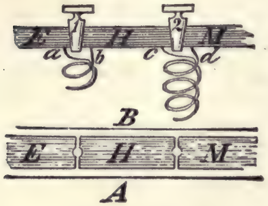
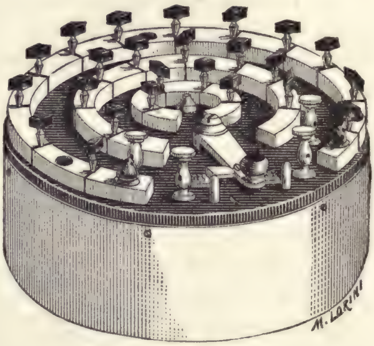


FIG. 433.

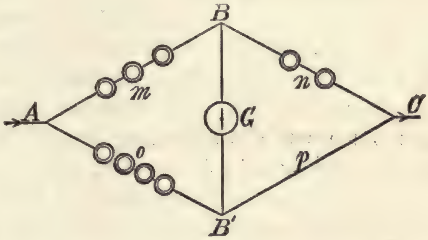


A set of Resistance Coils is shown in Fig. 433. A box may contain coils of the following convenient resistances in *ohms*: 1, 2, 2, 5, 10, 20, 20, 50, 100, 200, 200, 500, and so on to any desired total.

750. Wheatstone's Bridge.—If a current flows through a wire of uniform resistance, the potential at the end connected with the positive pole of the battery will be highest, and the fall in potential will be uniform to the negative pole where it will be least, so that when the current has traversed one-third of the

wire, that is to say, one-third of the resistance, the fall of potential will have been one-third of the total fall; in other words, the fall of potential is proportional to the resistance overcome. A constant current arriving at A (Fig. 434), will divide, one portion going through the wires m and n to C, and the other through o and p to C, at which point the currents again unite and form a single current as before. At A the potential of the current is less than when it left the battery pole, and the fall continues through both m and n to C, and also through o and p to C, at which point the two potentials are equal, the fall in these *arms*, as they are called, being proportional to the resistances which they offer. If the resistance of the arm m be 80 ohms, of the arm n 20 ohms, of the arm o 4 ohms, and of p 1 ohm, then, since the divided currents have the same potential on leaving A, and the same *lower* potential on arriving at C, it follows that $\frac{4}{5}$ of the total fall will have taken place at B on the upper circuit, and also $\frac{4}{5}$ of the same total fall at B' on the lower circuit; hence the potential at B is the same as that at B', and a galvanometer on a branch connecting these two points would show that no current flows between B and B'. Resistances and a galvanometer thus arranged were used by Wheatstone, and the device is commonly known as *Wheatstone's Bridge* or *Balance.*

FIG. 434.



751. Measurements of Resistance.—Suppose we wish to determine the resistance of one mile of a given wire. Let us arrange our *balance* so that the wire to be measured shall be the arm p, and suppose we have made $m = 100$ ohms and $n = 10$ ohms, and that we find it necessary to make $o = 110$ ohms to bring the needle of G to zero; then,

$$m : n :: o : p, \text{ or } p = \frac{n \times o}{m},$$

which gives $p = \frac{1100}{100} = 11$ ohms.

To measure the internal resistance of a cell, a number of resistance coils and a galvanometer are included in the circuit. If now a second cell, in all respects like the first, be joined with the first in multiple arc, the internal resistance will be only one-half as great as before, and added resistance must be introduced into the circuit to bring the needle back to its former reading. Call the

electromotive force of the cell E, its resistance R, the resistance of the external circuit in the first case r and in second r', and then, since the galvanometer showed the strength of the current to be the same in both cases we have, according to *Ohm's Law*,

$$\frac{E}{R + r} = \frac{E}{\frac{1}{2} R + r'},$$

from which we find

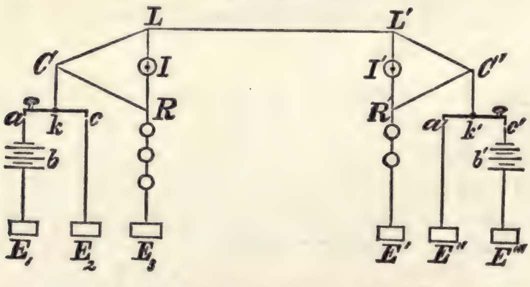
$$R = 2\,(r' - r).$$

To compare the electromotive force of a given cell with that of a standard cell, introduce into a circuit resistance coils and a galvanometer; first pass a current through from the standard cell, and so arrange the resistance that the galvanometer shall read any convenient number of degrees, which call d; then add resistance, say 100 ohms, so that the reading shall fall to d'; now substitute for the standard cell that whose relative electromotive force we wish to determine, and arrange the resistances so that the reading may be d, as before, and then add enough resistance, say 190 ohms, to bring the reading down to d' again. Now since the resistances which will reduce the galvanometer readings by the same amount are proportional to the electromotive forces we have, calling E the electromotive force of the standard cell and E' that of the other,

$$E : E' :: 100 : 190, \text{ or } E' = E\,\tfrac{190}{100}.$$

752. Duplex Telegraphy.—Let Fig. 435 represent two stations connected by the line wire $L\ L'$. $C\ L\ R$ is a Wheatstone

FIG. 435.



Bridge, modified to suit the conditions of the case, I the register or sounder, R resistance coils, k a key working upon the center and having forward and back contacts at a and c, b the battery, and E the earth connections. The same letters, accented, represent like parts at the second station. When not in use the keys make back contact by the action of a spring. The ratio of the resistance $C\ R$ and $R\ E_3$ is made equal to that of $C\ L$ and the line wire $L\ L'$, including the back contact earth connection at the

second station. If now, a' being closed, a be closed, a current will flow through a and k to C, where it will divide, one part going to earth through R and E_3, and the other through $L\ L'$. As the potentials were made equal at L and R, no current will pass through the indicator I; that part of the current which flows through $L\ L'$ divides at L', part going through $C'\ k'\ a'$ to E'', and part through I' (giving signal) and R' to E'. Thus the closing of a gives a signal at I' but none at I.

If now the second operator should close his key while a was closed, a current from b' would flow through c' and k' to C', where it would divide, part going to earth through R' and E' (joining the current already flowing through from $L\ L'$), and part would flow to L' and oppose the current from the other station; this opposing current will have the same effect as increased resistance in the line wire $L\ L'$, and hence the balance $C\ L\ R$ will be disturbed, the potential of L rising above that of R, and resulting in a current from L through I to R, giving a signal at I. Thus the register at each station will respond to the key of the other, and only to that, whether one or both operators be signalling.

The above explanation of the principle of this particular mode of sending simultaneous messages in opposite directions on a single wire, does not pretend to describe the *actual* arrangement of wires or earths in use. For a full description of the various modes of duplex and quadruplex telegraphy the student is referred to works on practical telegraphy.

753. Atlantic Telegraph Cable.—This cable stretches a distance of 3,500 miles, and from the nature of the case is a continuous wire, so that it cannot be advantageously worked by the Morse apparatus. The indicator employed is a sensitive galvanometer needle which is made to oscillate on opposite sides of the zero point by the passage through it of currents in opposite directions. But to reverse the direction of the current throughout the whole length of the cable is a slow process. *For the cable is an immense Leyden jar*, the surface of the copper wire (amounting to 425,000 sq. feet) answering to the *inner* coating, the water of the ocean to the *outer*, and the gutta-percha between the two to the *glass* of an ordinary jar. A current passing into it is therefore detained by electricity of the contrary kind induced in the water, and no effect will be produced at the further end until it is *charged.*

This very circumstance, at first considered a misfortune, is now taken advantage of in a very simple and ingenious manner to facilitate the transmission of signals. The current is allowed to pass into the cable till it is *charged*—then, *without breaking the circuit*, by depressing a key for an instant, a connection is made

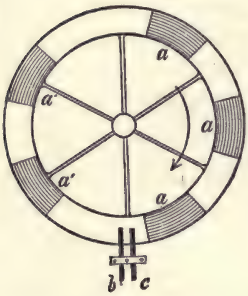
between it and a wire running out into the sea; that is, between the inner and outer coatings. *This partially discharges it,* and the needle at the other end is deflected. When the key is raised the discharge ceases, the current flows on as before, and the needle is deflected in the opposite direction.

It is said that after this plan was adopted, twenty words could be sent through the cable per minute, whereas only four per minute could be sent before. The greatest speed thus far attained on land wires is believed to have been the transmission in one instance of 1,352 words in thirty minutes between New York and Philadelphia in 1868.

754. Fire-Alarm Telegraph. — Recurring again to the standard (Fig. 425), the end E may be so connected with machinery as to cause the striking of a bell in a distant tower whenever the circuit is closed at n p. In our large cities boxes are placed at convenient points, each containing a crank, or lever, by which the circuit may be closed and the fire-bell rung.

Fig. 436 represents a device by which the number 32 may be signalled to a central station by setting free the wheel so that it may turn in the direction of the arrow. Strips, a a a and a' a', of insulating material are fastened upon the rim of a metallic wheel; the two wires, b and c, of the general circuit, press upon the metallic rim and the circuit is closed. If the wheel be turned the three strips a a a, passing under b and c, cause three successive interruptions of the circuit, followed after a longer interval by the two breaks due to a' a'. Thus, by previously arranged signals, the locality of the fire is immediately made known at the various engine-houses.

Fig. 436.

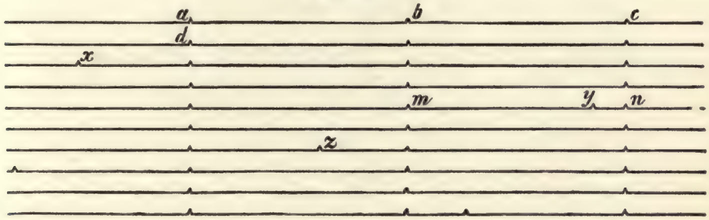


755. Chronograph.—This is used in observatories for recording the passage of stars across the meridian. Imagine the circuit of Fig. 427 to be closed and instantly broken again, by a clock pendulum at the end of every second. As the paper, R R, moves uniformly, dots are made on it at *equal distances from each other,* each of which distances, therefore, represents one second. The observer has a key, by which also he closes the circuit for an instant when a certain star passes the meridian. The dot thus made shows, by its situation between the two nearest second dots, at what fraction of the second the transit occurred.

In practice, however, the record is more conveniently made on

a large sheet of paper, which is wrapped tightly around a cylinder. The clock-work, which revolves the cylinder, also moves the recording pen in a line parallel to its axis. By these two motions, a spiral ink-line is traced on the paper. At the end of every beat of the observatory clock, the closing of the circuit gives the pen a momentary lateral movement, by which a slight notch is made in the line. A similar notch is made by the touch of the key, when the observer perceives the star on the meridian wire of the telescope. Fig. 437 represents a portion of the sheet after its removal

FIG. 437.



from the cylinder; *a*, *b*, *c*, *d*, &c., are the second marks; *x*, *y*, *z*, &c., are transit records. The ratio $m\,y : m\,n$ shows what fraction of the second $m\,n$ has elapsed when the transit *y* occurs.

APPENDIX.

APPLICATIONS OF THE CALCULUS.

I. Fall of Bodies.

1. Differential Equations for Force and Motion.—These are three in number, as follows:

$$1. \quad v = \frac{d\,s}{d\,t}.$$

$$2. \quad f = \frac{d\,v}{d\,t} = \frac{d^2\,s}{d\,t^2}.$$

$$3. \quad f\,d\,s = v\,d\,v.$$

These equations are readily derived from the elementary principles of mechanics. In Art. 6 we have $v = \frac{s}{t}$. Reducing the numerator and denominator to infinitesimals, v remains finite, and the equation becomes $v = \frac{d\,s}{d\,t}$; which is Equation 1st. Therefore, if the space described by a body is regarded as a function of the time, the *first* differential coefficient expresses the *velocity.*

Again (Art. 12), $f = \frac{v}{t}$, where f represents a *constant* force. Making velocity and time infinitely small, we get the intensity of the momentary force, $f = \frac{d\,v}{d\,t}$. But, by Equation 1st, $v = \frac{d\,s}{d\,t}$; $\therefore f = \frac{d^2\,s}{d\,t^2}$; which is Equation 2d. Hence we learn that the *first* differential coefficient of the *velocity* as a function of the time, or the *second* differential coefficient of the *space* as a function of the time, expresses the *force.*

Equation 3d is obtained by multiplying the 1st and 2d crosswise, and removing the common denominator.

We proceed to apply these equations to the preparation of formulæ for falling bodies.

2. Bodies falling through Small Distances near the Earth's Surface.—In this case, let the accelerating force, which

is considered *constant*, be called g. Then, by Eq. 2, $g = \frac{dv}{dt}$, $\therefore dv = g\,dt$. Integrating, we have $v = g\,t + C$. But, since $v = 0$ when $t = 0$, $\therefore v = g\,t$, and $t = \frac{v}{g}$, as in Art. 27.

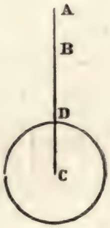
Again, substituting $g\,t$ for v in Eq. 1, $ds = g\,t\,dt$; and by integration, $s = \frac{1}{2}g\,t^2 + C$; but $C = 0$, for the same reason as before; $\therefore s = \frac{1}{2}g\,t^2$, and $t = \sqrt{\frac{2s}{g}}$.

Once more, equating the two foregoing values of t, we have $v = \sqrt{2\,g\,s}$, and $s = \frac{v^2}{2g}$.

If, in the equation, $s = \frac{1}{2}g\,t^2$, v be substituted for $g\,t$, we have $s = \frac{1}{2}v\,t$, or $v\,t = 2\,s$; that is, the acquired velocity multiplied by the time of fall gives a space twice as great as that fallen through (Art. 21).

3. Bodies falling through Great Distances, so that Gravity is Variable, according to the Law in Art. 16.—

FIG. 1.



Suppose a body to fall from A to B (Fig. 1), toward the centre C. Let $A\,C = a$; $B\,C = x$; $D\,C = r$, the radius of the earth.

The force f at B, is found by the principle, Art. 16,

$$x^2 : r^2 :: g : f = g\,r^2\frac{1}{x^2} = g\,r^2\,x^{-2}.$$

4. To find the Acquired Velocity.—Substitute $g\,r^2\,x^{-2}$ for f, and $a - x$ for s, in Equation 3d, and we have $g\,r^2\,x^{-2}\,.\,d\,(a - x) = v\,d\,v$; $\therefore$ by integration $\frac{1}{2}v^2 = \int -g\,r^2\,x^{-2}\,d\,x = g\,r^2\,x^{-1} + C$. But $v = 0$, when $x = a$; $\therefore C = -g\,r^2 a^{-1}$; and

$$\frac{1}{2}v^2 = g\,r^2\,x^{-1} - g\,r^2\,a^{-1};$$

$$\therefore \quad v^2 = \frac{2\,g\,r^2\,(a - x)}{a\,x};$$

$$\therefore \quad v = \left\{\frac{2\,g\,r^2\,(a - x)}{a\,x}\right\}^{\frac{1}{2}}$$

This is the general formula for the acquired velocity. If the body falls to the earth, $x = r$, and the formula becomes

$$v = \left\{\frac{2\,g\,r\,(a - r)}{a}\right\}^{\frac{1}{2}}$$

Again, if the body falls to the earth through so small a space that $\frac{r}{a}$ may be regarded as a unit, the formula reduces to

$$v = \{2g(a-r)\}^{\frac{1}{2}} = (2gs)^{\frac{1}{2}};$$

the same as obtained by other methods.

If a body falls to the earth from an infinite distance, it does not acquire an infinite velocity. For then, as we may put a for $a-r$,

$$v = \left\{\frac{2gr\,.\,a}{a}\right\}^{\frac{1}{2}} = (2gr)^{\frac{1}{2}} =$$

$$(2\,.\,32\tfrac{1}{6}\,.\,3956\,.\,5280)^{\frac{1}{2}} \text{ feet} = 6.95 \text{ miles}.$$

Therefore, the greatest possible velocity acquired in falling to the earth is less than *seven miles;* and a body projected upward with that velocity would never return.

5. To find the Time of Falling.—From equation first we obtain $dt = \frac{ds}{v}$; in this, substitute $d(a-x)$ for ds, and $\frac{\{2gr^2(a-x)\}^{\frac{1}{2}}}{(ax)^{\frac{1}{2}}}$ for v, as found in the preceding article; then

$$dt = \frac{(ax)^{\frac{1}{2}}\,.\,d(a-x)}{\{2gr^2(a-x)\}^{\frac{1}{2}}} = \left(\frac{a}{2gr^2}\right)^{\frac{1}{2}}.\frac{-x^{\frac{1}{2}}dx}{(a-x)^{\frac{1}{2}}};$$

$\therefore$ by integration $t = \left(\frac{a}{2gr^2}\right)^{\frac{1}{2}}.\int -x^{\frac{1}{2}}dx\,(a-x)^{-\frac{1}{2}}.$

By the formula in the calculus for reducing the index of x we obtain

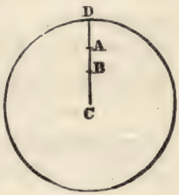
$$\int -x^{\frac{1}{2}}dx\,(a-x)^{-\frac{1}{2}} = (ax-x^2)^{\frac{1}{2}} - \frac{a}{2}\,vers^{-1}\left(\frac{2x}{a}\right) + C.$$

Now, when $t = 0, x = a$; $\therefore C = \frac{a\pi}{2}$;

hence, $t = \left(\frac{a}{2gr^2}\right)^{\frac{1}{2}}\left\{(ax-x^2)^{\frac{1}{2}} - \frac{a}{2}\,vers^{-1}\left(\frac{2x}{a}\right) + \frac{a\pi}{2}\right\}.$

6. Bodies falling within the Earth (supposed to be of uniform density), where Gravity Varies as the Distance from the Centre.—

FIG. 2.



Suppose a body to fall from A to B (Fig. 2); and let $DC = r$, $AC = a$, and $BC = x$. Then

$$r : x :: g : f = \frac{g}{r}x = \text{force at B}.$$

To find the velocity acquired.—By Eq. 3d,

$$v\,d\,v = f\,d\,s;\ \therefore v\,d\,v = \frac{g}{r}x\,.\,d\,(a-x) = -\frac{g\,x\,d\,x}{r};$$

$$\therefore \tfrac{1}{2}v^2 = -\frac{g\,x^2}{2\,r} + C;\ \text{but } v = 0 \text{ when } x = a;$$

$$\therefore C = \frac{g\,a^2}{2\,r},\ \text{and } \tfrac{1}{2}v^2 = \frac{g\,(a^2 - x^2)}{2\,r};\ \therefore v = \left\{\frac{g}{r}(a^2 - x^2)\right\}^{\frac{1}{2}}.$$

If the body falls from the surface to the centre, $x = 0$, and this formula becomes $v = (g\,r)^{\frac{1}{2}} = (32\frac{1}{6} \times 3956 \times 5280)^{\frac{1}{2}} = 25{,}904$ feet per second.

To find the time of falling.—By Equation 1st, and substitutions, we obtain $d\,t = \frac{d\,s}{v} = \frac{d\,(a-x)}{v} = -\frac{d\,x}{v} = \frac{-\,d\,x}{\left\{\frac{g}{r}(a^2 - x^2)\right\}^{\frac{1}{2}}}$

$$= \left(\frac{r}{g}\right)^{\frac{1}{2}} \times \frac{-\,d\,x}{(a^2 - x^2)^{\frac{1}{2}}};\ \therefore t = \left(\frac{r}{g}\right)^{\frac{1}{2}}\int\frac{-\,d\,x}{(a^2 - x^2)^{\frac{1}{2}}} = \left(\frac{r}{g}\right)^{\frac{1}{2}}\cos^{-1}\frac{x}{a} + C$$

When $t = 0, x = a, \frac{x}{a} = 1$, and the arc, whose cosine is $1 = 0$:

$$\therefore C = 0.\quad \therefore t = \left(\frac{r}{g}\right)^{\frac{1}{2}} \times \cos^{-1}\frac{x}{a}.$$

If the body falls to the centre, $x = 0$, and $t = \left(\frac{r}{g}\right)^{\frac{1}{2}} \times \frac{\pi}{2}$; in which a does not appear at all; so that the time of falling to the centre from any point within the surface is the same; and equals $\left(\frac{3956 \times 5280}{32\frac{1}{6}}\right)^{\frac{1}{2}} \times 1.570796$ in seconds, or 21m. 5.8s.

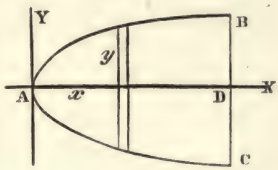
II. Centre of Gravity.

7. Principle of Moments.—In order to apply the processes of the calculus to the determination of the centre of gravity, the principle is used, which was proved (Art. 78), that if every particle of a body be multiplied by its distance from a plane, and the sum of the products be divided by the sum of the particles. the quotient is the distance of the common centre from the same plane. The product of any particle or body by its distance from the plane, is called its *moment* with respect to that plane.

8. General Formulæ.—Let $B\ A\ C$ (Fig. 3) be any symmetrical curve, having $A\ X$ for its axis of abscissas, and $A\ Y$, at right

angles to it, for its axis of ordinates. It is obvious that the centre of gravity of the line $B\,A\,C$, of the area $B\,A\,C$, of the solid of revolution around the axis $A\,X$, and of the surface of the same solid, are all situated on $A\,X$, on account of the symmetry of the figure. It is proposed to find the formula for the distance of the centre from $A\,Y$, in each of these cases. Let G in every instance represent the distance of the general centre of gravity from the axis $A\,Y$, or the plane $A\,Y$, at right angles to $A\,X$. The distance G would plainly be the same for the *half* figure $B\,A\,D$, as for the whole $B\,A\,C$; expressions may therefore be obtained for either, according to convenience.

FIG. 3.



1. *The line A B.*—Let x be the abscissa, and y the ordinate; then $(d\,x^2 + d\,y^2)^{\frac{1}{2}}$ is the differential of the line $A\,B$. For brevity, let $s =$ the line, and $d\,s$ its differential. If we now multiply this differential by its distance from $A\,Y$, $x\,d\,s$ is the moment of a minute portion of the line; and the integral of it, $\int x\,d\,s$, is the moment of the whole. Dividing this by the line itself, i. e. by s, we have $\frac{\int x\,d\,s}{s}$ for the distance G.

2. *The area B A D.*—The differential of the area is $y\,d\,x$; the differential of its moment is $x\,y\,d\,x$; hence the moment itself is $\int x\,y\,d\,x$; and the distance $G = \frac{\int x\,y\,d\,x}{\text{area}}$.

3. *The solid of revolution.*—The differential of the solid, generated by the revolution of $A\,B$ on $A\,X$, is $\pi\,y^2 d\,x$; the differential of its moment is $\pi\,x\,y^2 d\,x$; and the moment is $\int \pi\,x\,y^2 d\,x$; hence the distance $G = \frac{\int \pi\,x\,y^2 d\,x}{\text{solid}}$.

4. *The surface of revolution.*—The differential of the surface is $2\,\pi\,y\,d\,s$; the differential of its moment is $2\,\pi\,x\,y\,d\,s$; and therefore the moment is $\int 2\,\pi\,x\,y\,d\,s$; and the distance $G = \frac{\int 2\,\pi\,x\,y\,d\,s}{\text{surface}}$.

9. Application of Formulæ.—We proceed to determine the centre of gravity in a few cases by the aid of these formulæ:

1. *A straight line.*—Imagine the line placed on $A\,X$, with one extremity at the origin A. The moment of a minute part of it is $x\,d\,x$, and that of the whole is $\int x\,d\,x$, while the length of the whole is x; $\therefore\ G = \frac{\int x\,d\,x}{x} = \frac{\frac{1}{2}\,x^2 + C}{x} = \frac{1}{2}\,x$, as it evidently should

be. In all the cases considered here, $C = 0$, because the function vanishes when x does.

2. *The arc of a circle.*—By formula 1st we have $G = \frac{\int x\,d\,s}{s}$. but $d\,s = (d\,x^2 + d\,y^2)^{\frac{1}{2}}$; by the equation of the circle, $y^2 = 2\,a\,x - x^2$;

$$\therefore y\,d\,y = (a - x)\,d\,x; \therefore d\,y^2 = \frac{(a-x)^2\,d\,x^2}{y^2} = \frac{(a-x)^2\,d\,x^2}{2\,a\,x - x^2};$$

$$\therefore (d\,x^2 + d\,y^2)^{\frac{1}{2}} = \frac{a\,d\,x}{(2\,a\,x - x^2)^{\frac{1}{2}}};$$

$\therefore \frac{\int x\,d\,s}{s} = \int \frac{x}{s} \times \frac{a\,d\,x}{(2\,a\,x - x^2)^{\frac{1}{2}}} = \frac{a}{s}\int \frac{x\,d\,x}{(2\,a\,x - x^2)^{\frac{1}{2}}} = \frac{a}{s}\left\{ \text{vers}^{-1}\,x - (2\,a\,x - x^2)^{\frac{1}{2}} \right\} = \frac{a}{s}(s - y) = a - \frac{a\,y}{s} = a - \frac{a\,c}{t}$, if the arc is doubled and called t, and c (chord) put for $2\,y$. As $a - \frac{a\,c}{t}$ is the distance from the origin A, and a = radius of the arc; $\therefore$ the distance from the centre of the circle to the centre of gravity of the arc, is $\frac{a\,c}{t}$, which is a fourth proportional to the *arc*, the *chord*, and the *radius.*

When the arc is a semi-circumference, $c = 2\,a$, and $t = \pi\,a$; $\therefore$ the distance of the centre of gravity of a semi-circumference from the centre of the circle is $\frac{2\,a}{\pi}$.

3. *The area of a circular sector.*—Suppose the given sector to be divided into an infinite number of sectors; then each may be considered a triangle, and its centre of gravity therefore distant from the centre of the circle by the line $\frac{2\,a}{3}$. Hence the centres of gravity of all the sectors lie in a circular arc, whose radius is $\frac{2\,a}{3}$; so that the centre of gravity of the whole sector coincides with the centre of gravity of that arc. The distance of the centre of gravity of the arc from the centre of the circle, by the preceding case, is $\frac{2}{3}a \times \frac{2}{3}c \div \frac{2}{3}t = \frac{2\,a\,c}{3\,t}$, which is therefore the distance of the centre of gravity of the sector from the centre of the circle. When the sector is a semicircle the distance becomes $\frac{2\,a \times 2\,a}{3\,\pi\,a}$ $= \frac{4\,a}{3\,\pi}$

4. *The area of a parabola.*—The equation of the curve is

$$y^2 = p\,x, \text{ or } y = p^{\frac{1}{2}}\,x^{\frac{1}{2}};$$

therefore the formula 2 for moment,

$$\int x\,y\,d\,x = \int p^{\frac{1}{2}}\,x^{\frac{3}{2}}\,d\,x = \tfrac{2}{5}p^{\frac{1}{2}}\,x^{\frac{5}{2}}\,(+C=0);$$

but the area of the half parabola $= \frac{2}{3}\,p^{\frac{1}{2}}\,x^{\frac{3}{2}}$;

$$\therefore\ G = \tfrac{2}{5}\,p^{\frac{1}{2}}\,x^{\frac{5}{2}} \div \tfrac{2}{3}\,p^{\frac{1}{2}}\,x^{\frac{3}{2}} = \tfrac{3}{5}\,x.$$

To find the distance of the centre of gravity of the semi-parabola from the axis $A\,X$, proceed as follows: The differential of the area, as before, equals $y\,d\,x$; and the distance of its centre from $A\,X$ is $\frac{1}{2}\,y$. Therefore its moment with respect to $A\,X$ is $\frac{1}{2}\,y^2\,d\,x = \frac{1}{2}\,p\,x\,d\,x$; and the moment of the whole is $\int \frac{1}{2}\,p\,x\,d\,x = \frac{1}{4}\,p\,x^2$; ∴ the distance of the centre from

$$A\,X = \tfrac{1}{4}\,p\,x^2 \div \tfrac{2}{3}\,p^{\frac{1}{2}}\,x^{\frac{3}{2}} = \tfrac{3}{8}\,p^{\frac{1}{2}}\,x^{\frac{1}{2}} = \tfrac{3}{8}\,y.$$

5. *The area of a circular segment.*—The equation of the circle is, $y = (2\,a\,x - x^2)^{\frac{1}{2}}$. Therefore (formula 2),

$$\int x\,y\,d\,x = \int x\,(2\,a\,x - x^2)^{\frac{1}{2}}\,d\,x.$$

Add and subtract $a\,(2\,a\,x - x^2)^{\frac{1}{2}}\,d\,x$, and it becomes

$$\int a\,(2\,a\,x - x^2)^{\frac{1}{2}}\,d\,x - \int (a - x)\,(2\,a\,x - x^2)^{\frac{1}{2}}\,d\,x =$$

$$a\int y\,d\,x - \frac{(2\,a\,x - x^2)^{\frac{3}{2}}\,(a-x)\,d\,x}{\frac{3}{2}\,(2\,a - 2\,x)\,d\,x} = a\,.\,\text{area } A\,B\,D - \tfrac{1}{3}\,(2\,a\,x - x^2)^{\frac{3}{2}}.$$

$$\therefore\ G = a - \frac{(2\,a\,x - x^2)^{\frac{3}{2}}}{3\text{ area } A\,B\,D}.$$

When $x = a$, $G = a - \dfrac{4\,a}{3\,\pi}$; and the distance of the centre of gravity of a semicircle from the centre of the circle $= \dfrac{4\,a}{3\,\pi}$. When $x = 2\,a$, $G = a$, as it plainly should be.

6. *A spherical segment.*—The equation of the circle is $y^2 = 2\,a\,x - x^2$. Therefore (formula 3),

$$\int \pi x y^2 d\,x = \int \pi\,x\,d\,x\,(2ax - x^2) = \int 2a\pi x^2 dx - \int \pi x^3 dx = \tfrac{2}{3}a\pi x^3 - \tfrac{1}{4}\pi x^4;$$

$$\therefore\ G = \frac{\frac{2}{3}\,a\,\pi\,x^3 - \frac{1}{4}\,\pi\,x^4}{a\,\pi\,x^2 - \frac{1}{3}\,\pi\,x^3} = \frac{8\,a\,x - 3\,x^2}{12\,a - 4\,x}.$$

When $x = a$, $G = \frac{5}{8}\,a$; that is, the centre of gravity of a hemisphere is $\frac{5}{8}$ of radius from the surface, or $\frac{3}{8}$ of radius from the centre of the sphere. If $x = 2\,a$, $G = a$.

7. *A right cone.*—In this case $A\,B$ (Fig. 3), is a straight line, and its equation is $y = a\,x$, where a is any constant.

$$\therefore y^2 = a^2x^2; \therefore \int \pi xy^2 dx = \int \pi a^2 x^3 dx = \frac{\pi}{4}a^2x^4; \therefore G = \frac{\frac{1}{4}\pi a^2 x^4}{\frac{1}{3}\pi a^2 x^3} = \tfrac{3}{4}x.$$

Hence the centre of gravity of a cone is three-fourths of the axis from the vertex. See Art. 75.

8. *The convex surface of a right cone.*—The equation is

$$y = ax; \therefore dy^2 = a^2 dx^2; \text{ and } (dx^2 + dy^2)^{\frac{1}{2}} = (a^2 + 1)^{\frac{1}{2}} dx.$$

Therefore (formula 4),

$$\int 2\pi xy\, ds = \int 2\pi xy (dx^2 + dy^2)^{\frac{1}{2}} = \int 2\pi a x^2 (a^2+1)^{\frac{1}{2}} dx = \tfrac{2}{3}\pi a x^3 (a^2+1)^{\frac{1}{2}}$$

= the moment of the surface. The surface itself,

$$= \pi y (x^2 + y^2)^{\frac{1}{2}} = \pi a x^2 (a^2 + 1)^{\frac{1}{2}}. \quad \therefore G = \frac{\frac{2}{3}\pi a x^3 (a^2+1)^{\frac{1}{2}}}{\pi a x^2 (a^2+1)^{\frac{1}{2}}} = \tfrac{2}{3}x.$$

The centre of gravity of the convex surface of a right cone is on the axis, at a distance equal to two-thirds of its length from the vertex.

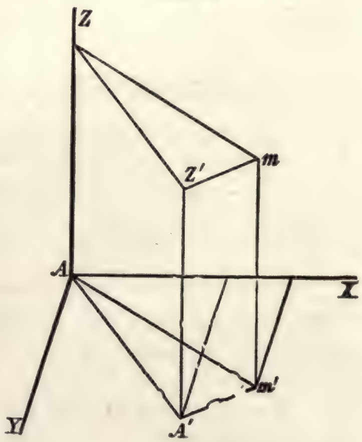
III. Centre of Oscillation.

10. To find the Moment of Inertia of a Body for any given Axis.—To render the formula $l = \frac{S(mr^2)}{Mk}$ suitable to the application of the calculus, we have simply to substitute the sign of integration for S, and dM for m, and we have

$$l = \frac{\int r^2 dM}{Mk}. \quad . \quad . \quad . \quad . \quad (1)$$

It is useful to know how to find the *moment of inertia* with respect to any axis by means of the *known moment* with respect to *another* axis parallel to it and passing through the centre of gravity of the body.

Fig. 4.



Let AZ (Fig. 4) be the axis passing through the centre of gravity of the body for which the moment of inertia is $\int r^2 dM$, and let $A'Z'$ be the axis parallel to it, for which the moment of inertia, $\int r'^2 dM$ of the same mass M, is to be determined. For every particle m of the body the corresponding value of Am' is $r^2 = x^2 + y^2$. In like man-

ner, if we denote the co-ordinates of A' by α and β, and the distance between the axes by a, we shall have $a^2 = \alpha^2 + \beta^2$. Now the distance of the particle m from $A'Z'$ is $r'^2 = (x - \alpha)^2 + (y - \beta)^2 = x^2 + y^2 + \alpha^2 + \beta^2 - 2\,\alpha\,x - 2\,\beta\,y = r^2 + a^2 - 2\,\alpha\,x - 2\,\beta\,y$; $\therefore$ $\int r'^2\,d\,M = \int r^2\,d\,M + a^2 \int d\,M - 2\,\alpha \int x\,d\,M - 2\,\beta \int y\,d\,M = a^2 M + \int r^2\,d\,M$, ... (2)
since $A\,Z$ passes through the centre of gravity of the body. Hence, *the moment of inertia of a body with respect to any axis is equal to the moment of inertia with respect to a parallel axis through the centre of gravity, plus the mass of the body multiplied by the square of the distance between the two axes.*

Put $C =$ the moment of inertia with respect to an axis through the centre of gravity; then the distance from the axis of suspension to the centre of oscillation, the axes being parallel, will be

$$l = \frac{C + a^2 M}{M\,k} \qquad (3)$$

11. Examples.—

1. Find the centre of oscillation of a slender rod or straight line suspended at any point.

Let a and b be the lengths on opposite sides of the axis of suspension, then by (1)

$$l = \frac{\int r^2\,d\,M}{M\,k} = \frac{\int r^2\,d\,r}{(a + b)\frac{1}{2}\,(a - b)} = \frac{2\,(a^3 + b^3)}{3\,(a^2 - b^2)} = \frac{2\,(a^2 - a\,b + b^2)}{3\,(a - b)}$$

between the limits $r = + a$ and $r = - b$.

If the rod is suspended at its extremity, $b = 0$, and $l = \frac{2}{3}\,a$. If it is suspended at its middle point, $a = b$ and $l = \infty$.

2. Find the centre of oscillation of an isosceles triangle vibrating about an axis in its own plane passing through its vertex.

Put b and h for the base and altitude of the triangle; then by

(1), $l = \dfrac{\int_0^h r^2 \cdot \frac{b}{h}\,r\,d\,r}{\frac{1}{2}\,b\,h \cdot \frac{2}{3}\,h} = \frac{3}{4}\,h.$

If the axis of suspension coincides with the base of the triangle, then $l = \dfrac{\int_0^h r^2 \cdot \frac{b}{h}\,(h - r)\,d\,r}{\frac{1}{2}\,b\,h \cdot \frac{1}{3}\,h} = \dfrac{h}{2}.$

3. Find the centre of oscillation of a circle vibrating about an axis in its own plane.

$$C = \int r^2\,d\,M = 2 \int x^2\,y\,d\,x = 2 \int x^2\,(R^2 - x^2)^{\frac{1}{2}}\,d\,x =$$
$$- x\,\frac{(R^2 - x^2)^{\frac{3}{2}}}{2} + \frac{\mathrm{R}}{2} \int (R^2 - x^2)^{\frac{1}{2}}\,d\,x.$$

Taking this integral between $x = -r$ and $x = +r$, we have

$$C = \frac{R^2}{2} \cdot \frac{\pi R^2}{2} = \frac{\pi R^4}{4}.$$

Substituting this value of C in (3) we have

$$l = \frac{\frac{\pi R^4}{4} + a^2 \pi R^2}{a \pi R^2} = a + \frac{R^2}{4a}.$$

4. Find the centre of oscillation of a circle vibrating about an axis perpendicular to it.

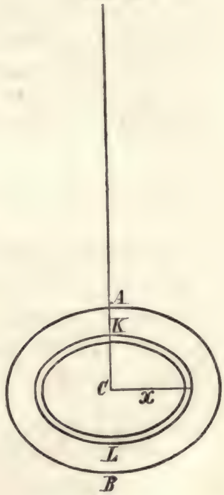
Let $K L$ (Fig. 5) be an elementary ring whose radius is x and whose breadth is $d x$; then

$$d M = 2 \pi x \, d x, \text{ and } C = \int_0^R x^2 . 2 \pi x \, d x$$

$$= \frac{\pi R^4}{2}; \therefore l = \frac{\frac{\pi R^4}{2} + a^2 \pi R^2}{\pi R^2 a} = a + \frac{R^2}{2a}.$$

As $a + \frac{R^2}{2a}$ is greater than $a + \frac{R^2}{4a}$, a circular pendulum will vibrate faster when the axis of suspension is in its plane, than when it is perpendicular to it.

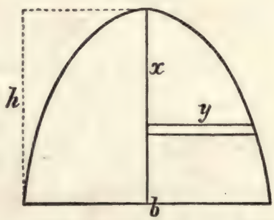
FIG. 5.



IV. Centre of Hydrostatic Pressure.

12. General Formula.—Let the surface pressed upon be plane and vertical; and let the water level be the plane of reference. Suppose the surface to have a symmetrical form with reference to a vertical axis, x, whose ordinate is y (Fig. 6). A horizontal element of the surface is $2\, y\, d\, x$, and (since the pressure varies as the depth) the pressure on that element $2\, x\, y\, d\, x$. Hence the whole pressure to the depth x is $\int 2\, x\, y\, d\, x = 2 \int x\, y\, d\, x$. The moment of the pressure on the element of surface is $2\, x^2\, y\, d\, x$; and the sum of all the moments to the same depth is $\int 2\, x^2\, y\, d\, x = 2 \int x^2\, y\, d\, x$. Therefore, putting p for the depth of the centre of pressure, $p = \frac{\int x^2\, y\, d\, x}{\int x\, y\, d\, x}$.

FIG. 6.



13. Examples.

1. *A rectangle.*—Let its height $= h$, and its base $= b$; then $2y$ everywhere equals b, and a horizontal element at the depth x is $b\,d\,x$, the pressure on it is $b\,x\,d\,x$, and the moment of that pressure is $b\,x^2\,d\,x$; $\therefore$ the depth of the centre of pressure $p = \frac{\int b\,x^2\,d\,x}{\int b\,x\,d\,x} = \frac{\frac{1}{3}\,b\,x^3 + c}{\frac{1}{2}\,b\,x^2 + c'}$. Since the pressure and area is each zero, when x is zero, c and c' both disappear, and $p = \frac{2}{3}\,x$, which for the whole surface becomes $p = \frac{2}{3}\,h$. That is, the centre of pressure on a vertical rectangular surface reaching to the water level, is two-thirds of the distance from the middle of the upper side to the middle of the lower.

2. *A triangle whose vertex is at the surface of the water, and its base horizontal.*—Let the triangle be isosceles, its height $= h$, and its base $= b$; then $h : b :: x : 2\,y = \frac{b}{h}\,x$. Therefore $p = \frac{\int \frac{b}{h}\,x^3\,d\,x}{\int \frac{b}{h}\,x^2\,d\,x}$ $= \frac{\frac{1}{4}\,x^4}{\frac{1}{3}\,x^3} = \frac{3}{4}\,x$; and for the whole height, $\frac{3}{4}\,h$.

If the triangle is not isosceles, it may be easily shown that the centre of pressure is on the line joining the vertex and the middle of the base, at a distance from the vertex equal to three-fourths of the length of that line.

3. *A triangle whose base is at the water level.*—Then $h : b : : h - x : 2\,y = b - \frac{b}{h}\,x$. Therefore the pressure is $\int - b\,x\,d\,x - \int - \frac{b}{h}\,x^2\,d\,x$, because $d\,x$ is negative. The moment of the pressure is $\int - b\,x^2\,d\,x - \int - \frac{b}{h}\,x^3\,d\,x$.

$$\text{Therefore } p = \frac{-\int b\,x^2\,d\,x + \int \frac{b}{h}\,x^3\,d\,x}{-\int b\,x\,d\,x + \int \frac{b}{h}\,x^2\,d\,x} = \frac{-\frac{1}{3}\,x^3 + \frac{1}{4\,h}x^4}{-\frac{1}{2}\,x^2 + \frac{1}{3\,h}\,x^3} =$$

$\frac{4\,h\,x^3 - 3\,x^4}{6\,h\,x^2 - 4\,x^3} = \frac{4\,h\,x - 3\,x^2}{6\,h - 4x}$; and, when $x = h$, this becomes $\frac{1}{2}\,h$. In general, the centre of pressure is at the middle of the line joining the vertex and the middle of the base.

4. *A parabola whose vertex is at the surface.*—As $y = p^{\frac{1}{2}}\,x^{\frac{1}{2}}$, therefore $p = \frac{\int x^2\,p^{\frac{1}{2}}\,x^{\frac{1}{2}}\,d\,x}{\int x\,p^{\frac{1}{2}}\,x^{\frac{1}{2}}\,d\,x} = \frac{\int x^{\frac{5}{2}}\,d\,x}{\int x^{\frac{3}{2}}\,d\,x} = \frac{\frac{2}{7}\,x^{\frac{7}{2}}}{\frac{2}{5}\,x^{\frac{5}{2}}} = \frac{5}{7}x$; or $\frac{5}{7}\,h$, for the whole area.

5. *A parabola whose base is at the surface.*—As $h - x$ is the depth of an element, $d\,x$ is negative. $p = \dfrac{-\int (h-x)^2 x^{\frac{1}{2}}\,d\,x}{-\int (h-x)\, x^{\frac{1}{2}}\,d\,x} =$

$$\frac{\int (h^2 x^{\frac{1}{2}}\,d\,x - 2\,h\,x^{\frac{3}{2}}\,d\,x + x^{\frac{5}{2}}\,d\,x)}{\int (h\ x^{\frac{1}{2}}\,d\,x - x^{\frac{3}{2}}\,d\,x)} = \frac{\frac{2}{3}\,h^2 x^{\frac{3}{2}} - \frac{4}{5}\,h\,x^{\frac{5}{2}} + \frac{2}{7}\,x^{\frac{7}{2}}}{\frac{2}{3}\,h\,x^{\frac{3}{2}} - \frac{2}{5}\,x^{\frac{5}{2}}} =$$

$\dfrac{\frac{1}{3}\,h^2 - \frac{2}{5}\,h\,x + \frac{1}{7}\,x^2}{\frac{1}{3}\,h - \frac{1}{5}\,x}$; and when $x = h$, the expression becomes

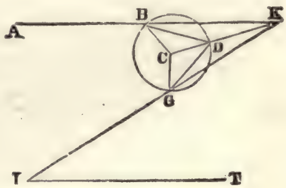
$$\frac{\frac{1}{3}\,h^2 - \frac{2}{5}\,h^2 + \frac{1}{7}\,h^2}{\frac{1}{3}\,h - \frac{1}{5}\,h} = \frac{4}{7}\,h.$$

V. Angular Radius of the Primary and Secondary Rainbow and the Halo.

14. The Primary Rainbow.—Since the primary bow is formed by those rays which, on emerging after one reflection, make the largest angle with the incident rays, proceed to find what angle of incidence will cause the largest deviation of the emerging rays.

In Fig. 7, let $x =$ angle of incidence; $y =$ angle of refraction; $z =$ angle of deviation; $n =$ index of refraction. Then, in the quadrilateral $B\,D\,G\,K$, $D\,B\,K = D\,G\,K = x - y$; angle at $D = 360 - 2\,y$; $\therefore K = z = 4\,y - 2\,x$;

Fig. 7.



$$\therefore \frac{d\,z}{d\,x} = \frac{4\,d\,y}{d\,x} - 2 = 0.$$

But $$\sin x = n \sin y;$$

$$\therefore \cos x\,d\,x = n \cos y\,d\,y, \text{ and } \frac{d\,y}{d\,x} = \frac{\cos x}{n \cos y}.$$

By substitution, $\dfrac{4 \cos x}{n \cos y} = 2.$

$$\therefore 2 \cos x = n \cos y;\ \text{and } 4 \cos^2 x = n^2 \cos^2 y.$$

But $$\sin^2 x = n^2 \sin^2 y;$$

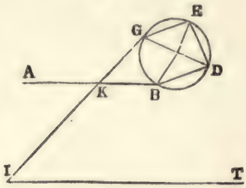
$$\therefore 3 \cos^2 x + 1 = n^2;\ \text{since } \sin^2 + \cos^2 = 1.$$

$$\therefore \cos x = \sqrt{\frac{n^2 - 1}{3}}$$

If 1.33 and 1.55, the values of n for extreme red and violet, be used in this formula, we obtain x, and therefore y and z, for the limiting angles of the primary bow.

15. The Secondary Bow.—To find the angle of minimum deviation. Using the same notation as before, we have in the pentagon $G\,E\,D\,B\,K$ (Fig. 8), $G = B = 180 - x + y$; $E = D = 2\,y$; $\therefore K = z = 180 + 2x - 6\,y$;

FIG. 8.



$$\therefore \frac{d\,z}{d\,x} = 2 - \frac{6\,d\,y}{d\,x} = 0.$$

$$\therefore \frac{6 \cos x}{n \cos y} = 2; \text{ and } 3 \cos x = n \cos y;$$

$$\therefore 9 \cos^2 x = n^2 \cos^2 y;$$

but $\quad \sin^2 x = n^2 \sin^2 y$;

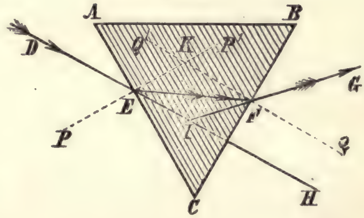
$$\therefore 8 \cos^2 x + 1 = n^2;$$

$$\therefore \cos x = \sqrt{\frac{n^2 - 1}{8}};$$

which, as before, will furnish z for each limiting color of the secondary bow.

16. The Common Halo.—Let $D\,E$ (Fig. 9) be the ray from the sun, and $F\,G$ the emergent ray. Let $D\,E\,p = x$; $K\,E\,F = y$; $K\,F\,E = x'$; $G\,F\,p = y'$; $I = z = x - y + y' - x'$. Now, $y + x' = p'\,K\,F = C = 60°$.

FIG. 9.



$$\therefore z = x + y' - C.$$

$$\sin x = n \sin y,$$

and $\sin y' = n \sin x'$;

$$\therefore x = \sin^{-1} (n \sin y),$$

and $\quad y' = \sin^{-1} (n \sin x') = \sin^{-1} \{n \sin (C - y)\}$.

By substitution,

$z = \sin^{-1} (n \sin y) + \sin^{-1} \{n \sin (C - y)\} - C$. Therefore z is a function of y; and, by differentiating, we have

$$\frac{d\,z}{d\,y} = \frac{n \cos y}{\sqrt{1 - n^2 \sin^2 y}} - \frac{n \cos (C - y)}{\sqrt{1 - n^2 \sin^2 (C - y)}} = 0.$$

$$\therefore \frac{n^2 \cos^2 y}{1 - n^2 \sin^2 y} = \frac{n^2 \cos^2 (C - y)}{1 - n^2 \sin^2 (C - y)};$$

$$\therefore \frac{1 - \sin^2 y}{1 - n^2 \sin^2 y} = \frac{1 - \sin^2 (C - y)}{1 - n^2 \sin^2 (C - y)};$$

$$\therefore (n^2 - 1) \sin^2 y = (n^2 - 1) \sin^2 (C - y);$$

$$\therefore y = C - y, \text{ and } y = \tfrac{1}{2}\,C;$$

$$\text{and } x' = \tfrac{1}{2}\,C.$$

Hence, the minimum deviation occurs when the ray within the crystal is equally inclined to the sides. Knowing n, the index of refraction for ice, x, and its equal, y', can be obtained, and then z, the deviation required.

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